547

THERMO DYNAMICS

UNIT - I INTRODUCTION BASIC CONCEPTS

THERMODYNAMICS :-

- → It is the science which deals with Energy.
- → Energy can be viewed as the ability to cause changes.
- → Thermodynamics includes the aspects of energy & energy transformation, energy conversions, including power generation, refrigeration. & energy savings.
- → It studies the relationship b/w the properties of system.
- → It is based upon the observation of common experience which have been formulated into thermodynamic laws.

Thermodynamics (Greek word) → Therme (heat)

Thermodynamics (Greek word) → Dynamics (power or force)

⊛ Thermodynamics is based on four laws namely zeroth, first, second & third law of thermodynamics.

- → Zeroth law builds the knowledge of thermal equilibrium and establishes the concept of temperature.
- → First law introduces the concept of internal energy.
- → Second law introduces the concept of Entropy.
- → Third law enables the evaluation of absolute entropy.

Applications of Thermodynamics :-

All activities in nature involve some interactions b/w energy & matter.

→ Human body, where heart is pumping the blood to all parts of human body. Various energy conversions occur in body cells & the heat generated by the body is constantly rejected to the environment.

→ Many house hold appliances are designed by using the principles of thermodynamics. Some of the examples are

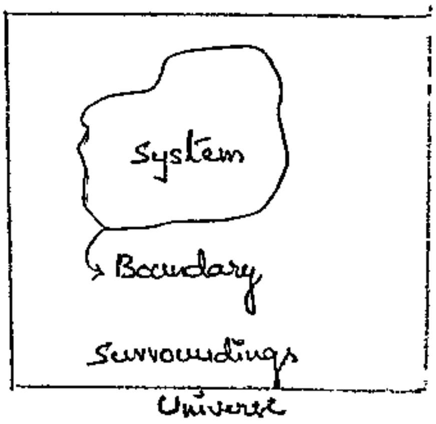
- o→ Heating & air conditioning systems
- o→ Refrigerators, humidifiers & pressure cookers
- o→ Water heater, Shower & the iron
- o→ Even the computer & T.V.

→ Thermodynamics plays a major part in the design & analysis of

- o→ Automotive engines
- o→ Rockets & jet engines
- o→ Conventional or nuclear power plants & Solar collectors
- o→ Design of vehicles from car to aeroplanes.
- o→ Design of turbines
- o→ In chemical process plants & many industrial applications etc.

SYSTEM

A system is defined as a quantity of matter or a region in space chosen for study.



Surroundings :-

The mass or region outside the system is called surroundings.

Boundary :- The real or imaginary surface that seperates the system from its surroundings is called the boundary. It may be fixed or movable.

Boundary has zero thickness, and thus it can neither contain any mass nor occupy any volume in space.

Universe :-

System & surroundings together comprises as Universe.

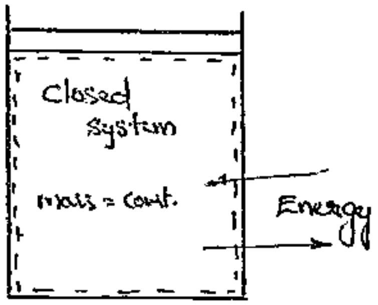
Universe = System + Surroundings.

* In order to understand this concept, best example is piston & cylinder arrangement. It has 3 fixed & one movable boundary.

Types of Systems :-

① closed system ② open system ③ Isolated system.

① Closed systems :- In this system, mass remains constant within the boundary of the system & energy



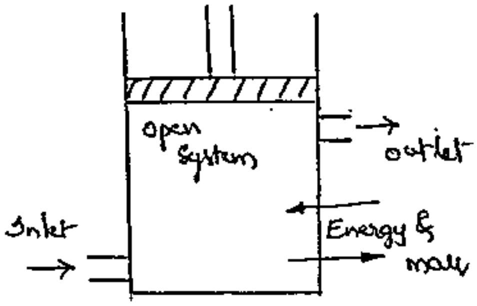
may transfer across its boundary.

Ex:- Gas contained in piston cylinder arrangement with non-flow process.

In this case when the gas is heated, no mass transfers across the boundary, but the volume may get changed. So energy transfer takes place.

② Open System:- (Control volume)

Both mass & energy transfer across the boundary.



Ex:- Compressor, Gas turbines, engines etc.

③ Isolated System:-

Neither mass nor energy get transfer across its boundaries.

Ex:- A fluid enclosed in a perfectly insulated closed vessel. (thermoflask).

A system may be classified as
- Homogeneous
- Heterogeneous

(i) Homogeneous System:- If the system consists of single phase it is called as homogeneous system. It may be either solid, liquid or gaseous state.

Ex:- Mixture of gases, water + nitric acid

(ii) Heterogeneous System:- If the system consists of more than one phase it is called heterogeneous system.

Ex:- water & steam, ice & water, water & oil etc.

Properties of a System :-

Any measurable characteristics which describes a physical condition or state of a system or state of a body is known as a property. Some of the observable characteristics such as pressure, temp, volume etc.

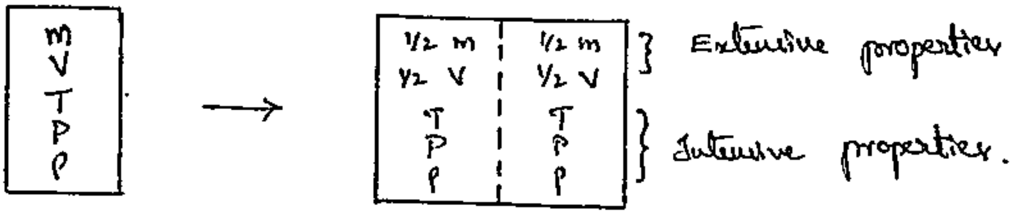
Types of Properties :-

→ Intensive properties

→ Extensive properties.

(i) **Intensive properties** :- These properties are those that are independent of the mass of a system such as temperature, pressure & density.

(ii) **Extensive properties** :- These properties depends on mass of the system. such as total volume, total mass & total momentum; enthalpy, entropy etc.



An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition. Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.

Note:- Extensive properties per unit mass are called specific properties.

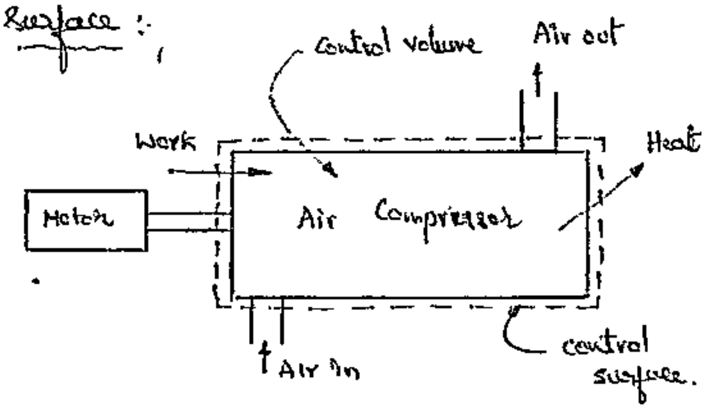
ex :- $v = V/m$ etc.

Difference b/w closed system & Open system :

Closed System	Open System
① Only energy crosses the boundary.	① Both mass & energy crosses the boundary.
② System contains fixed or constant amount of mass.	② System contains varying amount of mass.
③ It is not very complicated.	③ It is complicated due to the movement of fluids.
④ Boundary of the system doesnot allow the matter to flow in or out of the system.	④ Boundary of the system allows the matter to flow in or out of the system.
⑤ Oftenly used in analysis.	⑤ Used in most of applications.
⑥ Ex: Piston cylinder arrangement without valves.	⑥ Ex:- Engines, compressor, turbines, heaters, powerplants etc.

Control volume & Control surface :

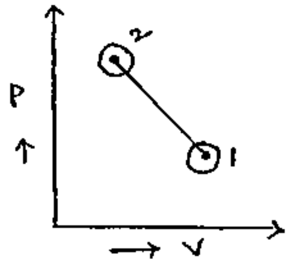
For thermodynamic analysis an open system (compressor) is considered.



If the volume of a system under study remains constant then that volume is called control volume. The control volume is surrounded by a control surface. Matter as well as energy crosses the control surface.

State :- State is the condition of a system at an instant of time described by its properties such as pressure, temp, density etc.

Points ① & ② in the figure shows two different states of a system.



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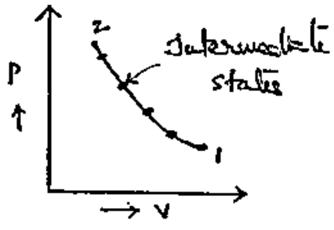
Path :- A thermodynamic system passing through a series of states constitutes a path. (or) It is a line through which system passes when it undergoes a change in state.



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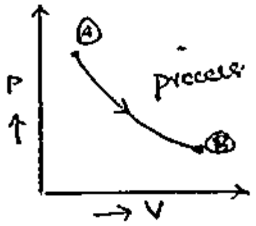
Process :- A process is defined as a transition in which a system changes from initial state to a final state.

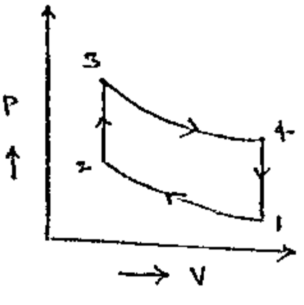


ⓒ

Ex:- For example state A is the initial state of the system. Due to expansion of the system the final state is B. The line AB is the process that has taken place. as shown in the figure ⓒ

Thermodynamic Cycling (or) Cyclic process :-

If a system undergoes a number of different processes and finally reaches to its initial state by forming a complete cycle then it is known as Cyclic process.



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Thermodynamic Equilibrium :-

A system is said to be in a state of thermodynamic equilibrium if there is no change in any property at all points in a system.

A system will be in a state of thermodynamic equilibrium if the following equilibrium are satisfied.

① Mechanical Equilibrium :- A system is said to be in a state of mechanical equilibrium if there no unbalanced forces within the system and or b/w system & surroundings. [pressure should be constant]

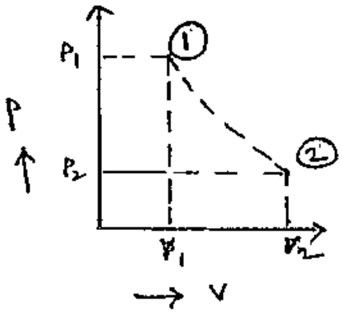
② Chemical Equilibrium :- There should not be any chemical reaction within the system and b/w system & surroundings.

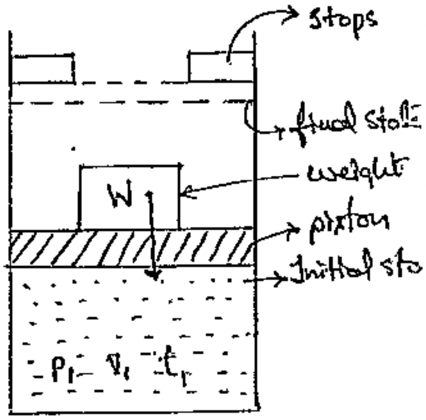
③ Thermal Equilibrium :- If there exists an uniformity of temperature (or) (Equality of temperature) throughout system or b/w the system & surroundings.

④ Electrical Equilibrium :- If there exists an uniformity of electrical potential through the system.

Quasi - Static Process :- (Quasi - Equilibrium process)

Let us consider a system of gas contained in a cylinder. The system initially is in an equilibrium state, represented by the properties p_1, V_1, t_1. The weight on the piston just balances the upward force exerted by the gas. If the weight is removed, there will be an unbalanced force b/w the system & surroundings and under gas pressure, the piston will move up till it hits the stops. The system again comes to an equilibrium state, being described by the properties P_2, V_2, t_2. But the intermediate states passed through by the system are non-equilibrium states which cannot be described by thermodynamic coordinates. Figure shows point ① & ② as the initial & final equilibrium states joined by a dotted line, which has got no meaning otherwise.





Such A process, which is but a locus of all the equilibrium points passed through by the system, is known as a quasi-static process, quasi means 'almost'. Infinite slowness is the characteristic feature of a quasi static process. A quasi-static process is thus a succession of equilibrium states. A quasi-static process is also called as reversible process.

MICROSCOPIC & MACROSCOPIC APPROACHES :-

Let us consider a gas filled in the container which is containing number of molecules. These molecules will have certain characteristics like position, momentum, velocity, pressure etc.

The analysis of thermodynamic systems can be explained in two ways.

→ Microscopic view

→ Macroscopic view.

Microscopic Approach :-

This approach considers a system being composed of large no. of particles known as molecules. Each molecule is having different energy & velocity.

These values constantly changes with time as the molecules collide with one another. So the microscopic approach is considered with structure of matter hence it is also called as Statistical thermodynamics.

Ex:- velocity, K:E, momentum etc.

Macroscopic Approach :-

A macroscopic approach is concerned with certain quantity of matter by taking into account which is going at guage level or overall level behaviour. Hence it is refferred as classical thermodynamics.

Ex:- pressure, volume, temperature etc.

If a pressure gauge is fitted to the gas filled container the pressure can be measured at any instant.

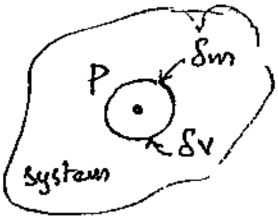
It is very difficult to measure the physical parameters such as pressure, temperature, volume, impulse etc., with microscopic approach.

MICROSCOPIC VIEW	MACROSCOPIC VIEW
① Every molecule in the matter have its own properties & they move with independent velocities.	① It is assumed that all the molecules behave in the same manner in the system.
② Large no. of variables are required for describing a system.	② Only few variables are required to describe the system
③ Measurement of variable is very difficult.	③ The properties are easily measured at any point of time with the help of guages.
④ Time required for measuring the variables is more.	④ Time required for measuring the variables is less.
⑤ Both mathematical & statistical methods are required for describing a system.	⑤ Simple mathematical formulae are sufficient for analysis.
⑥ It explains the structure of matter.	⑥ It don't explain the structure of matter.
⑦ It is limited to few applications of engineering problems.	⑦ It provides an easy way to the solution of engineering applicati
⑧ It is also known as statistical thermodynamics.	⑧ It is also known as classical thermodynamics.

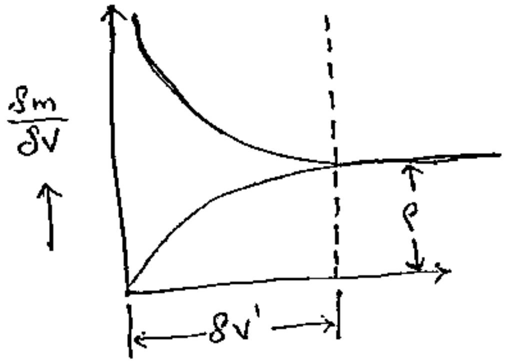
Concept of Continuum :-

It is very much useful in classical thermodynamics. From the continuum point of view, the matter is seen as being distributed through space & not as in the particle view, localised. Matter exists in big chunks having mass, energy & momentum, as do particles, but it also possesses some additional continuum characteristics such as volume, density & temp. etc.

In otherwords continuum idealisation treats the substance as being continuous disregarding the action of individual molecules.



Let us consider the mass δm in a volume δv surrounding the point P. The ratio $\delta m/\delta V$ is the avg mass density of the system within the volume δV. We suppose that at first δV is rather large, and is subsequently shrunk about the point P. If we plot $\delta m/\delta V$ against δV, the avg density tends to approach an asymptote as δV increases. However when δv becomes so small as to contain relatively few molecules, ~~pass~~ ~~into~~ & ~~out~~ the avg density fluctuates substantially with time as molecules pass into & out of the volume in random motion & so it is impossible to speak of a definite value of $\delta m/\delta V$. The smallest volume which may be regarded as continuous is $\delta V'$.



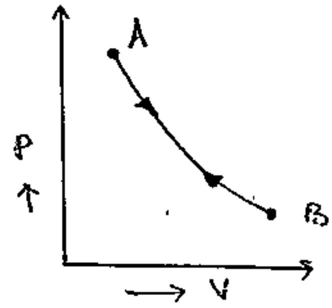
Mathematically, density at a point may be expressed as,

$$\rho = \lim_{\delta V \to \delta V'} \frac{\delta m}{\delta V} \quad kg/m^3$$

where $\delta V'$ is the smallest volume about point P in which the fluid can be called in continuum.

REVERSIBILITY (or) IDEAL PROCESS :-

A reversible process is one which is performed in such a way that at conclusion of the process, both the system & surroundings may be restored to their initial states, without producing any changes in the rest of the universe.



Let the state of a system be represented by A & let the system be taken to state B by following the path A-B. If the system & also the surroundings are restored to their initial states & no change in the universe is produced, then the process A-B will be a reversible process.

A reversible process is carried out infinitely slowly with an infinitesimal gradient, so that every state passed through by the system is an equilibrium state. So a reversible process coincides with a quasi-static process.

Ex:-
→ Frictionless relative motion
→ Electrolysis process
→ Polytropic expansion or compression of a fluid.
→ Isothermal expansion or compression.

Irreversible Process (Irreversibility) :-

Any natural process carried out with a finite gradient is an irreversible process. A reversible process which consists of a succession of equilibrium states, is an idealized hypothetical process, approached only as a limit. It is said to be an asymptote to reality. All spontaneous processes are irreversible. Time has an important effect on reversibility. If the time allowed for a process to occur is infinitely large, even though the gradient is finite, the process becomes reversible. However, if this time is squeezed to a finite value, the finite gradient makes the process irreversible.

Ex :-

→ Fluid flow with friction

→ combustion of air & fuel.

→ Plastic deformation

→ Heat transfer ... etc.

Causes of Irreversibility :-

The irreversibility of a process may be due to either one or both of the following.

→ Lack of equilibrium during the process.

→ Involvement of dissipative effects.

Irreversibility due to lack of equilibrium :-

The lack of equilibrium (mechanical, thermal or chemical) b/w the system & its surroundings, or b/w two systems or two parts of the same system, causes a spontaneous change which is irreversible.

The following are specific examples in this regard.

ⓐ Heat transfer through a finite temp difference.

ⓑ Lack of pressure equilibrium within the interior of the system or b/w the system & the surroundings.

ⓒ Free expansion.

Irreversibility due to Dissipative Effects:-

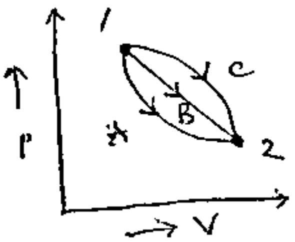
The irreversibility of a process may be due to the dissipative effects in which work is done without producing an equivalent increase in the kinetic or potential energy of any system. The transformation of work into molecular internal energy either of the system or of the reservoir takes place through the agency of such phenomena as friction, viscosity, inelasticity, electrical resistance & magnetic hysteresis. These effects are known as dissipative effects & work is said to be dissipat

Ex:- ⓐ Friction

ⓑ Paddle-wheel work transfer

ⓒ Transfer of electricity through a ~~reservoir~~. resistor.

Point function & path functions:-



→ Thermodynamic properties are the point function for a given state where there is one & only one value for each property. This can be located on a graph with a point. This is also known as state functions. These are exact differentials.

Ex: pressure, vol, temp etc.

$$\int_1^2 dp = P_2 - P_1 \; ; \; \int_1^2 dV = V_2 - V_1 \; ; \; \int_1^2 dT = T_2 - T_1$$

→ Path functions are certain quantities which cannot be located on a graph by a point. These are generally given by area under the curve. These are inexact or partial differentials.

Ex:- Heat, work, ~~energy~~ etc.

$\int_1^2 \delta Q \neq Q_2 - Q_1$: $\int_1^2 \delta w \neq w_2 - w_1$

Point function	Path function
① change in function depends only on initial & final state of a process ie., point function	① The change in a function depends on a path followed by a system ie., path function.
② Function does not depend on path followed by the system.	② Function depends on path followed by the system..
③ Differentials are exact & perfect.	③ Differentials are inexact (or) partial
④ pressure, vol, temp etc.	④ Ex: work, energy etc.

Work :-

Work is the energy transferred (with transfer of mass) becoz of a property difference (other than temp) that exists b/w the system & surroundings.

work = force × displacement N-m (or) Joules.

Energy:-

Energy is the capacity to do work. In other words, energy is defined as capacity to exert a force through a distance.

It can be classified into

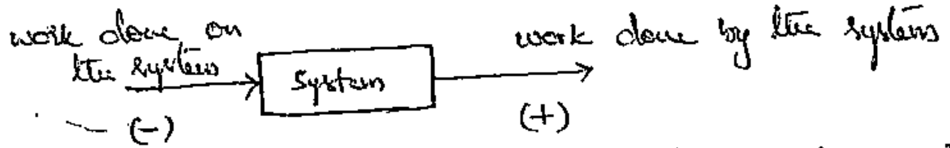
→ Stored energy (PE, KE, IE etc)
→ Energy in transit. (Heat, work, electricity etc)

Energy possessed by a system when it is static is called energy in state ie., stored energy. It is also known as internal energy.

Energy in transition are mainly

ⓐ Work transfer
ⓑ Heat transfer

Work transfer :-



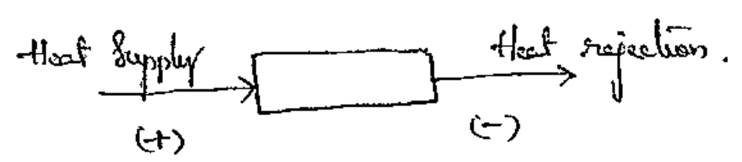
Work transfer from the system to the outside is positive ie., power producing systems.

Ex:- Power plants, automobiles, ships, aeroplanes, rockets etc.

Work transfer from outside to the system is negative ie., power consuming systems.

Ex:- pumps, motors, iron box, freezers, air conditioners, compressors, etc

Heat transfer :-

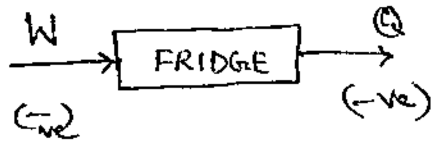


Heat transfer from the system to outside is negative.

Ex:- Air conditioners, refrigerators etc.

Heat transfer from outside to system is positive.

Ex:- Heat given to a boiler drum, heat given inside the engine of an automobile etc.



Energy Vs Heat :-

Energy	Heat
① It is the capacity to do work (or) exert the force through a distance.	① Heat is one form of energy that exists due to temperature difference b/w system & surroundings.
② It is available in different forms. It changes from one form to another. ex: Mechanical to electrical	② It transfer from high temp to low temp system without mass transfer.
③ Energy is divided into two types. ⓐ Internal energy ⓑ Energy in transition	③ Heat belongs to energy in transition.
④ Energy is present in the system & at the boundary of the system.	④ Heat is present only at the boundary of the system.

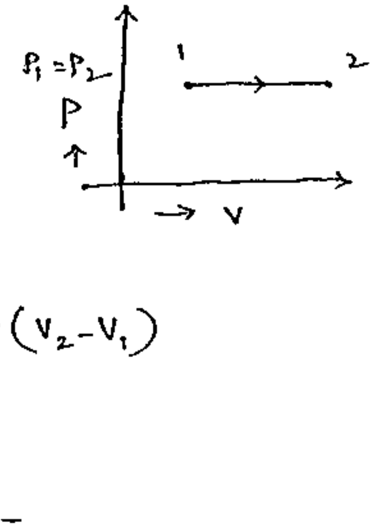
Types of TD Process :-

Process	law	Relation
① Const. pressure process	$P = C$	$P_1 = P_2 = P_3$ at all state points
② Const. vol. process	$V = C$	$V_1 = V_2 = V_3$, at all state points
③ const. temp process	$T = C$; $PV = C$	$P_1V_1 = P_2V_2$ .....
④ Adiabatic process	$PV^{\gamma} = C$	$P_1V_1^{\gamma} = P_2V_2^{\gamma} = P_3V_3^{\gamma}$ ....
⑤ Polytropic process	$PV^{n} = C$	$P_1V_1^{n} = P_2V_2^{n}$ ......

Workdone is given by $W = \int p.dv$

Derivation for work :

① Const. Pressure process (Isobaric) :-



$P_1 = P_2$

$W = \int p\,dv$

$W_1 = \int_1^2 p.dv$

$= P\int_1^2 dV = P\left[V\right]_1^2 = P(V_2 - V_1)$

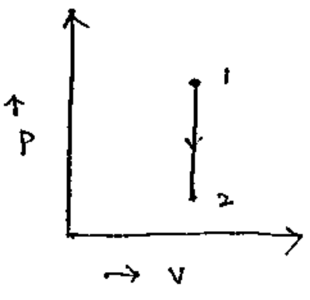
$W = P_1(V_2 - V_1)$

② constant volume process (Isochoric) :-

$V_1 = V_2$

$W = \int p.dv$

$W_{1-2} = \int_1^2 p.dv = P\int_1^2 dv = P\left[V\right]_1^2$



$= P(V_2 - V_1) = P(V - V) = 0$

$W = 0$ [work done by a closed system in a constant vol. process is zero]

③ Constant temperature process (Isothermal) :- $[T_1 = T_2 = T_3 = C]$

$P_1V_1 = P_2V_2 \ldots\ldots = C$

$PV = C$ (or) $P = \frac{C}{V}$

log = $\log_{10}(x)$
ln = $\log_e(x)$

$$W = \int_1^2 P \cdot dV$$

$$= R_o \int_1^2 \frac{c}{V} dV = c\int_1^2 \frac{1}{V} \cdot dV$$

$$= c\,[\log V]_1^2 = c\,[\log V_2 - \log V_1]$$

$$= c \log\left(\frac{V_2}{V_1}\right)$$

$P_1V_1 = P_2V_2$

$\frac{P_1}{P_2} = \frac{V_2}{V_1}$

$$\boxed{W = PV \log\left(\frac{V_2}{V_1}\right)} \quad \text{(or)} \quad \boxed{W = PV \log\left(\frac{P_1}{P_2}\right)}$$

① A mass of 1.5 kg of air is compressed in a Quasi-static process from 0.1 MPa to 0.7 MPa for which $PV = C$ i.e., isothermal process. The initial density of air is 1.16 kg/m³. Find the workdone by the piston to compress the air.

Sol. Given :

$m = 1.5$ kg
$P_1 = 0.1$ MPa
$P_2 = 0.7$ MPa
$\rho_1 = 1.16$ kg/m³

req. : workdone = ?

$PV = C$

$$\text{Density} = \frac{m}{V} \Rightarrow \rho_1 = \frac{m}{V_1} \Rightarrow 1.16 = \frac{1.5}{V_1}$$

$$V_1 = 1.293 \text{ m}^3$$

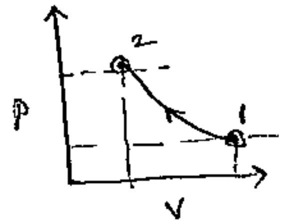
$\therefore P_1V_1 = P_2V_2$

$$V_2 = \frac{P_1V_1}{P_2} = \frac{0.1 \times 1.293}{0.7}$$

$$W = P_1V_1 \ln\left(\frac{P_1}{P_2}\right) = 0.1 \times 1.293 \overset{\ln}{\cancel{\log}}\left(\frac{0.1}{0.7}\right) \times 10^3$$

$$= -252.38 \text{ kJ}$$

Compression process:-



In compression process pressure increases & volume decreases. Movement of piston in upward direction increases the pressure of gas acting on piston.

In compression process $\frac{V_2}{V_1} < 1$

$\therefore$ work = Negative i.e., workdone on the system.

Expansion process:-

In this process pressure decreases & volume increases.

In expansion process $\frac{V_2}{V_1} > 1$

$\therefore$ work = Positive i.e., workdone by the system.

② The properties of closed system change following relation b/w pressure & vol. as $pV = 3$, where P is in bar & V is in m^3. Calculate the workdone when pressure increases from 1.5 bar to 7.5 bar.

Sol: Given, $P_1 = 1.5$ bar
$P_2 = 7.5$ bar

$P_1V_1 = 3$
$1.5\,V_1 = 3 \Rightarrow V_1 = 2\ m^3$

$P_2V_2 = 3$
$7.5\,V_2 = 3 \Rightarrow V_2 = 0.4\ m^3$

$$W = \int_{V_1}^{V_2} p\,dV = \int_{2}^{0.4} \frac{3}{V}\,dV$$

$$\int \frac{1}{V}\,dV = [\ln V]$$

$$= 3\left[\overset{\ln}{\cancel{\log}}(0.4) - \overset{\ln}{\cancel{\log}}(2)\right]$$

$$= -4.828 \times 10^3\ \text{bar}\ m^3$$

$$= -4.828 \times 100\ \text{KPa} \times m^3$$

$$= -483\ \text{KJ}$$

③ A gas undergoes reversible non flow process according to the relation $P = -3V + 15$ where V is volume in m^3 and P is the pressure in bar. Determine the workdone when the volume changes from $3\ m^3$ to $6\ m^3$.

Sol:- $P = -3V + 15$

$$W = \int_{V_1}^{V_2} p.dv = \int_3^6 (-3V+15)\,dV$$

$$= \left[-\frac{3V^2}{2} + 15V\right]_3^6$$

$$= \left[-\frac{3(6)^2}{2} + 15(6)\right] - \left[-\frac{3(3)^2}{2} + 15(3)\right]$$

$$= 4.5 \text{ bar} \Rightarrow 4.5 \times 100 \text{ KJ}$$

$$= 450 \text{ KJ}$$

$\int (x+1)dv$

$=$

④ During non flow reversible process a gas enclosed in a cylinder piston assembly expands from $2\ m^3$ to $4\ m^3$ with the relation $p = V^2 + \frac{6}{3}$ where P is the pressure in bar. Determine the work.

Sol:- $W = \int_{V_1}^{V_2} pdv$

$$= \int_2^4 \left(V^2 + \frac{6}{3}\right)dv = \left[\frac{V^3}{3} + \frac{6V}{3}\right]_2^4$$

$$= \left[\frac{(4)^3}{3} + \frac{6(4)}{3}\right] - \left[\frac{(2)^3}{3} + \frac{6(2)}{3}\right] = 18.66 + 4 = 22.66 \ \cancel{KJ}\ \text{bar}\,m^3$$

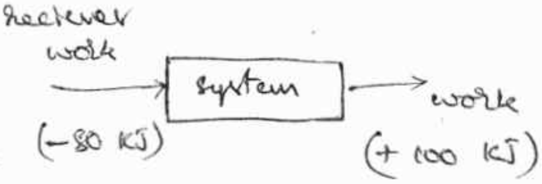
$$= 2266 \text{ KJ}$$

⑤ A gas contained in a piston cylinder arrangement expands from $0.75\ m^3$ to $1.35\ m^3$ volume while the pressure remains constant at 200 kPa. If the gaseous system receives 80 KJ of work from a wheel, determine the net workdone by the system.

Sol:- $V_1 = 0.75\ m^3$ $P = 200$ kPa

$V_2 = 1.25\ m^3$

pressure remains constant



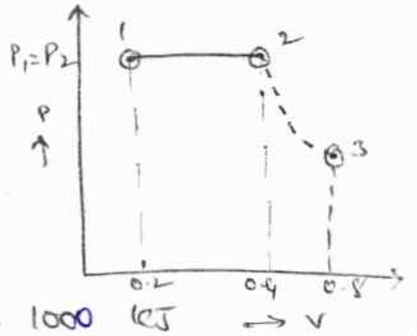
$W = P_1 [V_2 - V_1]$

$= 200 [1.25 - 0.75] = 100$ KJ

Since it is receiving 80 KJ of work from outside.

∴ Net workdone $= 100 - 80 = \underline{20}$ KJ

⑥ Air at pressure of 50 bar & $0.2\ m^3$ is expanded at const. pressure until the volume is doubled, it is then expanded according to the polytropic process $pv^{1.3}$ = const., until the volume is $0.8\ m^3$. Calculate the workdone in each process. Indicate the process on p-v diagram.



Sol:- $P_1 = 50$ bar × 100

$V_1 = 0.2\ m^3$

$V_2 = 0.4\ m^3$

$V_3 = 0.8\ m^3$

$pv^{1.3}$ = const.

$W = P_1 (V_2 - V_1)$

$= 5000 (0.4 - 0.2) = 1000$ KJ

(1−2) is constant

(2−3) is polytropic

$P_1 = P_2$

~~$P_1 V_1^{1.3} = P_2 V_2^{1.3} \Rightarrow P_2 =$~~

$P_2 V_2^{1.3} = P_3 V_3^{1.3} \Rightarrow P_3 = \cancel{2.3 \times 10^3}\ 20 \times 10^2$ KJ

$P_3 = 123.11$ bar $P_3 = \left(\frac{V_2}{V_3}\right)^n \times P_2$

Polytropic process

$$W = \frac{P_2 V_2 - P_3 V_3}{n-1}$$

$=$ ~~26126.67 KJ~~

$W =$ ~~1251.7~~ KJ

1253 ✓

⑦ A gas undergoes two processes that are in series. The 1st process is an expansion that is carried out according to the law pv = const. Second process is a constant pressure process that returns the gas to the initial volume of the 1st process

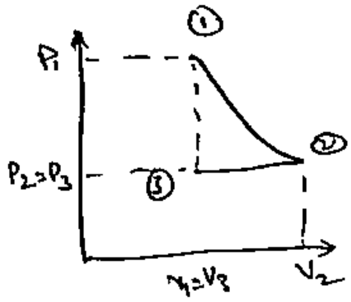
The starting of the 1st process is at 400 KPa & 0.025 m³ with the expansion to a pressure of 200 KPa. Sketch the process on PV diagram & determine work of combined process.

Sol:- 2 process (1-2) → PV constant

(2-3) → pressure is constant.

$P_1 = 400$ kpa $\qquad V_1 = 0.025\ m^3$

$P_2 = 200$ kpa



$$P_1V_1 = P_2V_2 \Rightarrow \frac{P_1V_1}{P_2} = V_2 = 0.05\ m^3$$

$P_2 = P_3 = 200$ kpa

$V_3 = V_1 = 0.025\ m^3$

workdone in 1st process

$$W = P_1V_1 \log\left[\frac{P_1}{P_2}\right] = 6.93\ KJ$$

2nd process

$$W = P_2(V_3 - V_2)$$
$$= 200(0.025 - 0.05) = -5\ KJ$$

∴ Total workdone $= 6.93 - 5 = 1.93$ KJ

⑧ A fluid at a pressure of 3 bar in the sp. volume 0.18 m³/kg contained in a cylinder behind piston expand reversably to a pressure of 0.6 bar it follows the law $pv^2 = $ const. calculate the workdone by the fluid on the path.

Sol:- $P_1 = 3$ bar $\qquad pv^2 = c$

$P_2 = 0.6$ bar

$V_1 = 0.18\ m^3/kg \qquad p = \frac{c}{v^2}$

$$W = \int_1^2 p \cdot dv \qquad = c\left[\frac{v^{-2+1}}{-2+1}\right]_1^2$$
$$= \int_1^2 \frac{c}{v^2}\,dv \qquad = c\left(\frac{v^{-1}}{-1}\right)_1^2$$
$$= \int c v^{-2} \cdot dv \qquad = -c\left[v^{-1}\right]_1^2$$

$$= -c\left[V_2^{-1} - V_1^{-1}\right]$$

$$W = c\left[\frac{1}{V_1} - \frac{1}{V_2}\right]$$

(or)

$$\frac{P_1V_1 - P_2V_2}{2-1}$$

$$P_1V_1^2 = c$$

$$8 \times (0.18)^2 = c \Rightarrow c = 0.0972 \text{ bar}\cdot m^3/kg.$$

$$= 9.72 \text{ KJ}$$

$$P_1V_1^2 = P_2V_2^2$$

$$V_2^2 = \frac{P_1V_1^2}{P_2} = \sqrt{0.162} = 0.4024 \; m^3$$

$$W = c\left[\frac{1}{V_1} - \frac{1}{V_2}\right]$$

$$= 0.2984 \text{ bar } m^3/kg \Rightarrow 0.298 \times 100 \text{ KPa } m^3/kg.$$

$$= 29.8 \frac{KN}{\cancel{m^2}} \times \frac{m^{\cancel{3}}}{Kg} \qquad [NM = J]$$

$$= 29.8 \text{ KJ/kg}.$$

Relations b/w volume, pressure & temperature :-

① Constant pressure process :-

$$\boxed{\frac{V_1}{V_2} = \frac{T_1}{T_2}}$$ where P is constant.

② constant volume process :-

$$\boxed{\frac{P_1}{P_2} = \frac{T_1}{T_2}}$$ where V is constant.

③ Constant temperature process (Isothermic) :-

$$\frac{P_1V_1}{P_2V_2} = \frac{\cancel{m}R\cancel{T_1}}{\cancel{m}RT_2} \Rightarrow \boxed{P_1V_1 = P_2V_2} \qquad PV^{\gamma} = c$$

④ Adiabatic process :-

$$\frac{P_1V_1}{P_2V_2} = \frac{\cancel{m}RT_1}{\cancel{m}RT_2} \Rightarrow \frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)\left(\frac{V_1}{V_2}\right) \longrightarrow ①$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^{\gamma} \rightarrow ②$$ substitute in eq ①

$$\therefore \frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{\gamma}\left[\frac{V_2}{V_1}\right]^{-1} = \left[\frac{V_2}{V_1}\right]^{\gamma-1}$$

$$\boxed{\frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{\gamma-1}}$$

⑤ Polytropic process :- Similar to the above derivation.

$$\boxed{\frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{n-1}}$$

Q. A quantity of air having a volume of 0.03 m³ at a temp of 200°C & a pressure of 150 N/cm² is expanded at a constant pressure to a volume of 0.06 m³, it is then expanded adiabatically to a volume of 0.12 m³. Calculate workdone during each process, temp & pressure at the end of adiabatic process. assume γ is equal to 1.41, also draw the process on p-V diagram.

Sol:

$V_1 = 0.03 \text{ m}^3$

$T_1 = 200^\circ C + 273 = 473 \text{ K}$

$P_1 = 150 \text{ N/cm}^2$

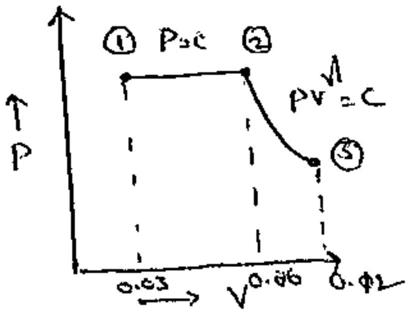
$= 1500 \text{ kPa}$

$V_2 = 0.06 \text{ m}^3$

$V_3 = 0.12 \text{ m}^3$

$\gamma = 1.41$

$\therefore PV^{\gamma} = C$



Process (1-2):

$W = P_1(V_2 - V_1)$

$= 1500(0.06 - 0.03)$

$W_{(1-2)} = \underline{45 \text{ kPa}}$

process (2-3) : $pV^{\gamma} = C$

$$P_2V_2^{\gamma} = P_3V_3^{\gamma}$$

$$\frac{P_2V_2^{\gamma}}{V_3} = P_3 = \underline{564 \text{ kPa}}$$

$$W_{(2-3)} = \frac{P_2V_2 - P_3V_3}{\gamma - 1}$$

$$= \frac{1500 \times 0.06 - 564 \times 0.12}{1.41 - 1}$$

$$= 54.4 \text{ kJ}$$

$$\frac{T_1}{T_2} = \frac{V_1}{V_2} \Rightarrow T_2 = \frac{V_2T_1}{V_1} = 946 \text{ K}$$

$$\frac{T_3}{T_2} = \left[\frac{V_2}{V_3}\right]^{\gamma-1} \Rightarrow T_3 = 946\left[\frac{0.06}{0.12}\right]^{(1.41-1)}$$

$$\underline{T_3 = 711.98 \text{ K}}.$$

workdone at const. pressure $W = 45$ kPa

pressure at final process $P_3 = 564$ kPa.

Temperature at final process $T_3 = 711.98$ K

Adiabatic process :-

In adiabatic process heat transfer is zero.

The condition for adiabatic process. is $pV^{\gamma} = C \Rightarrow p = \frac{C}{V^{\gamma}}$

$$W = \int p.dv$$

$$W_{1-2} = \int_1^2 p.dv = \int_1^2 \frac{C}{V^{\gamma}} dv$$

$$= C\int_1^2 \frac{1}{V^{\gamma}} dv = C\int_1^2 V^{-\gamma} dv = C\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_1^2$$

$$\int x^n = \frac{x^{n+1}}{n+1}$$

$$W = C\left[\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1}\right]$$

$$= \frac{(P_2V_2^{\gamma})V_2^{-\gamma+1} - (P_1V_1^{\gamma})V_1^{-\gamma+1}}{-\gamma+1} = \frac{P_2V_2 - P_1V_1}{-\gamma+1}$$

$$\boxed{W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}}$$

Polytropic Process :-

In this process the condition is $pV^n = C$

where 'n' is known as polytropic index.

$$pV^n = c \Rightarrow p = \frac{c}{V^n}$$

$$w = \int p dV \Rightarrow w_{1-2} = \int_1^2 p.dv$$

$$= \int_1^2 \frac{c}{V^n} dv = c\int_1^2 V^{-n} dv$$

$$= c\left[\frac{V^{-n+1}}{-n+1}\right]_1^2 = c\left[\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1}\right]$$

$$= \frac{(P_2V_2^n)V_2^{-n+1} - (P_1V_1^n)V_1^{-n+1}}{n-1}$$

$$= \frac{P_2V_2 - P_1V_1}{-n+1} \Rightarrow \boxed{W = \frac{P_1V_1 - P_2V_2}{n-1}}$$

Imp ⊛ State or show that work is a path function & not a point function:-

In the figures shown, the system is at state ①. To move the system point ① & ②, it can follow any path such as A (or) B (or) C. etc. The area under the curve represents the workdone.

In the diagrams, the area under each curve is not equal even though initial & final points are same.

Hence work is a path function and not a point function.

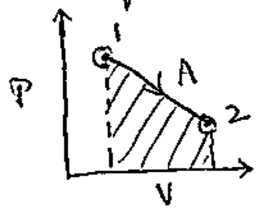
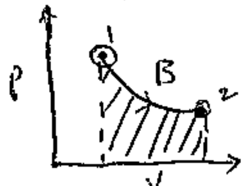


fig ⓐ



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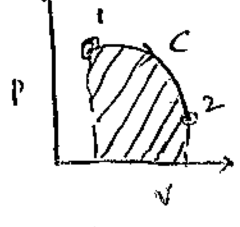
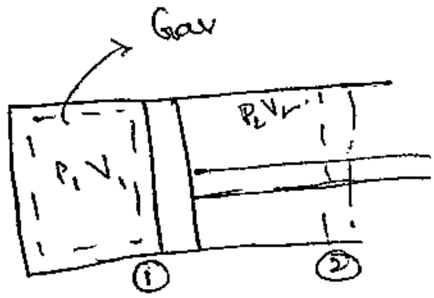


fig. ⓒ

Pdv - Work (or) Displacement Work :

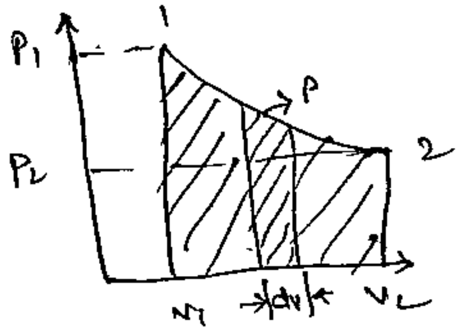


pdv work.

let P_1V_1 are initial press & Vol

& P_2V_2 are final " "

when the piston moves an infinitesimal distance dl, & if 'a', be the area of piston, the force F acting on piston,



$F = p \cdot a$

And the infinitesimal amount of workdone by the gas on the piston

$đW = F \cdot dl = pa\,dl = pdv$ where $dv =$ ~~p~~ adl = infinitesimal disp. vol.

with change in vol from v_1 to v_2,

Amount of WD by the system

$$\boxed{W_{1-2} = \int_{v_1}^{v_2} pdv,}$$

The ~~infini~~ integration $\int pdv$ can be performed only on a quasistatic path.

1 Pa = 1 N/m^2

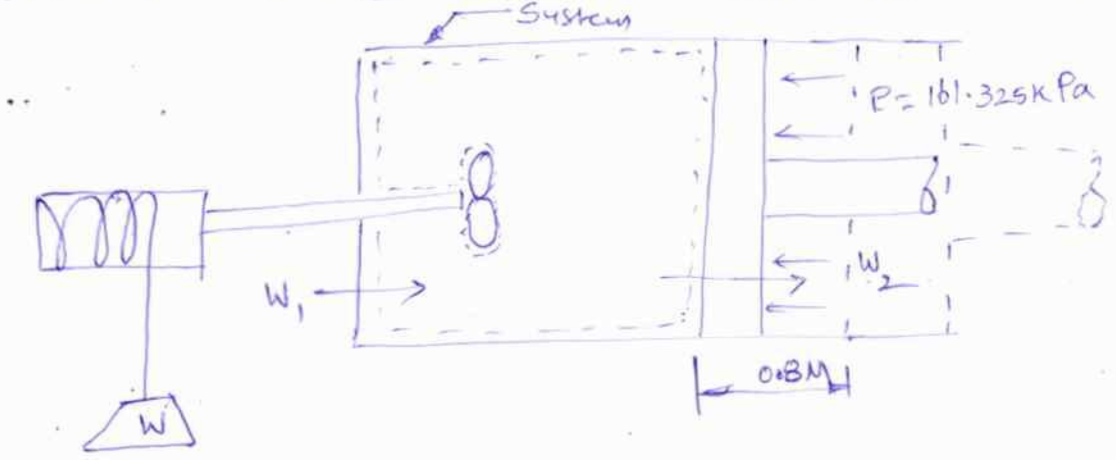
1 bar = 10^5 Pa = 100 kPa

1 atm = 101.325 kPa = 1.01325 bar

1 Joule = 1 Nm .

watt = J/s

P) A piston and cylinder machine containing a fluid system has a stirring device on the cylinder. The piston is frictionless, and is held down against the fluid due to atm. pressure of 101.325 KPa. The stirring device is turned 10000 revolutions with an average torque against the fluid of 1.275 N.m. Mean while the piston of 0.6m diameter moves out 0.8 m. Find the net work transfer for the system



Work done by the stirring device upon system

$w_1 = 2\pi NT$

$= 2\pi \times 10000 \times 1.275 = 80$ kJ

This is the Negative work for system

Work done by the system upon surrounding.

$w_2 = (PA) \cdot L$

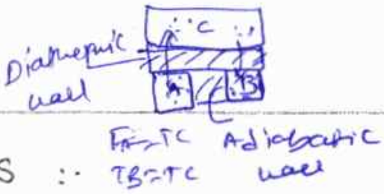
$= 101.325 \times \frac{\pi}{4}(0.6)^2 \times 0.8 = 22.9$ KJ

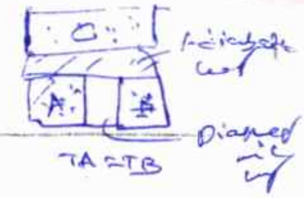
This is positive work for the system.

Hence the net work transfer for the system

$W = W_1 + W_2 = -80 + 22.9 = \underline{-57.1 \text{ KJ}}$

UNIT - 2

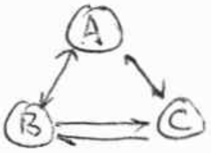




ZEROTH LAW OF THERMODYNAMICS :-

When two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with eachother.

This can be stated as, when a body A is in thermal equilibrium with a body B & when B is seperately in thermal equilibrium with a body C, then A & C will also be in thermal equilibrium with each other. i.e., A & C will not undergo any physical changes when they brought in contact.



$\therefore$ $A = B$
$B = C$ then $A = C$

Note :- Zeroth law is the basis of temperature measurement.

Equality of temperature :-

Temperature is generally defined as the degree of hotness or intensity of heat. The physical meaning of temperature is that it describes whether the body is hot or cold.

Let two bodies, one hot (at temp θ_1) & one cold (at temp θ_2) be placed in contact with each other ($\theta_1 > \theta_2$) while is isolated from all other bodies. After some time, we observe that hot body becomes colder whereas the colder one becomes hotter. It indicates that both the bodies have experienced a change in one or more of their properties. We also find that both the bodies are now at same temp θ_m. ($\theta_m < \theta_1$; $\theta_m > \theta_2$). The state, that these two bodies have attained same temp. is a state of thermal equilibrium.

The condition of equality of temp is stated as follows:

"Two systems have equal temperatures if there are no changes in their properties when they are brought in thermal contact with each other."

Thermometry :- It is the science of ~~measurement~~ of temperature.

Principles of Thermometry :-

In order to obtain a quantitative measure of temp, a reference body is used. Besides this a certain physical characteristics is to be selected, which changes with temp. This reference body is called thermometer. & its selected characteristics is called thermo~~dynamic~~metric property. The Principle of thermometer lies in the thermometric property of fluid used in it. A very common thermometer consists of a small amount of mercury in an evacuated capillary tube. The expansion of mercury in the tube when it comes in contact with hot body, is used as thermometric property.

The following properties are generally used.

1) Pressure exerted by gases.
2) volume expansion of solids, liquids & gases
3) Vapour pressure of liquids
4) Electrical resistance of solids
5) Thermo electricity.

Reference Points :-

Reference points are the fixed points of fixed temp's., marked on the thermometer. Universally adopted reference points are (i) Ice point
(ii) steam point.

The ice point is the equilibrium temp of ice & air-saturated water under a pressure of one atmosphere. In otherwords it refers to melting of ice at atmospheric pressure.

The steam point is the equilibrium temp of water & water vapour under a pressure of one atmosphere. In brief, it refers to boiling point (saturation temp) of water at atm. pressure

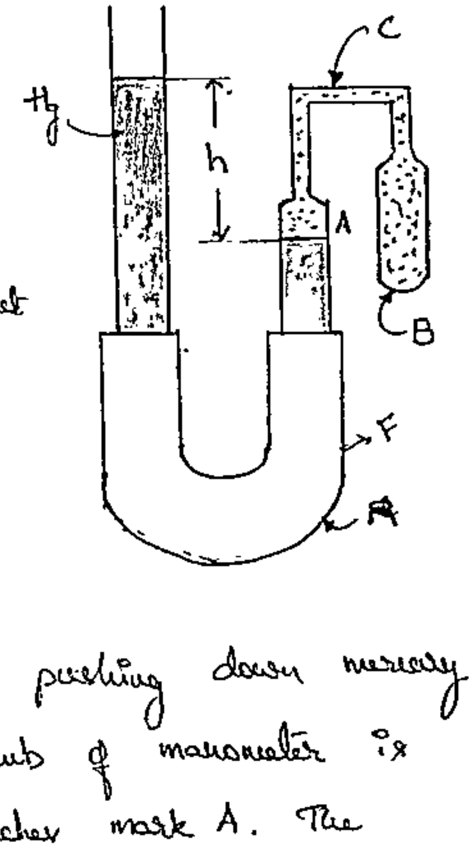
The thermometer is graduated into equal no. of divisions in b/w these reference points. The number assigned to the reference points are arbitary. Following table shows the most popular reference points on different scales.

S.NO	SCALE	ICE POINT	STEAM POINT	NO. OF DIVISIONS
1)	CELSIUS	0° C	100° C	100
2)	FAHRENHEIT	32° F	212° C	180
3)	REAUMER	0° R	80° R	80

Constant volume Gas Thermometer :-

The figure below shows constant vol. gas thermometer. Bulb (B) (platinum-iridium) contains a small amount of gas (hydrogen). The bulb is connected to one end of U-tube mercury manometer (M) by means of a glass capillary tube (C). The other end of the manometer is open to atmosphere. This end can be moved vertically becoz of flexible tubing (F).

The thermometric property that is made use of, is 'pressure' in the bulb. The gas in the bulb is kept at const. vol by keeping the mercury level in the tube at mark A. This is done by raising or lowering the left limb of the tube. The difference 'h' in mercury columns in two limbs gives the guage pressure (P) as



$$P = P_{atm} + \rho_m g h \longrightarrow ①$$

where P_{atm} = atmospheric pressure

ρ_m = density of mercury.

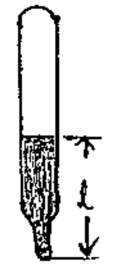
When the bulb is brought in contact with the system whose temp is to be measured, the bulb in course of time, comes in thermal equilibrium with the system. The gas in the bulb expands when it absorbs heat of given system, pushing down mercury column in right limb. The flexible limb of manometer is then adjusted so that mercury again touches mark A. The difference in mercury levels 'h' is recorded & pressure 'P' of the gas in the bulb is estimated using eq ①.

The temperature can be calculated by using the ideal gas equation $\boxed{PV = mRT}$

Scales of temperature :-

To give a numerical value to the temp of the body we have to define a scale of temperature. The earlier method is to choose two fixed point temp's, one the melting point of ice (ice point) & the other boiling point of water (steampoint) at 1 atm pressure as fixed temp points. We assign arbitrarily 't_1' to ice point & 't_s' to steam point. Correspondingly let the length of Hg column are l_1 & l_2.

The temp corresponding to any length 'l' may be defined assuming linear relation:,



Mercury thermometer

$$\boxed{t = a l + b} \rightarrow ① \ [a, b \text{ are constants}]$$

Thus a change of one degree in temp means a change of $\frac{l_2 - l_1}{t_2 - t_1}$ in the length of mercury column. Thus we can graduate scale.

Different scales of temperature :-

① Centigrade Scale (°C)

→ It is also known as Celsius scale.

→ Melting of ice is marked as 0°C & boiling point of water as 100°C under a pressure of 760 mm of Hg

② Absolute (or) Kelving Scale (°K)

→ Melting point of ice is 273 K & boiling point is 373 K

③ Fahrenheit Scale (°F)

→ Melting point of ice is 32°F & boiling point 212°F.

④ Reaumur Scale (°R)

→ Melting point of ice is 0°R & boiling of water as 80°R.

Relation for Centigrade System : & Fahrenheit System

The centigrade system assumes ice point as $0^\circ C$ & boiling point as $100^\circ C$. Substitute in eq ①.

$$0 = a l_1 + b$$
$$100 = a l_2 + b$$

$$\therefore a = \frac{100}{l_2 - l_1} \; ; \quad b = -a l_1$$
$$b = -\frac{100 l_1}{l_2 - l_1}$$

$$\therefore t = a l + b$$
$$t^\circ C = \frac{100}{l_2 - l_1} l - \frac{100 l_1}{l_2 - l_1}$$

$$\boxed{\therefore t^\circ C = \frac{100(l - l_1)}{l_2 - l_1}} \longrightarrow ② \qquad \frac{t^\circ C}{100} = \frac{(l - l_1)}{l_2 - l_1}$$

Similarly, in Fahrenheit scale it is assumed that ice point is $32^\circ F$ & $212^\circ F$ as steam point.

$$\therefore t = a l + b$$
$$32 = a l_1 + b$$
$$212 = a l_2 + b$$

$$\therefore a = \frac{180}{l_2 - l_1} \qquad \therefore b = \frac{32 - 180 l_1}{l_2 - l_1}$$

$$\therefore t^\circ F = \frac{180}{l_2 - l_1} l + \frac{32 - 180 l_1}{l_2 - l_1}$$

$$\boxed{t^\circ F = \frac{32 + 180(l - l_1)}{l_2 - l_1}} \longrightarrow ③ \qquad t^\circ F = 32 + \frac{180(l - l_1)}{l_2 - l_1}$$

$$= 32 + \frac{180\, t^\circ C}{100}$$

Relation b/w $^\circ C$ & $^\circ F$,

$$\text{take } \cancel{\frac{t^\circ F}{t^\circ C}} = \frac{32 + 180(l - l_1) / \cancel{l_2 - l_1}}{100\cancel{(l - l_1)} / l_2 - l_1}$$

$$\boxed{t^\circ F = 32 + \frac{9}{5} t^\circ C}$$

$$= \frac{32 + 180(l-l_1)}{100(l-l_1)}$$

$$\frac{t°F}{t°C} = \frac{9}{5} + 32 \Rightarrow t°F = \frac{9}{5}\, t°C + 32$$

① If a human body temp is 101°F then what is human body temp in °C.

Sol:
$$101 = \frac{9}{5}\,°C + 32$$
$$101 - 32 = \frac{9}{5}\,°C$$
$$°C = 69 \times \frac{5}{9}$$
$$= 38.33°C.$$

② Define a new temp scale say °M with ice & steam point temp's are 80°M, 300°μ resp. Calculate with Centigrade scale

Sol:
$$M° : t = al + b$$
$$80 = al_1 + b \rightarrow ①$$
$$300 = al_2 + b \rightarrow ②$$
$$\therefore a = \frac{220}{l_2 - l_1} \quad ; \quad b = 80 - al_1$$
$$b = 80 - \frac{220\, l_1}{l_2 - l_1} = \frac{80(l_2 - l_1) - 220\, l_1}{l_2 - l_1}$$
$$b = \frac{80\, l_2 - 300\, l_1}{l_2 - l_1}$$

① $t°_{\mu} = al + b$
$$= \frac{220\, l}{l_2 - l_1} + \frac{80\, l_2 - 300\, l_1}{l_2 - l_1} = \frac{220\, l}{l_2 - l_1} + 80 - \frac{220\, l_1}{l_2 - l_1}$$
$$= \frac{220\, l + 80\, l_2 - 80\, l_1 - 220\, l_1}{l_2 - l_1}$$
$$t°M = \frac{220(l - l_1)}{l_2 - l_1} + 80 \rightarrow ③$$

$t = al + b$

$0 = al_1 + b \qquad \therefore a = \frac{100}{l_2 - l_1} \; ; \; b = \frac{-100 l_1}{l_2 - l_1}$

$100 = al_2 + b$

$$t^\circ C = \left[\frac{100}{l_2 - l_1}\right] l - \frac{100 l_1}{l_2 - l_1}$$

$$t^\circ C = \frac{100(l - l_1)}{l_2 - l_1} \longrightarrow ④ \Rightarrow \frac{l - l_1}{l_2 - l_1} = \frac{t^\circ C}{100}$$

$$\frac{③}{④} = \frac{t^\circ M}{t^\circ C} = \frac{\frac{220\cancel{(l-l_1)}}{\cancel{l_2-l_1}} + 80}{100\cancel{(l-l_1)}/\cancel{l_2-l_1}}$$

$$t^\circ M = \frac{11}{5} t^\circ C + 80$$

$$\boxed{{}^\circ M = \frac{11}{5} {}^\circ C + 80}$$

$$t^\circ M = 80 + \frac{220(l - l_1)}{(l_2 - l_1)}$$

$$= 80 + \frac{\overset{11}{\cancel{220}}\, t^\circ C}{\underset{5}{\cancel{100}}}$$

$$\boxed{t^\circ M = \frac{11}{5} t^\circ C + 80}$$

③ A temp scale in °N is to be define the boiling point is 400°N & freezing point is 100°N & temp scale,

(i) corelate the new scale with Centigrade scale.

(ii) " " " " Fahrenheit scale

(iii) what will be the new scale reading corresponding to 60°C.

Sol Case (i): for Centigrade scale & New scale.

$$t^\circ C = \frac{100(l - l_1)}{l_2 - l_1} \longrightarrow ①$$

where $a = \frac{100}{l_2 - l_1}$

$b = \frac{-100 l_1}{l_2 - l_1}$

Case (i): Fahrenheit scale

$a = \frac{180}{l_2 - l_1}$, $b = 32 - \frac{180}{l_2 - l_1} \Rightarrow t^\circ F = \frac{180(l - l_1)}{l_2 - l_1} + 32 \longrightarrow ②$

Case ③ :- for new scale $\quad t = a\,l + b$

$$100 = a\,l_1 + b$$
$$400 = a\,l_2 + b$$
$$300 = a(l_2 - l_1) \Rightarrow a = \frac{300}{l_2 - l_1}$$

$$\therefore b = 100 - \frac{300\,l_1}{l_2 - l_1}$$, substitute

$$°N = \cancel{400} = \frac{300\,l}{l_2 - l_1} + 100 - \frac{300\,l_1}{l_2 - l_1}$$

$$t°N = \frac{300(l - l_1)}{l_2 - l_1} + 100 \longrightarrow ③$$

From ③ & ① $\Rightarrow$
$$\frac{t°N}{t°C} = \frac{\frac{300(l-l_1)}{\cancel{l_2 - l_1}} + 100}{\frac{100\cancel{(l-l_1)}}{\cancel{l_2 - l_1}}}$$

$$t°N = \frac{300(l-l_1)}{l_2 - l_1} + 100$$
$$= \frac{300 \times t°C}{100} + 100$$
$$\boxed{t°N = 3t°C + 100}$$

$$t°N = \frac{\frac{300(l-l_1)}{l_2 - l_1} + 100}{\frac{100(l-l_1)}{l_2 - l_1}} \times \cancel{t°C} \Rightarrow t°N = \frac{300(l-l_1)}{l_2 - l_1} + 100$$

$$= 3 \times \frac{100(l-l_1)}{l_2 - l_1} + 100$$

$$\boxed{\begin{array}{l} t°N = 3 \times t°C + 100 \\ °N = 3°C + 100 \end{array}}$$

From ③ & ② $\Rightarrow$
$$\frac{t°N}{t°F} = \frac{\frac{300(l-l_1)}{l_2 - l_1} + 100}{\frac{180(l-l_1)}{l_2 - l_1} + 32}$$

$$\Rightarrow t°N = \frac{300(l-l_1)}{l_2 - l_1} + 100$$

$$= \frac{300}{180} \cdot \frac{180(l-l_1)}{l_2 - l_1} + 32 - 32 + 100 \Rightarrow t°N = \frac{\overset{5}{\cancel{300}}}{\underset{3}{\cancel{180}}}(t°F - 32) + 100$$

$$t°N = \frac{5}{3}(t°F - 32) + 100$$

$$\boxed{°N = \frac{9}{3}°C + 100}$$ a.

Case(iii):- New scale reading in Centigrade of 60°C

$$°N = 3°C + 100$$

$$°N = 3 \times 60 + 100 \Rightarrow °N = \underline{280°N}$$

④ Certain is calibrated using ice & steam point as fixed points at a temp of 0°C & 100°C resp. The function of the thermometric substance is taken as $t = a\log_e x + b$ instead of usual relation $t = al + b$ (or) $t = ax + b$. Show that a new scale is given by $t = 100\left[\frac{\log\left(\frac{x}{x_i}\right)}{\log_e\left(\frac{x_i}{x_s}\right)}\right]$.

Sol: Given that $t = al + b$

$$0° = al_1 + b$$
$$100° = al_2 + b$$
$$\overline{100 = a(l_2 - l_1)}$$

$a = \frac{100}{l_2 - l_1}$ $b = -\frac{100l}{l_2 - l_1}$

Similarly $t = a\log_e x + b$

$$0 = a\log_e x_i + b$$
$$100 = a\log_e x_s + b$$
$$\overline{100 = a\log_e\left(\frac{x_i}{x_s}\right)}$$

where $a = \frac{100}{\log_e\left(\frac{x_i}{x_s}\right)}$

$b = -\frac{100\log_e(x_i)}{\log_e\left(\frac{x_i}{x_s}\right)}$

$$t = a\log_e x + b$$

$$t^\circ c = \frac{100 \log_e x}{\log\left(\frac{x_i}{x_s}\right)} - \frac{100 \log(x_i)}{\log\left(\frac{x_i}{x_s}\right)}$$

$$\boxed{t^\circ c = \frac{100 \log_e\left(\frac{x}{x_i}\right)}{\log\left(\frac{x_i}{x_s}\right)}}$$

proved

Note :- Relation b/w °C , °F & °R

$$\boxed{\frac{C}{100} = \frac{F-32}{180} = \frac{Re}{80}}$$

Types of thermometers.

Thermometer	Thermometric property	Range.
1) Hg in Glass thermometer	length	-40 to $525^\circ C$
2) Const. Vol gas thermometer	pressure	-200 to $1500^\circ C$
3) const. pressure gas thermometer	volume	-200 to $1500^\circ C$
4) Electrical resistance thermometer	Resistance (wheat stone bridge)	-200 to $120^\circ C$
5) Thermo couple	emf (See back effect)	0 to $1500^\circ C$
6) Radiation pyrometer	wave length	$> 400^\circ C$

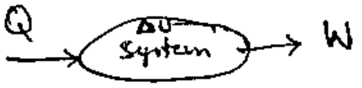
FIRST LAW OF THERMODYNAMICS :-

This is also known as law of conservation of energy. It states that energy can neither be created nor destroyed but can be convertible from one form to another form. It means two forms of energy heat & work are mutually convertable.

whenever heat is absorbed by a system it increases its internal energy plusto do some external work.

This can be represented as follows.

$$Q = \Delta U + W$$



where Q is heat energy entering a system

ΔU is internal energy,

W is external work.

First law can also be stated as,

If a system undergoes a cyclic change (the end state being precisely the same as the initial state), then the algebraic sum of the work delivered to the surroundings is proportional to the algebraic sum of heat taken from the surroundings. It is expressed as

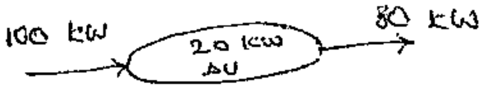
$$\oint dQ = \oint dW \quad (\text{or}) \quad \Sigma Q = \Sigma W$$

<u>Internal Energy</u> :-

Energy stored in a system which is neither heat nor work is called internal energy.

<u>Ex</u>:- when a gas is heated it expands producing some work but the amount of work produced will be less than heat supply. This difference b/w heat & work is known as internal energy which is stored in the system.

$$\boxed{\Delta U = Q - W}$$



Ideal gas temp Scale :-

Assume that the bulb of a constant vol. gas thermometer contains certain amount of gas such that when the bulb is surrounded by water at its triple point, the pressure P_t is 1000 mm of mercury. (Triple point is the temp at which substance exists in all three states.) Keeping the vol. of gas V constant let the following procedure be carried out.

$$\boxed{T = CP \quad \text{where } C = \frac{273.16}{P_{tr}}}$$

(i) Bulb of the thermometer is surrounded by steam condensing at 1 atm & pressure P_1 is determined. Then calculate the temp as $\theta_1 = 273.16 \frac{P_1}{P_t} = 273.16 \frac{P_t}{1000}$

(ii) Some amount of gas is removed from the bulb so that when it is surrounded by water at its triple point, the pressure P_t is 500 mm of Hg. At this instant, pressure is determined say P_2. Then θ_2 for steam condensing at 1 atm is calculated

$$\theta_2 = 273.16 \frac{P_2}{500}$$

Similarly $\theta_3, \theta_4 \ldots \theta_n$ are found from the experimentally determined values of $P = P_3, P_4 \ldots P_n$ by going on reducing the vol. of gas against $P_t = 250, 100 \ldots$ mm Hg.

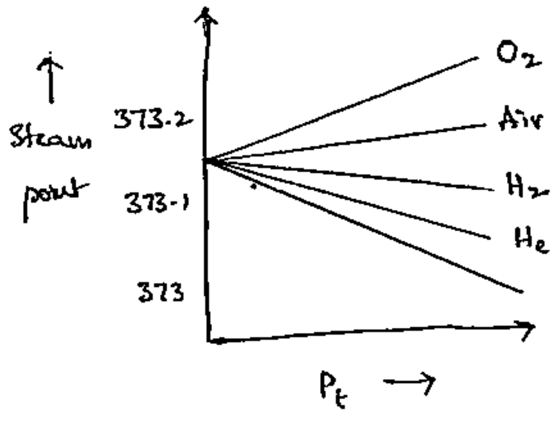
Note that $\theta_t = 273.16 \frac{P}{P_t}$

θ Vs P_t is plotted & explorate the curve to the axis where $P_t = 0$. Read from the graph $\lim_{P_t \to 0} \theta$

[The temp of triple point of water is 273.16 K]

It may be shown that all the gases indicate the same temp as P_t is lowered & made to approach zero.

A similar series of tests can be conducted with a const pressure gas thermometer.



$$\therefore \theta = 273.16 \frac{V}{V_t}$$

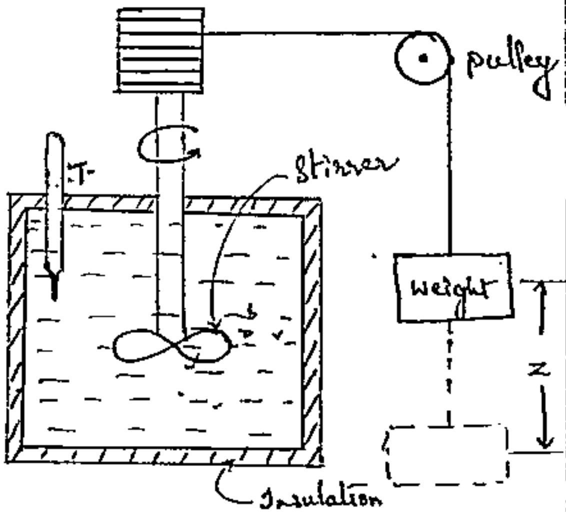
θ vs P is plotted.

Replacing θ by T, the ideal gas temp is given by

$$\boxed{T = 273.16 \lim_{P_t \to 0} \frac{P}{P_t}} \quad \text{(or)} \quad \boxed{T = 273.16 \lim_{V_t \to 0} \frac{V}{V_t}}$$

JOULE'S EXPERIMENT :

A quantity of liquid is taken in an insulated vessel. Work is done on the fluid by stirring the paddle wheel. This work input to the fluid causes a rise in temp of fluid. The amount of work done is calculated by the product of weight & the vertical height of weight displaced.



Next by removing the insulation the system is immersed in a water bath, such that heat is transferred from the fluid till the original state of fluid is reached which will be indicated by temp & pressure. In this way system undergoes a complete cycle. The amount of heat rejected by the fluid is equal to the increase of energy of water bath.

Conclusion: The net work input W was directly proportional to the net heat Q transferred from the system regardless of the kind of workdone, the rate at which the work was done & the scheme used for transferring it into heat.

Thus for a closed system undergoing cyclic process

$$\oint \delta Q = \oint \delta W$$

$$Q_{1-2} + Q_{21} = W_{1-2} + W_{2-1}$$

Perpetual Motion Machine I : PMM-I [Perpetual → continuing forever]

PMM-I refers to the Perpetual Motion machine of first kind. It is hypothetical (theoretical) machine that will continuously churn (produce) out work but without absorbing heat from its surroundings.

But such a machine is not feasible from ~~the~~ a practical point of view, for it violates law of conservation of energy (1st law TD).

$$Q=0 \longrightarrow \boxed{\text{PMM I}} \longrightarrow W$$

The reverse of PMM I is also not true. It is a hypothetical machine which is not feasible as it violates the 1st law of TD.

$$W=0 \longrightarrow \boxed{\text{PMM I (reverse)}} \longrightarrow Q$$

Corollaries of First law :- (Corollary: practical consequence that follows naturally)

Cor. 1 :- There exists a property of a closed system, such that a change in its value during any change of state is given by the difference b/w the heat supplied & workdone. And this property is called 'Internal energy' of the system. Mathematically

$$\boxed{\Delta U = \Delta Q - \Delta W}$$

[ΔU increase or decrease - with change of state]

This ~~energy~~ equation is called Non flow energy equation (NFEE)

Cor. 2 :-

"The energy of an isolated system is constant." By isolated system we mean a system which exchanges neither heat nor work with its surroundings. In otherwords both Q & W are zero and $\Delta U = 0$,

Alternatively, the second corollary may be stated as, that the "internal energy of a closed system remains unchanged if the system is isolated from the surroundings."

In a way this is Law of Conservation of Energy.

Cor.3 :-

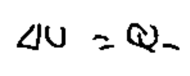
"A perpetual machine of 1st kind is impossible": It is a machine that produces & supplies 'work' by itself without absorbing any energy from the surroundings.

Internal Energy :-

Energy stored in a system which is neither heat nor work is called internal energy.

Ex:- when a gas is heated it expands producing some work, but amount of work produced will be less than heat supply. This difference b/w heat & work is known as internal energy which is stored in a system.

$$\boxed{\Delta E = Q - W}$$



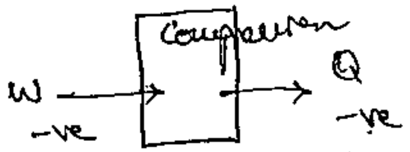
$\Delta U = Q - W$

⑤ In general compression process 2 kJ of mechanical work is supplied to 5 kg of working substance. 800 J of heat is rejected to the cooling water jacket. Calculate change in specific internal energy.

Soln.

$Q = \Delta E + W$

$\Delta E = Q - W$



$W = 2\ kJ = 2000\ J/5\ kg$

$= -400\ J/kg$

$Q = 800\ J/5kg = -160\ J/kg$

$\therefore\ \Delta E = Q - W$

$= -160 + 400$

$\Delta E = 240\ J/kg.$

⑥ Piston & cylinder m/c contains a fluid system which possesses through complete cycle of 4 process. During the cycle sum of all heat transfers is -170 kJ. The system completes 100 cycles per minute. Complete following table showing method for each item. Calculate the net work o/p in kW.

Process	δQ (kJ/m)	δW (kJ/m)	ΔE (kJ/m)
(a-b)	0	2170	-2170
(b-c)	21000	0	21000
(c-d)	-2100	34500	-36,600
(d-a)	-35,900	-53,670	17770

Soln

$\Sigma Q = -170\ kJ/m \times 100 = -17000$

$\Sigma Q = Q_{a-b} + Q_{b-c} + Q_{c-d} + Q_{d-a}$

$Q_{d-a} = \Sigma Q - Q_{a-b} - Q_{b-c} - Q_{c-d}$

$= -17000 - 0 - 21000 + 2100 = -35900\ kJ/min$

for Q_{a-b} $\Rightarrow$ $Q = \Delta E + W$

$\Delta E = Q - W$

$\Delta E = 0 - 2170 = -2170$ KJ/min

For Q_{b-c} $\Rightarrow$ $\Delta E = Q - W$

$\Delta E = 21000 - 0 = 21000$ KJ/min

For Q_{c-d} $\Rightarrow$ $\Delta E = Q - W$

$Q = \Delta E + W \Rightarrow W = Q - \Delta E$

$= -2100 + 36600 = 34500$ KJ/min

$\Sigma W = \Sigma Q$ for cyclic process.

$\Sigma W = W_{a-b} + W_{b-c} + W_{c-d} + W_{d-a}$

$W_{d-a} = \Sigma W - W_{a-b} - W_{b-c} - W_{d-a} = -53670$ KJ/min.

For W_{d-a} $\Rightarrow$ $\Delta E = Q - W = -35900 + 53670$

$\Delta E = 17770$ KJ/min

A/c to Ist law of TD $\Sigma Q = \Sigma W$

$\Delta E_{cyclic} = 0$

Total work $\Rightarrow$ -17000 KJ/min

$$-\frac{17000}{60} = -283.33 \text{ KW}$$

⑦/ A gas undergoes a thermodynamic cycle consisting of 3 process of beginning at initial stage where $P_1 = 1$ bar, $V_1 = 1.5$ m^3 & $U_1 = 512$ KJ. The process are as follows.

ⓐ process (1-2) $\Rightarrow$ Compression with PV = const, $P_2 = 2$ bar, $U_2 = 690$ KJ

ⓑ process (2-3) $\Rightarrow$ $W_{2-3} = 0$, $Q_{2-3} = -150$ KJ

ⓒ process (3-1) $\Rightarrow$ $W_{3-1} = 50$ KJ

Neglecting KE & PE changes, determine & values, Q_{1-2}, Q_{3-1}

Sol: ⓐ PV = const

$P_1V_1 = P_2V_2 \Rightarrow 1 \times 1.5 = 2 \times V_2$

$V_2 = 0.75$

$Q_{1-2} = \Delta E_{1-2} + W_{1-2}$

$= (690-512) + -103.972$

$Q_{1-2} = 74.03$ kJ

$W_{1-2} = p_1V_1 \ln\left[\frac{p_1}{p_2}\right]$

$= 100 \times 1.5 \ln(1/2)$

$W_{1-2} = -103.972$ kJ.

ⓑ process (2-3) $\Rightarrow W_{2-3} = 0$

$Q_{2-3} = -150$ kJ (given)

~~$= -150$ kJ~~

$\Delta U_{2-3} = Q_{2-3} - W_{2-3} = -150 - 0 = -150$ kJ

$U_3 - U_2 = -150$ kJ

$U_3 - 690 = -150 \Rightarrow U_3 = 540$ kJ

ⓒ $W_{3-1} = 50$ kJ

$Q_{3-1} = \Delta U_{3-1} + W_{3-1}$

$= U_1 - U_3 + W_{3-1}$

$= 512 - 540 + 50 \Rightarrow Q_{3-1} = 22$ kJ.

⑧ A system receives 200 kJ of heat at const. vol. process & rejects 220 kJ of heat at const. pressure & 40 kJ of work is done on the system. The system is brought to its initial condition by adiabatic. Calculate adiabatic work.

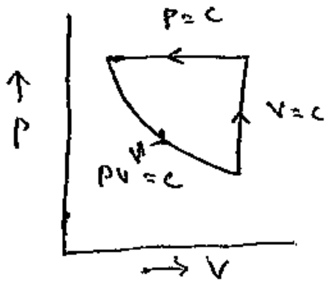
If initial internal energy is 240 kJ then what is I.E at all points. Represent cycle on p-v diagram.

1-2 $\quad Q_{1-2} = 200$ kJ

2-3 $\quad Q_{2-3} = -220$ kJ

3-1

$U_1 = 240$ kJ



① 1-2 :- $W_{1-2} = 0$ [∵ const vol. process]

$Q_{1-2} = 200$ kJ

∴ $Q = \Delta U_{1-2} + W$

$U_2 - U_1 = 200$

$U_2 = 200 + U_1$

$U_2 = 200 + 240 = 440$ kJ.

② 2-3 :- Const. pressure.

$Q_{2-3} = -220$ kJ ; $W_{2-3} = -40$ kJ

$Q = \Delta U_{2-3} + W$

$-220 + 40 = U_3 - U_2 \Rightarrow U_3 = -220 + 40 + U_2$

$= -220 + 40 + 440$

$U_3 = 260$ kJ

③ 3-1 :- Adiabatic process.

$pV^{\gamma} = C$, For cyclic process $\Sigma Q = \Sigma W$ & ΣE or $\Sigma U = 0$

$Q_{3-1} = \Delta U_{3-1} + W_{3-1}$

$= U_1 - U_3 + W_{3-1}$

In adiabatic process $\boxed{\text{heat transfer} = 0}$

$\therefore Q_{3-1} = 0 \quad \therefore U_1 - U_3 = -W_{3-1}$

$W_{3-1} = 20$ kJ

$\Sigma Q = -220 + 200$

$= -20$ k

$\Sigma W = 0 + -40 + W_{3-1}$

$W_{3-1} = -40 + 20$

$= 20$ kJ

∴ Adiabatic process, $W_{3-1} = 20$ kJ

∴ $U_2 = 440$ kJ, $U_3 = 260$ kJ.

④ The system is composed of 2 kg of fluid expands in a piston cylinder machine from initial state of 1 MPa, 100° C to final temp of 30° C. If there is no heat transfer find net work for process. Assume R = 0.287 kJ/kg K

Sol: $m = 2$ kg $\quad T_1 = 100°C \quad R = 0.287$ kJ/kg k.

$P = 1$ MPa $\quad T_2 = 30°C$

Adiabatic process :- (Heat transfer is zero)

$pV^{\gamma} = C$

$pV = mRT$

(1) $V = 2 \times 0.287 \times 100°c$

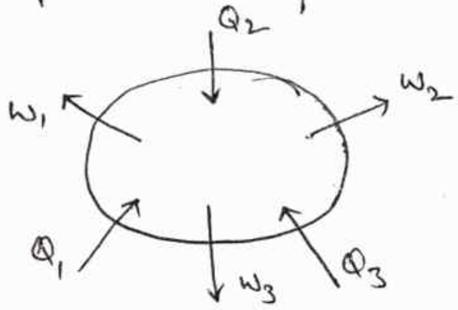
$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{mRT_1 - mRT_2}{\gamma - 1}$$

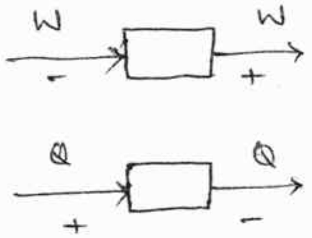
$$= \frac{mR(T_1 - T_2)}{\gamma - 1} \quad [\gamma = 1.4]$$

$$= \frac{2 \times 0.287 (100 - 30)}{1.4 - 1}$$

$W = 100.45$ kJ

① Write first law of TD equation for the following system.





Sol:

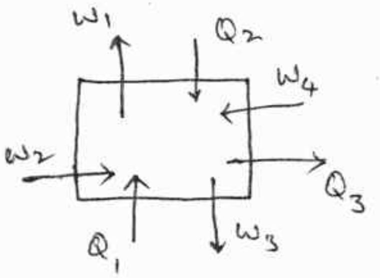
$Q = \Delta U + W$

$\Delta U = \cancel{W - Q}$
$Q - W$

$\Rightarrow \Delta U = (W_1 + W_2 + W_3) - (Q_1 + Q_2 + Q_3)$

$\therefore (Q_1 + Q_2 + Q_3) = \Delta U + (W_1 + W_2 + W_3)$

② Give the relation b/w heat energy, work & internal energy for the following system of multiple energy conversion.



Sol:- $Q_1 + Q_2 - Q_3 = \Delta U + (W_1 - W_2 + W_3 - W_4)$

③ The system undergoes a cyclic process composed of 4 processes. The hea
transfers in each process are 400 KJ, − 365 KJ, − 200 KJ, 250 KJ
the respective work transfers are 140 KJ, 0 KJ, − 55 KJ & 0. Is th
above data is consistant with 1st law of TD?

Sol:- For cyclic process, $\Sigma Q = \Sigma W$

$\Sigma Q = 400 - 365 - 200 + 250 = 85$ KJ

$\Sigma W = 140 + 0 - 55 + 0 = 85$ KJ

$\because \Sigma Q = \Sigma W = 85$ KJ

The given data is consistant of the cyclic process.
& follows 1st law of TD.

(4) A closed system undergoes a TD cycle consisting of four seperate processes. The heat & work transfers are given in the following table. Show that the data is consistent with 1st law of TD. Calculate net work output in kW & change in internal energy.

process	Heat (KJ/m)	work (KJ/m)
1-2	20000	0
2-3	-10000	30000
3-4	0	20000
4-1	15000	-25000

$\Sigma Q = 20000 - 10000 + 0 + 15000 = 25000$ KJ/m

$\Sigma W = 0 + 30000 + 20000 - 25000 = 25000$ KJ/m

$\therefore \Sigma Q = \Sigma W$

The data is consistent. It follows 1st law of TD.

Net work = 25000 KJ/m

= 416.66 KW

change in ΔU,

process (1-2) : $Q_{1-2} = \Delta U_{1-2} + W_{1-2}$

$\Delta U_{(1-2)} = 20000 - 0 = 20000$ KJ/m → ①

process (2-3) $\Delta U_{(2-3)} = -10000 - 30000 = -40000$ KJ/m → ②

process (3-4) $\Delta U_{(3-4)} = 0 - 20000 = -20000$ KJ/m → ③

process (4-1) $\Delta U_{(4-1)} = 15000 + 25000 = 40000$ KJ/m → ④

$\Delta U_{(1-2-3-4-1)} = \Sigma Q - \Sigma W = 25000 - 25000 = 0$

First law applied to flow process : (Open System)

In an open system, we know that energy (heat & work) as well as matter cross the boundary. The matter may take internal, kinetic, potential, chemical & magnetic energies along with it while crossing the boundary. However, chemical & magnetic energies are generally neglected.

In open system there are 2 types of flow process, namely.

1) Steady flow process
2) Unsteady flow process

1) In steady flow process the mass flowing into the system is equal to the mass flowing out of the system. In a given time & there is no change in the storaged energy of the system.

Ex:-

Imagine a bath tub with a closed drain & turn on water which fills the tub & starts overflowing.

2) In unsteady flow process, the mass flowing into the system is not equal to the mass flowing out from the system in a given time & there is change in storaged energy of the system i.e., the rate of net flow of matter is not constant.

Ex:- Imagine a bath tub with a closed drain at the bottom or with an open drain & turn on water. As the quantity of matter in the tub is not constant, it is an unsteady process.

Joules law - Internal Energy of perfect gas :-

Joules law states that internal energy of a perfect gas is independent of vol & pressure. It only depends on temperature. Thus $dU = mC_v dT$ is applicable for any change of state b/w 1 & 2 irrespective of the process since internal energy is a point function.

From above definition we can express the sp. heat at constant vol. as $\boxed{C_v = \frac{\partial u}{\partial T}}$

where du is specific change in internal energy in KJ/kg

Heat Transfer in various Non-flow process :-

First law for a closed system is given by

$$Q_{1-2} - W_{1-2} = \Delta U$$

For an ideal gas internal energy is a function of temp alone.

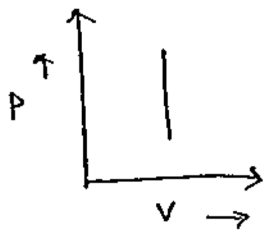
$$\therefore \boxed{\Delta U = mc_v(T_2 - T_1)} \rightarrow \text{eq.}$$

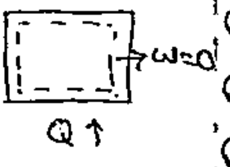
This eq. is applicable for any process since it is a point function & does not depends on path of the process and depends only on the state temp's.

Let us Apply this eq. to various possible process of a closed system which changes from one state to another to develop relations for heat transfer.

ⓐ Constant Volume process.

Applying 1st law,





$$Q_{1-2} - W_{1-2} = \Delta U$$

$$\because W_{1-2} = 0$$

$$Q_{1-2} = \Delta U = m c_v(T_2 - T_1)$$

It indicates that the total amount of energy supplied in the form of heat is stored by the system as internal energy & thus

$$\boxed{(Q_{1-2})_v = mc_v(T_2-T_1)}$$

ⓑ Constant Pressure process :-

Applying 1st law,

$$Q_{1-2} - W_{1-2} = \Delta U$$

$$W_{1-2} = P_1(V_2-V_1) \ ; \quad \Delta U = mc_v(T_2-T_1)$$

$$\therefore Q_{1-2} = mc_v(T_2-T_1) + P_1(V_2-V_1)$$

$$= mc_v(T_2-T_1) + (P_2V_2 - P_1V_1) \quad [\because P_1 = P_2]$$

$$= mc_v(T_2-T_1) + mR(T_2-T_1) \quad [\because P_1V_1 = mRT_1]$$

$$mc_p(T_2-T_1) = m(T_2-T_1)(c_v+R) \quad [\because Q_{1-2} = mc_p(T_2-T_1)]$$

$$c_p = c_v + R$$

$$\boxed{c_p - c_v = R} \longrightarrow \text{eq.}$$

This indicates that c_p is always greater than c_v & the amount of heat required in a const. pres. process is greater than the const. vol. process to bring the same change in temp.

ⓒ Isothermal process :- $[PV = C]$

$$Q_{1-2} - W_{1-2} = \Delta U$$

$$\Delta U = mc_v(T_2-T_1) \quad [\because \text{Temp} = \text{const}]$$

$$\Delta U = 0$$

$\therefore\ Q_{1-2} = W_{1-2}$

$$\boxed{Q_{1-2} = P_1V_1 \ln\frac{V_2}{V_1}}$$

i.e., In isothermal process the system will not store any energy & total amount of heat supplied will convert into work.

Though the temp is constant in isothermal process $Q \neq 0$, moreover it is equal to W.

④ Adiabatic Process:- (Reversible)

$Q_{1-2} - W_{1-2} = \Delta U$

$\Delta U = mC_V(T_2-T_1)$; $W_{1-2} = \dfrac{P_1V_1 - P_2V_2}{\gamma-1} = \dfrac{mR(T_1-T_2)}{\gamma-1}$

$W_{1-2} = mC_V(T_1-T_2)$ $\left[\because \dfrac{R}{\gamma-1} = C_V \right.$ from $C_p - C_V = R$, $\left. C_p/C_V = \gamma \right]$

$\therefore\ Q_{1-2} = \Delta U + W_{1-2}$

$= mC_V(T_2-T_1) + mC_V(T_1-T_2)$

$$\boxed{Q_{1-2} = 0}$$

In adiabatic process heat transfer is zero i.e., the system does work without any heat interaction but at the expense of its own internal energy. So the temp changes.

" In adiabatic process heat transfer is zero but temp changes."

ⓒ Polytropic process :-

$$Q_{1-2} = W_{1-2} + \Delta U$$

$$= \frac{P_1V_1 - P_2V_2}{n-1} + mc_v(T_2-T_1)$$

$$= \frac{mR(T_1-T_2)}{n-1} + \frac{mR}{\gamma-1}(T_2-T_1) \qquad \left[\because c_v = \frac{R}{\gamma-1}\right]$$

$$= mR(T_1-T_2)\left[\frac{1}{n-1} - \frac{1}{\gamma-1}\right]$$

$$= mR(T_1-T_2)\left[\frac{(\gamma-1)-(n-1)}{(n-1)(\gamma-1)}\right]$$

$$= \frac{mR(T_1-T_2)}{n-1}\left[\frac{\gamma-n}{\gamma-1}\right]$$

$$Q_{1-2} = \frac{\gamma-n}{\gamma-1} \times \text{work done}$$

$$= \frac{mR}{\gamma-1} \cdot \frac{\gamma-n}{n-1}(T_1-T_2)$$

$$= mc_v \frac{n-\gamma}{n-1}(T_2-T_1)$$

$$Q_{1-2} = mc_N(T_2-T_1) \qquad \left[C_N = c_v\left[\frac{n-\gamma}{n-1}\right] \text{ is polytropic specific heat}\right]$$

$$\boxed{Q = mC_N \Delta T}$$

First law for an isolated system :

In an isolated system $dQ = 0$ & $dW = 0$ & hence $\Delta E = 0$. There E = constant. Then it states that in an isolated system energy remains constant. For example universe is an isolated system for which the energy remain constant.

⑩ A domestic refrigerator is loaded with vegetables, food, ~~extra~~ etc and then door is closed, during a certain period of the m/c consumed 1 kw-hr of energy & I.E of a system drops by 5000 KJ. Find the net heat transfer for the system ?

Sol:- Given Internal energy $\Delta E = -5000$ KJ

$W = 1$ kw-hr

$= -3600$ KJ

$\Delta Q = \Delta E + W$

$= -5000 - 3600 = -8600$ KJ

⑪ An air reservoir of vol. 5.5 m^3 contains air at 16 bar & 42°C, a valve cock is opened & some quantity of air escapes to atmosphere. The pressure of air in the ~~receiver~~ reservoir reduced to 12 bar, when the valve is closed. Calculate the mass of the air left ~~in the reservoir receiver~~ from the reservoir. Assume adiabatic process.

Sol:

$V_1 = 5.5\ m^3$

$P_1 = 16$ bar $= 1600$ KPa

$P_2 = 12$ bar $= 1200$ KPa

$T_1 = 42 + 273 = 315$ K

$PV^\gamma = \text{Const.}$

$P_1V_1^\gamma = P_2V_2^\gamma$

$P_1V_1 = mRT_1$

$\frac{T_1}{T_2} = \frac{P_1}{P_2} \times \frac{V_1}{V_2}$

$\frac{T_2}{T_1} = \frac{P_2}{P_1} \times \frac{V_2}{V_1}$

$= \frac{P_2}{P_1} \times \left(\frac{P_1}{P_2}\right)^{1 - \frac{1}{\gamma}}$

$=$

① $m_1 = \frac{P_1V_1}{RT_1} = \frac{1600 \times 5.5}{0.287 \times 315} = 97.34$ kg.

② $P_1V_1^\gamma = P_2V_2^\gamma$

$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma \Rightarrow \left(\frac{V_2}{V_1}\right) = \left(\frac{P_1}{P_2}\right)^{1/\gamma}$

$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$

$\Rightarrow \frac{315}{T_2} = \left[\frac{1600}{1200}\right]^{\frac{1.4-1}{1.4}} \Rightarrow T_2 = 291.6$ K

$V_1 = V_2 = 5.5\ m^3$

$P_2V_2 = m_2RT_2$

$m_2 = \frac{1200 \times 5.5}{0.287 \times 291} = 79.02$ kg.

$m = m_1 - m_2 = 18.3$ kg (left mass)

Steady Flow Energy Equation :-

Consider a control vol of a thermodynamic open system through which working fluid is flowing at a steady state.

The fluid enters with an internal energy (U_1), pressure (P_1), vol (V_1), with a velocity (C_1) & at a height of (Z_1) from the datum. Due to its velocity it possesses K.E & due to its height it possesses P.E. Q is heat supplied & W is the work produced. If fluid leaves system with U_2, P_2, V_2 & C_2 at a height Z_2, A/c to 1st law of TD energy i/p must be equal to energy o/p. Assume m = 1kg of fluid.

Energy	Input	output
① Internal energy	U_1	U_2
② K E	$\frac{1}{2}C_1^2$	$\frac{1}{2}C_2^2$
③ P.E	gZ_1	gZ_2
④ flow work	P_1V_1	P_2V_2

Total energy entering = Total energy leaving

$$Q + \left[U_1 + P_1V_1 + \tfrac{1}{2}C_1^2 + gZ_1\right] = \left[U_2 + P_2V_2 + \tfrac{1}{2}C_2^2 + gZ_2\right] + W$$

$$\left[h_1 + \tfrac{1}{2}C_1^2 + gZ_1\right] + Q = \left[h_2 + \tfrac{1}{2}C_2^2 + gZ_2\right] + W \quad \left[\begin{array}{c}\text{where } h = U+PV \\ \text{enthalpy}\end{array}\right]$$

Enthalpy is defined as the sum of IE & flow work.

$h = U + PV$

Neglecting changes in K.E & P.E. i.e., $c_1 = c_2$, $z_1 = z_2$ we get

$$\therefore h_1 + Q = h_2 + W$$

$$Q = (h_2 - h_1) + W$$

$$\boxed{Q = \Delta H + W}$$

Applications of SFEE :-

Most of the engineering applications are steady flow process. All devices can be divided into 3 categories.

① work producing devices $[W = +ve]$

→ Automobiles, gas turbines, steam engines, turbines, rockets etc.

② work consuming devices $[W = -ve]$

→ Pumps, Air compressor, motors, fridges, A.C, fans etc.

③ No work interacted devices $[W = 0]$. These are also known as neutral devices mainly used heat exchanger etc.,

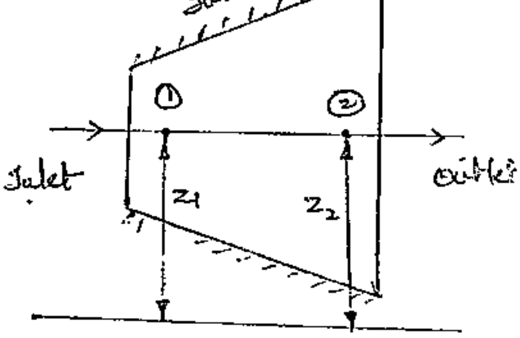
→ Boilers, condensers, radiators, cooling towers, nozzles, throttle valve etc.

Many Energy Analysis are derived steady energy flow equation for following devices :-

(1) Turbine (2) Compressor (3) Heat exchangers
④ Boiler (or) evaporator (or) Steam generator
ⓑ Condensor
(4) Nozzle
(5) Throttle valve.

Turbine :-

Turbine is a rotary device used for producing mechanical energy. Considering one kg of fluid & applying steady flow energy equation.



$$\left[h_1 + \tfrac{1}{2}\cancel{c_1^2} + g\cancel{z_1}\right] + Q = \left[h_2 + \tfrac{1}{2}\cancel{c_2^2} + g\cancel{z_2}\right] + w$$

Assume that change in velocity is very less & hence K.E changes will be neglected.

From diagram $z_1 = z_2$

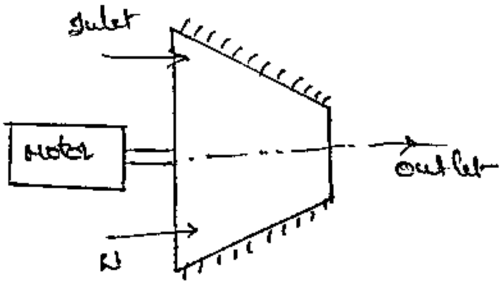
$$\therefore h_1 + Q = h_2 + W \Rightarrow Q = (h_2 - h_1) + w$$

Since system is insulated heat transfer is zero.

$$Q = 0$$

$$\boxed{w = h_1 - h_2}$$

② Compressor :-



Compressor is a device which is used to increase the pressure of air, considering 1 kg of fluid & applying SFEE.

$$(h_1 + \tfrac{1}{2}c_1^2 + gz_1) + Q = (h_2 + \tfrac{1}{2}c_2^2 + gz_2) - W$$

Neglecting K.E & P.E changes, then

$$h_1 + Q = h_2 - W \quad [\because \text{Bcoz of insulation } Q = 0]$$

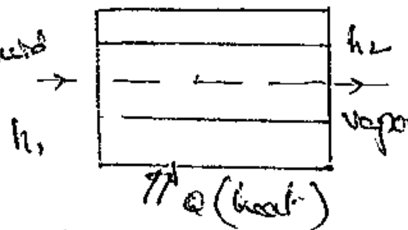
$$\therefore \boxed{W = h_2 - h_1}$$

③ Heat Exchangers :-

ⓐ Boiler or Evaporator

Boiler is a device which is used to convert liquid into vapour convection ie., to evaporate liquid (or) to produce steam.

$$\left[h_1 + \tfrac{1}{2}c_1^2 + gz_1\right] + Q = \left[h_2 + \tfrac{1}{2}c_2^2 + gz_2\right] + w$$



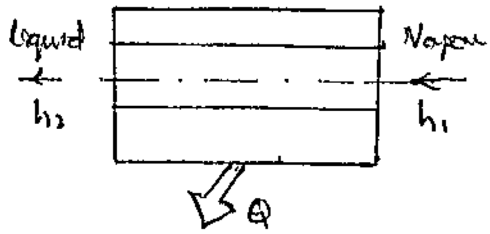
Neglecting KE & PE changes then

$$h_1 + Q = h_2 + W$$

Boiler is a non work interaction device. Hence workdone W=0.

$$\therefore \boxed{Q = h_2 - h_1}$$

③ Condensor :-



$$\left[h_1 + \frac{1}{2}C_1^2 + gz_1\right] - Q = (h_2 + \frac{1}{2}C_2^2 + gz_2) + W$$

Neglecting KE & PE,

$$h_1 - Q = h_2 + W$$

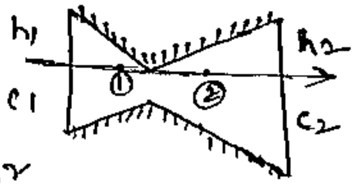
[W=0, since No work interaction device]

$$\therefore -Q = h_2 - h_1$$

$$\boxed{Q = h_1 - h_2}$$

Condensor is a device to convert vapour to liquid i.e., heat rejection process.

④ Nozzle :-



$$\frac{1}{2}C_1^2 + h_1 + Q = h_2 + W + \frac{1}{2}C^2$$

$\boxed{W=0}$ NO WORK

The function of nozzle is to increase velocity of fluid i.e K.E considering 1kg of fluid.

and Q=0 Since it is insulated heat Transfer is '0'

$$\frac{1}{2}C_1^2 + Q = h_2 - h_1 + \frac{1}{2}C_2^2$$

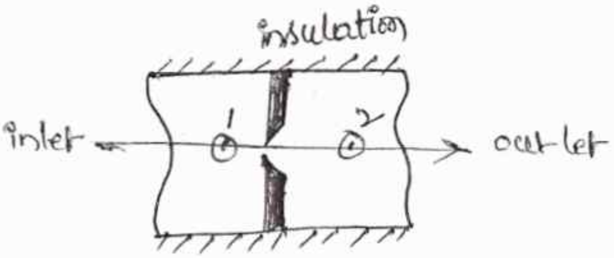
$$\frac{1}{2}C_1^2 + h_1 = \frac{1}{2}C_2^2 + h_2$$

$$h_1 - h_2 = \frac{1}{2}\left(C_2^2 - C_1^2\right)$$

$$C_1^2 + 2\left[h_1 - h_2\right] = C_2^2$$

$$\boxed{C_2 = \sqrt{2(h_1 - h_2) + C_1^2}}$$

⑤ Throttle valves:-



This is mainly used in Refrigirator and A.c's. throttle valve is used to reduce temp of fluid.

When fluid flows through a very small passage like orifice, there is decreasing pressure & flow is said to be "throttled."

Consider in 1 kg of fluid & applying energy equations.

$$\left[h_1 + \tfrac{1}{2}C_1^2 + g z_1\right] + Q = \left[h_2 + \tfrac{1}{2}C_2^2 + g z_2\right] + W$$

$Q = 0$ [∵ insulation]

$W = 0$

$$\boxed{\therefore h_1 = h_2}$$

Throttling process is also known as Constant enthalpy process (Isoenthalpy process)

① Steam enters the steam condenser with an enthalpy of 2090 kJ/kg & velocity 510 m/s & after the condensation process ... at enthalpy of 209 kJ/kg & with a velocity of 10 m/s. Determine heat rejected to cooling water per kg of steam ~~[illegible]~~.

Sol: $\left[h_1 + \tfrac{1}{2}C_1^2 + g z_1\right] + Q = \left[h_2 + \tfrac{1}{2}C_2^2 + g z_2\right] + W$

m/Sec

$Q = -ve$

$h_1 = 2090$ kJ/kg $\quad h_2 = 209$ kJ/kg

$C_1 = 510$ m/s $\quad C_2 = 10$ m/s

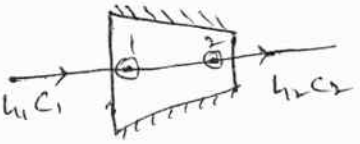
$2090 + \underset{= ⓙ}{\tfrac{1}{2}(510)^2} + Q = 209 + \tfrac{1}{2}(10)^2 + W \quad [W = 0]$

$Q = 2011$ kJ/kg

$Q = -2011$ kJ/kg

$\tfrac{1}{2} \times mv^2$
$= kg\,\frac{m^2}{sec^2}$
$= \frac{kg\,m}{sec^2} \cdot m$
$= N\text{-}m = J$

② A gas leaving turbojet engine flows into jet pipe with an enthalpy of 960 kJ/kg & velocity of 250 m/s. The exit from pipe is at an enthalpy of 800 kJ/kg. The exhaust is inline with intake. Neglecting heat losses from system, determine velocity of gas leaving the pipe.



Sol: $h_1 = 960$ kJ/kg $\quad C_1 = 250$ m/s

$h_2 = 800$ kJ/kg $\quad C_2 = ?$

$$\left[h_1 + \frac{1}{2}c_1^2 + gz_1\right] + \cancel{Q} = \left[h_2 + \frac{1}{2}c_2^2 + gz_2\right] + \cancel{W}$$

$\left[\begin{array}{l} Q = 0 \Rightarrow \text{insulation} \\ W = 0 \Rightarrow \text{No work interacting device} \end{array}\right]$

$$c_2 = \sqrt{c_1^2 + 2(h_1 - h_2) \times 1000}$$

$$= \sqrt{(250)^2 + 2(960 - 800) \times 10^3}$$

? $C_2 =$ ~~512.34~~ m/s. ~~258.63~~ m/s 618.46 618.4 m/s

③ A steam turbine operates under steady flow conditions, it receives 7200 kg/hr of steam from boiler. Steam enters turbine at an enthalpy of 2800 kJ/kg. The velocity of steam is 4000 m/min at an elevation of 4 m. ~~The velocity of~~ steam ~~is~~ leaves turbine at an enthalpy of 2000 kJ/kg with ~~an~~ velocity 8000 m/min at an elevation of 1 m. Due to radiation heat losses from the turbine to the surrounding is 1580 kJ/hr. Calculate o/p of turbine.

Sol:- $h_1 = 2800$ kJ/kg $\quad h_2 = 2000$ kJ/kg $\quad m = 7200$ kg/hr

$= 2$ kg/sec

$c_1 = 4000$ m/min $\quad c_2 = 8000$ m/min

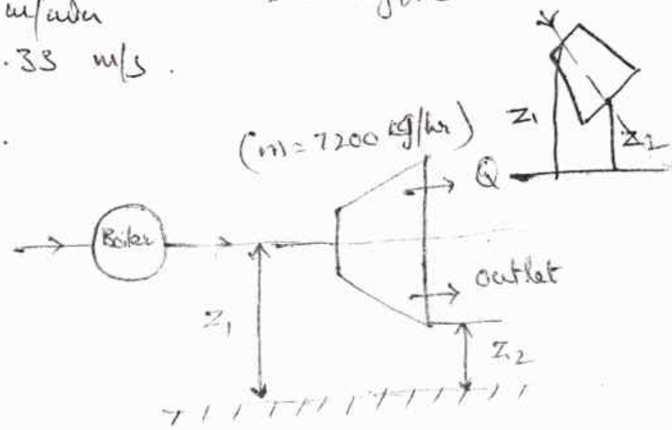
$= 66.67$ m/s $\quad = 133.33$ m/s.

$z_1 = 4$ m $\quad z_2 = 1$ m.

$Q = -1580$ kJ/hr

$= \frac{-1580}{3600}$ kJ/sec

$= -0.438$ kJ/sec



$W = ?$

$$\left[h_1 + \tfrac{1}{2}c_1^2 + gz_1\right] + Q = \left[h_2 + \tfrac{1}{2}c_1^2 + gz_2\right] + W$$

$$\therefore W = \left[(h_1 - h_2) + \frac{1}{2}\frac{(c_1^2 - c_2^2)}{1000}\frac{m}{s^2} + \frac{g(z_1 - z_2)\,m}{1000}\frac{m}{s^2}\right] + Q$$

Heat loss per kg of steam $= \frac{Q}{m} = \frac{-0.438}{2} = -0.219$ kJ/kg

$W = ~~800~~ + \frac{1}{2}($ ~~444~~

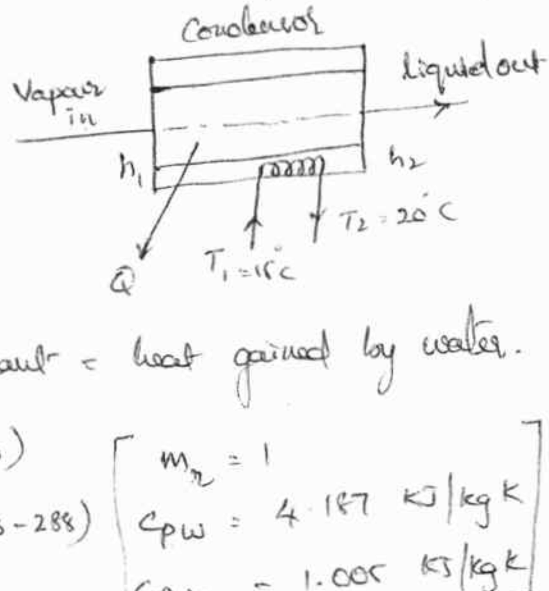
~~$W = 793.64$ kJ/kg~~ $\quad - 5832_{kJ/kg}$ kJ/kg

Output of the turbine $W = 793.6 \times 2$ kg/sec

$W = 1587.2$ kW.

07 Feb

③ Refrigerant vapour enters condenser of refrigeration plant with enthalpy of 223.75 kJ/kg & leaves with an enthalpy of 64.6 kJ/kg. Cooling water enters at 15°C & leaves at 20°C. Calculate mass flow rate of water per unit flow rate of refrigerant.



Sol:- $h_1 = 223.75$ kJ/kg

$h_2 = 64.6$ kJ/kg

$T_1 = 15 + 273 = 288$ K ; $T_2 = 20 + 273 = 293$ K

A/c to 1st law heat loss by refrigerant = heat gained by water.

$$m_r (h_1 - h_2) = m_w\, c_{pw} (T_2 - T_1)$$

$$1(64.6 - 223.75) = m_w (4.187)(293 - 288)$$

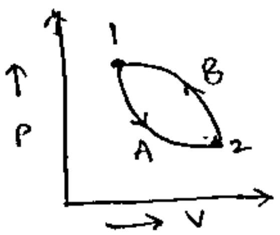
$$m_w = +7.602 \text{ kg/sec}$$

$\left[\begin{array}{l} m_r = 1 \\ c_{pw} = 4.187 \text{ kJ/kgK} \\ c_{pair} = 1.005 \text{ kJ/kgK} \end{array}\right]$

④ Show that I.E of a system is a point function & thermodynamic property of a system?

Sol:- Internal Energy:- Energy stored in the body is called I.E. It is neither heat nor work. It is associated with motion of molecules, atoms & sub-atomic particles.

Consider a closed system from ① to ② through path A & returned to its initial state through B. The cycle is 1-A-2-B-1



A/c to 1st law of TD for cyclic process

$$\Sigma Q = \Sigma W$$

$$Q_A + Q_B = W_A + W_B \longrightarrow ①$$

Applying 1st law for states A & B

$$Q_A = \Delta E_A + W_A \longrightarrow ②$$

$$Q_B = \Delta E_B + W_B \longrightarrow ③$$

$$Q_A + Q_B = (\Delta E_A + \Delta E_B) + (W_A + W_B)$$

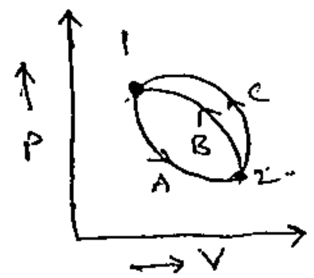
$$\cancel{W_A + W_B} = (\Delta E_A + \Delta E_B) + \cancel{(W_A + W_B)}$$

$$\Delta E_A + \Delta E_B = 0$$

$$\boxed{\Delta E_A = -\Delta E_B} \longrightarrow ④$$

Change in I.E of system of path A & B is same. ((-ve) sign is due to change in direction).

checking for one more path or process



considering a system returning to its initial state through path C. Now the cycle is 1-A-2-C-1.

$$\Sigma Q = \Sigma W$$

$$Q_A + Q_c = W_A + W_c \longrightarrow ⑤$$

$$Q_A = \Delta E_A + W_A \longrightarrow ⑥$$

$$Q_C = \Delta E_c + W_c \longrightarrow ⑦$$

$$Q_A \cancel{+} Q_c = (\Delta E_A + \Delta E_c) + (W_A \cancel{+} W_c)$$

$$\boxed{\Delta E_A = -\Delta E_c} \longrightarrow ⑧$$

From eq. ⑧ it is evident that change in I.E. is same for path A & path C.

From eq ④ & eq. ⑧, we can write as follows

$$\boxed{\Delta E_A = \Delta E_B = \Delta E_C}$$ (Neglecting sign)

Hence I.E is a point function. From above I.E is same b/w two fixed state point & it is not depend on path (or) process. Hence the I.E of system is point function & properties of system.

④ A cylinder contains 0.115 m³ of gas at 1 bar & 90°C. The gas is compressed to a volume of 0.0288 m³, the final pressure being 5.67 bar. Calculate ⓐ the mass of gas. ⓑ the value of index of compression ⓒ the increase in internal energy of gas ⓓ the heat transfer during the compression.

If after the above compression, the gas is to be cooled at constant pressure to its original temp of 90°C, find the further work of compression required, assume $\gamma = 1.4$ & $R = 0.3$ KJ/kg K.

Sol:

$V_1 = 0.115 \ m^3$

$P_1 = 1 \times 10^2$ KPa

$T_1 = 90°C = 363$ K

$V_2 = 0.0288 \ m^3$

$P_2 = 567$ KPa

$\gamma = 1.4$

$R = 0.3$ KJ/kg K

find $m = ?$

$n = ?$

$\Delta U_{1-2} = ?$

$Q_{1-2} = ?$

If $P_3 = P_2$

$T_3 = T_1 = 363$ K

$W_{2-3} = ?$

we have,

$$P_1V_1 = mRT_1$$

$$m = \frac{P_1V_1}{RT_1} = 1.056\times10^{-1}\ \text{kg}.$$

$$x^n = y$$

$$n = \frac{\log y}{\log x} = \frac{\log 5.67}{\log 3.99}$$

$$n = 1.253.$$

$$P_1V_1^n = P_2V_2^n \Rightarrow \left(\frac{V_1}{\cancel{V}_2}\right)^n = \left(\frac{P_2}{P_1}\right)$$

$$\left[n = \log_{(0.25)} 0.176\right]$$

$$\left(\frac{0.115}{0.0288}\right)^n = \left(\frac{567}{100}\right) \Rightarrow n = 1.253.$$

$$T_1V_1^{n-1} = T_2V_2^{n-1}$$

$$\frac{T_2}{T_1} \quad \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$\& \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{0.115}{0.0288}\right)^{0.253}$$

$$T_2 = 515.27\ \text{K}$$

$$C_v = \frac{R}{\gamma-1} = \frac{0.3}{1.4-1} = 0.75\ \text{KJ/kg K}$$

$$C_p = R + C_v = 1.05\ \text{KJ/kg K} \qquad [R = C_p - C_v]$$

$$\Delta U_{1-2} = mC_v(T_2 - T_1)$$

$$= 12.059\ \text{KJ}$$

$$W_{(1-2)} = \frac{P_1V_1 - P_2V_2}{n-1} = -19.08\ \text{KJ}$$

$$\therefore Q = \Delta U + W = 12.059 - 19.08 = -7.03\ \text{KJ}$$

$$W_{2-3} = \boxed{mC_p(T_3 - T_2)} \times = 1.056\times10^{-1}\times1.05(363 - 515.27)$$

$$= -24.94\ \text{KJ}\ //$$

(*) show that $pv^n = c$ for reversible adiabatic process.

sol:

$$PV = mRT$$

Differentiating on both sides, we get

$$pdv + Vdp = mRdT$$

$$mRdT = pdv + vdp$$

$$dT = \frac{pdv + vdp}{mR} \longrightarrow ①$$

In adiabatic, heat transfer $\delta Q = 0$

$$\delta Q = \delta u + \delta w \Rightarrow \delta w = -\delta u$$

$$pdv = -mC_v dT$$

$$pdv + mC_v dT = 0 \rightarrow ②$$

Substitute eq ① in ②

$$pdv + \cancel{m} c_v \left(\frac{pdv + vdp}{\cancel{m} R} \right) = 0$$

$$\frac{PRdv + c_v (pdv + vdp)}{R} = 0 \Rightarrow c_v(pdv + vdp) + PRdv = 0$$

$$c_v (pdv + vdp) + P(c_p - c_v) dv = 0 \quad \left[\because R = c_p - c_v \right]$$

$$c_v p\cancel{dv} + c_v vdp + P c_p dv - P \cancel{c_v dv} = 0$$

$c_v Vdp + c_p Pdv = 0$, Dividing both sides by $c_v PV$

$$\frac{\cancel{c_v} \cancel{V} dp}{\cancel{c_v} P \cancel{V}} + \frac{c_p \cancel{P} dv}{c_v \cancel{P} V} = 0 \qquad \left[\frac{c_p}{c_v} = \gamma \right]$$

$$\frac{dP}{P} + \gamma \frac{dv}{V} = 0 \quad (\text{Integrate})$$

$$\log_e P + \gamma \log_e V = C_1 \qquad \left[\text{Assume} \because C_1 = \log_e C \right]$$

$$\log_e P + \log_e V^{\gamma} = C_1$$

$$\log_e (pV^{\gamma}) = C_1$$

$$\log_e (pV^{\gamma}) = \log_e C$$

$$\boxed{\therefore PV^{\gamma} = C}$$

UNIT - III

Limitations of First law of TD :-

(i) According to 1st law of TD, heat & work are mutually convertable. This is not true in real practice.

(ii) There is no restriction on the direction of flow of work & heat which is not true.

(iii) In general heat is not completely converted into work.

(iv) High pressure gas expands to low pressure, but the reverse is not possible in general i.e., some external energy is required which violets the 1st law of TD.

(v) In an automobile, if brakes are applied the brake work is converted into heat & dissipated into the atmosphere. If reverse process is considered i.e., by giving the heat to brakes wheel, it should rotate, which is not possible.

(vi) A/c to Joule's experiment work can be completely converted into heat but in general complete conversion of heat into work is not possible becoz of losses.

$$W \xrightarrow[=]{} Q$$

$$Q \xrightarrow[>]{} W$$

Thermal Reservoir :-

→ It is a reservoir having infinite amount of heat quantity in it such that when we add or remove finite amount of heat from it. Its temp will not change.

→ Thermal reservoir is a body of infinite heat capacity, which is capable of absorbing or rejecting unlimited qty of heat without affecting its temperature.

→ All the process associated with a thermal reservoir are assumed to be quasi-static. [ex:- ocean water, atm.air]

Heat Source:

The thermal reservoir from which heat is transferred to a system during a cycle is called "heat source".

Ex: Boiler furnace, Sun, Nuclear reactors, combustion chamber of an engine or turbine etc.

Heat Sink:-

The thermal reservoir to which heat is rejected from the system during a cycle is called as "heat sink".

Ex: Atmospheric air, river, sea (or) ocean water etc.

Heat Engine:-

A heat engine is a device which converts heat energy (low grade) into mechanical work (High grade energy) [ex:- Steam power plant]

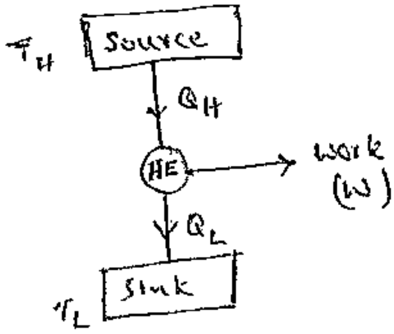
$$\text{Thermal efficiency } \eta_{th} = \frac{O/P}{I/P} = \frac{\text{Net workdone}}{\text{Heat supplied}}.$$

Q_H → heat supply from source

Q_L → heat rejected to sink

T_H → highest (or) hot body temp

T_L → lowest (or) cold body temp



$\eta = \frac{O/P}{I/P} = \frac{W}{Q}$ where $W = Q_H - Q_L$

$Q = Q_H$

$$\therefore \boxed{\eta = \frac{Q_H - Q_L}{Q_H}} \quad (\text{or}) \quad \boxed{\eta = \frac{T_H - T_L}{T_H}}$$

① A heat engine, one heat pump & a refrigerator are working b/w two heat reservoirs one at 600° K, the other at 300° K

calculate ① Efficiency of heat engine

② COP_{HP}

③ COP_R

Sol: ① Efficiency of HE = $\frac{T_H - T_L}{T_H} = \frac{600-300}{600} = \frac{3}{6} = 0.5 = 50\%$.

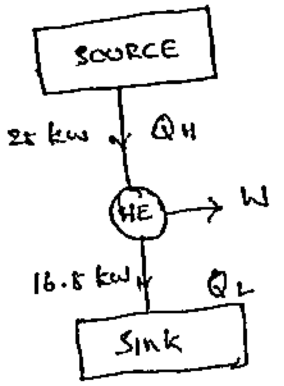
② $COP_{HP} = \frac{T_H}{T_H - T_L} = \frac{600}{800-300} = 2$.

③ $COP_R = \frac{T_L}{T_H - T_L} = \frac{300}{600-300} = 1$

② A Heat engine receives heat at the rate of 1500 KJ/min & gives an o/p of 8.2 kW. Determine

ⓐ Thermal efficiency

ⓑ Rate of heat rejection



Sol:

$Q_H = 1500$ KJ/min

$Q_H = \frac{1500}{60}$ KJ/sec $= 25$ kW

$W = 8.2$ KW

$\eta = \frac{Q_H - Q_L}{Q_H} = \frac{W}{Q_H} = \frac{8.2}{25} = \underline{\underline{32.8\%}}$.

$W = Q_H - Q_L \Rightarrow -Q_L = -16.8$ KW

$\boxed{Q_L = 16.8 \text{ KW}}$

③ If H.E, H.P & refrigerator receives 500 KJ of heat each but they reject 250 KJ, 800 KJ & 600 KJ of heat respectively.

Determine ① η of HE ② COP_R ③ COP_{HP}

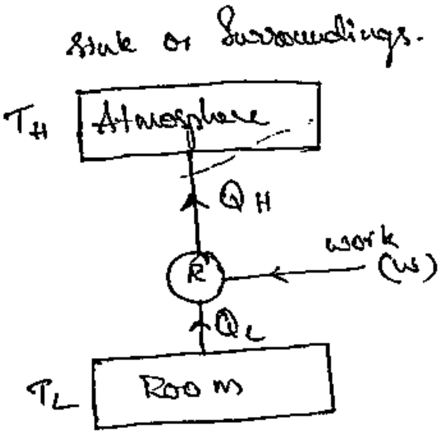
Represent these values with the help of diagrams.

Sol: Efficiency HE, (η) :-

Refrigerator :-

Refrigerator is a device which is used to produce cooling effect & maintaining at low temp.

The efficiency of refrigerator is generally expressed in terms COP (Coefficient of performance).



$$\text{Efficiency } \eta = \frac{o/p}{i/p}$$

$$= \frac{\text{cooling effect}}{\text{work}} = \frac{Q_L}{W}$$

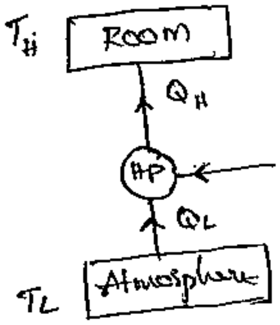
$$COP = \frac{Q_L}{Q_H - Q_L} \quad \left[\begin{array}{l} Q_L + W = Q_H \\ \therefore W = Q_H - Q_L \end{array}\right]$$

(or) $COP = \frac{T_L}{T_H - T_L}$ Ex: Refrigerators, A.c's etc.,

Heat Pump :-

Heat pump is a device which delivers heat energy from low temp to high temp while working in a cycle.

COP of heat pump can be represented as follows.



$$COP_{HP} = \frac{o/p}{i/p} = \frac{\text{heating effect}}{\text{work}} = \frac{Q_H}{W}$$

$$Q_H = Q_L + W$$

$$W = Q_H - Q_L$$

$\boxed{(COP)_{HP} = \frac{Q_H}{Q_H - Q_L}}$ (or) $\boxed{(COP)_{HP} = \frac{T_H}{T_H - T_L}}$

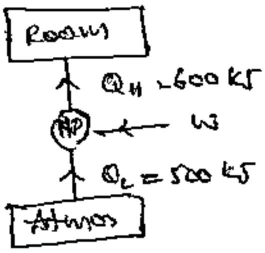
Note :- Derive the relation b/w COP of heat pump & COP of Refrigerator.

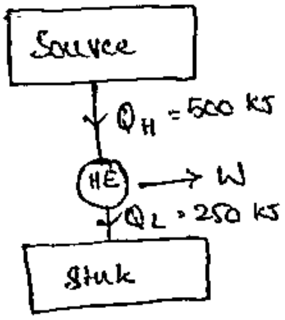
$$\frac{Q_H}{Q_H - Q_L} - \frac{Q_L}{Q_H - Q_L} = 1$$

$$\boxed{(COP)_{HP} - (COP)_R = 1}$$

$$\eta = \frac{O/P}{I/P} = \frac{Q_H - Q_L}{Q_H}$$

$$= \frac{500 - 250}{500} = 0.5$$

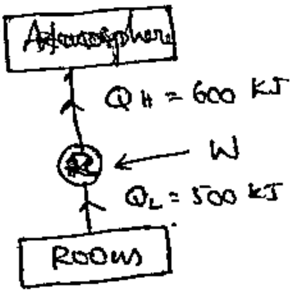




$\eta = 50\%$

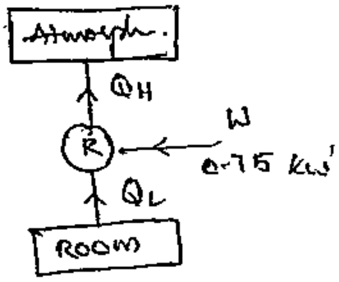
② $COP_R = \frac{O/P}{I/P} = \frac{Q_L}{Q_H - Q_L} = \frac{500}{100} = 5$

③ $COP_{HP} = \frac{O/P}{I/P} = \frac{Q_H}{Q_H - Q_L} = \frac{600}{100} = 6$



④ Calculate COP & Heat transfer to condensor of a refrigerator in KJ/hr which has refrigeration capacity 12000 KJ/hr when power input is 0.75 KW.

Sol:- $Q_L = 12000$ KJ/hr
$= 200$ KJ/min



$W = 0.75$ KW
$= 0.75 \times 3600 = 2700$ KJ/hr

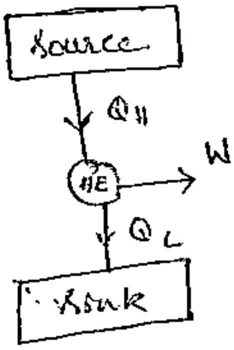
$Q_H = Q_L + W$
$= 12000 + 2700 = 14700$ KJ/hr

$COP = \frac{Q_L}{Q_H - Q_L} = \frac{12000}{2700} = 4.44$.

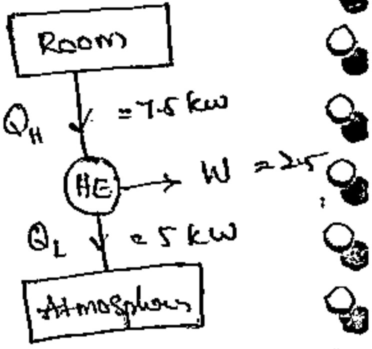
⑤ Reversible H.E operates b/w 2 reservoirs at 420 K & 280 K. If o/p from engine is 2.5 KJ. Determine efficiency of engine and its heat interaction with reservoirs. Subsequently engine is made to operate as a heat pump. b/w same reservoir. Calculate COP_{HP}. and power input required when heat transfer rate from 280 K reservoir is 5 KW.

Sol: $\eta = \frac{T_H - T_L}{T_H} = \frac{420 - 280}{420} = 0.33$

$\eta = 33.3\%$



W = 2.5 kJ



$Q_H = Q_L + W$

$\eta = \frac{O/P}{I/P}$

$0.333 = \frac{W}{Q_H} = \frac{2.5}{Q_H}$

$\therefore Q_H = 7.5$ kJ

$W + Q_L = Q_H \Rightarrow Q_L = Q_H - W = 5$ kJ

$COP_{HP} = \frac{420}{420 - 280} = 3.$

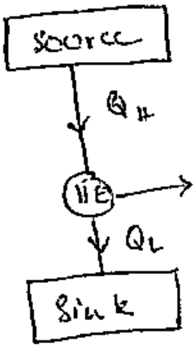
$3 = \frac{Q_H}{Q_H - Q_L} \Rightarrow Q_H = 3Q_H - 3Q_L$

$2Q_H = 3Q_L$

$Q_H = \frac{3}{2} \times 5 = 7.5$ kW.

⑥ A reversible heat engine operating b/w 900 K & 300 K is used to a reversible refrigerator for which temp limits are 300 K & 250 K. The engine absorbs 1800 kJ of energy as heat from reservoir at 900 K. The net output from engine & refrigerator system is 360 kJ. Make calculation for heat extracted from refrigerator cabinet & total heat rejected at 300 K.

Sol:

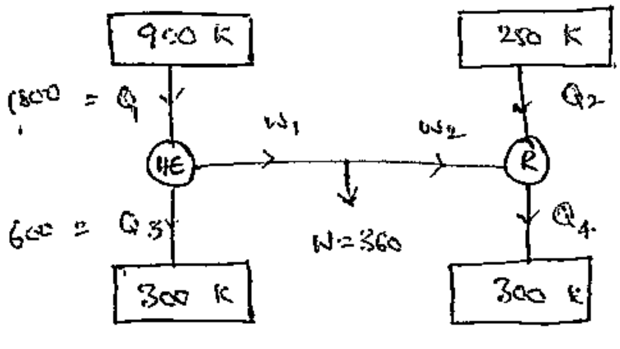


$\eta = \frac{T_H - T_L}{T_H} = \frac{900 - 300}{900}$

$\eta = 66.67\ \%.$

$0.667 = \frac{1800 - Q_L}{1800}$

$Q_L = 600$ kJ



$COP_R = \frac{T_L}{T_H - T_L} = \frac{250}{300 - 250}$

$COP_R = 5$

$$COP_R = \frac{Q_L}{Q_H - Q_L} \Rightarrow 5 = \frac{Q_2}{Q_4 - Q_2} \rightarrow ①$$

$W_1 = Q_1 - Q_3$

$W_1 = 1800 - 600 = \underline{1200 \text{ KJ}}$

Given, $W = W_1 - W_2$

$W_2 = 360 - 1200 = -840$

$W_2 = \underline{840} \text{ KJ}$

$\therefore Q_4 - Q_2 = W_2 = \underline{840 \text{ KJ}}$

$$\therefore 5 = \frac{Q_2}{Q_4 - Q_2} \Rightarrow 5 = \frac{Q_2}{840} \Rightarrow Q_2 = 4200 \text{ KJ} \rightarrow ②$$

$\therefore Q_4 - Q_2 = 840$

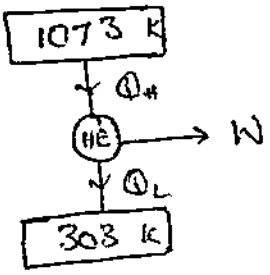
$Q_4 = 840 + 4200 = 5040 \text{ KJ} \rightarrow ③$

Heat extracted from refrigerator $Q_2 = 4200$ KJ → ④

Total heat rejected at 300 K reservoir,

$Q_{3_{HE}} + Q_{4_R} = 5040 + 600 = 5640 \text{ KJ} \rightarrow ⑤$

⑦ Cyclic H.E operates b/w source temp of 800°C & sink temp of 30°C, what is the least rate of heat of rejection per KW, net o/p of engine



Sol:
$$\eta = \frac{o/p}{i/p} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}$$

$$= \frac{T_H - T_L}{T_H} = \frac{1073 - 303}{1073}$$

$\eta = 71.76\%$

$\eta = \frac{W}{Q_H} \Rightarrow 0.7176 = \frac{W}{Q_H} \Rightarrow Q_H = 1.3927$

$W = Q_H - Q_L \Rightarrow Q_L = 1 - 1.3935$

$Q_L = 0.3927$ KW.

SECOND LAW OF TD :-

It is stated in two ways i.e., through

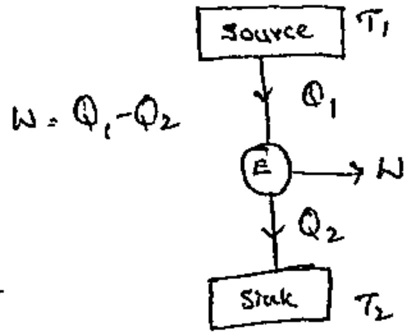
(i) Kelvin - Planck Statement

(ii) Clausius Statement

Kelvin - Planck's Statement :-

"It is impossible to construct an engine working on a cyclic process, whose sole purpose is to convert all the heat supplied to it into equivalent amount of work."

If $Q_2 = 0$ $\left(\text{ie., } W_{net} = Q_1 \text{ or } \eta = 100\% \right)$, the heat engine will produce net work in a complete cycle by exchanging heat with only one reservoir, thus violating the Kelvin - Plancks statement.

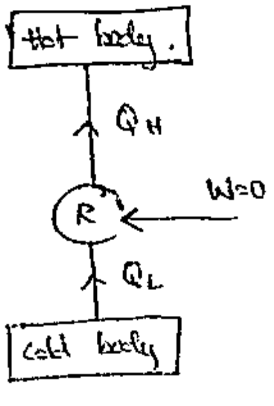


Such a heat engine is called a Perpetual motion machine of the second kind (PMM2). A PMM2 is impossible.

Clasius Statement :-

"It is impossible for the heat to flow from a body at a lower temp to a body at higher temp without aid of external agency". (or)

"Heat flows from a hot body to a cold body."



Corallaries of Second law :-

① NO engine, working on a cyclic process is more efficient than (Carnot's) reversible engine when working b/w the same two limits of temperature.

② All reversible engines have the same efficiency when working b/w the same two reservoirs of heat.

③ whenever a system executes a complete cyclic process the $\oint dQ/_T$ around the cycle is less than zero or in the limit is equal to zero. This shows the inequality of Clausius.

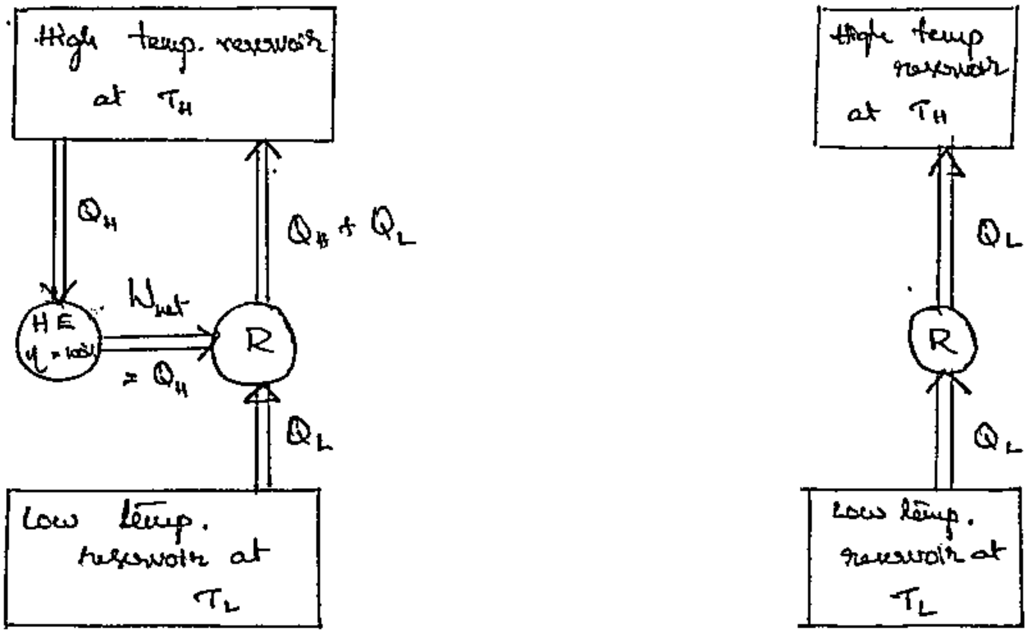
④ $\oint dQ/_T = 0$ for any reversible cyclic process & consequently for any reversible process b/w reference states ① & ② is a property of the system in state ②. This property is called 'entropy.'

⑤ A true absolute TD temp scale may be defined which is independent of the characteristics of the thermodynamic medium.

Equivalence of the two statements :-

The Kelvin Planck & the Clausius statements are equivalent in their consequence, & either statement can be used as the expression of II law of TD. Any device that violates the Kelvin-Plancks statement also violates the clausius statement & vice versa.

Consider the heat engine - refrigerator combination as shown in fig., operating b/w the same two reservoirs. The heat engine is assumed to have, in violation of Kelvin-Planck statement, a thermal η of 100%. & therefore it converts



ⓐ A refrigerator that is powered by a 100% efficient heat engine.

ⓑ The equivalent refrigerator

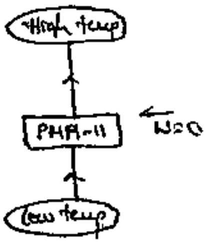
all the heat Q_H it receives, to work W. This work is now supplied to a refrigerator that removes heat in the amount of Q_L from the low temp reservoir & rejects heat in the amount of $Q_L + Q_H$ to the high temp reservoir. During this process, the high temp reservoir receives a net amount of heat Q_L (the diff b/w $Q_L + Q_H$ and Q_H). Thus, the combination of these two devices can be viewed as a refrigerator, that transfers heat in an amount of Q_L from a cooler body to a warmer one without requiring any input from outside. This is clearly a violation of the clausius statement.

It can also be shown in a similar manner that a violation of the Clausius statement leads to the violation Kelvin-Planck statement. Therefore, both are two equivalent expressions of II law of TD.

Perpetual Motion Machine of Second Kind (PMM II) :-

It is impossible to construct a PMM II. It is a hypothetical m/c that continuously supply heat from low temp reservoir to high temp without taking any input or work from surroundings.

But such a m/c is not feasible from practical point of view. It violates II law of TD. (Clausius statement)



Carnot Cycle / Engine :-

It is a reversible cycle which consists of all reversible process. i.e.,

- → Reversible isothermal heat addition
- → Reversible adiabatic expansion
- → Reversible isothermal heat rejection
- → Reversible adiabatic compression.

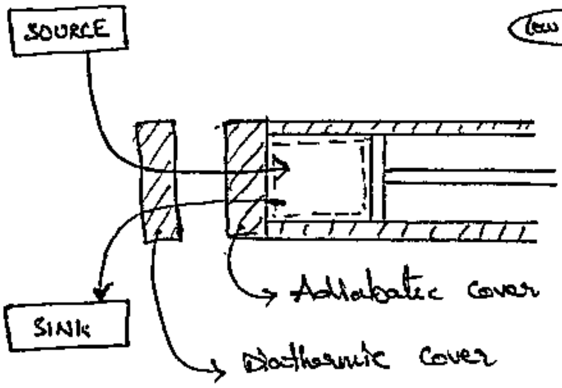
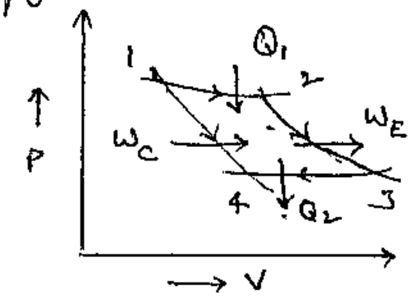


fig: CARNOT ENGINE

Carnot has proposed a piston cylinder arrangement with working medium & two covers as shown in fig.



Process (1-2) : $T_H = C$

By ~~applying~~ placing diathermic cover an amount of heat Q_1 is supplied from the source at constant temp T_1.

Process (2-3) :- (drops T_H to T_L) The diathermic cover is replaced by an adiabatic cover & the medium with its stored energy pushes the piston forward by developing work W_E adiabatically & reversibly & the temp of medium decreases from T_H to T_L

Process (3-4) :- $(T_L = C)$ The adiabatic cover is replaced by diathermic cover & Q_2 amount of heat is made to leave the system at T_2 to a sink at T_2 reversibly.

Process (4-1) :- $(T_L \text{ to } T_H)$ The diathermic cover is again replaced by adiabatic cover & pump work W_p is done on the medium reversibly & adiabatically till the temp rises to T_1 & system returns to its initial state, as shown in PV diagram.

For a steady flow process,

A/c to I law of TD for a cycle,

$$\Sigma Q = \Sigma W$$

$$Q_H - Q_L = W_E - W_c$$

$\therefore$ Efficiency of Carnot engine $= \dfrac{\text{Net work output}}{\text{Heat supply}}$

$$\eta_{carnot} = \frac{W_E - W_c}{Q_H} = \frac{Q_H - Q_L}{Q_H}$$

$$\boxed{\therefore \eta_{carnot} = \frac{T_H - T_L}{T_H}}$$

Reversible Carnot Heat Engine :-

Since all the process in the Carnot cycle are reversible, it is possible to reverse the engine. Let individually all the process are reversed then

Process (1-2) → Isothermal heat rejection

(2-3) → Adiabatic pumping

(3-4) → Isothermal heat addition

(4-1) → Adiabatic expansion.

Thus all the energy transfers associated with the process are reversed in direction but their magnitude remains the same, Thus the W_{net} is negative & heat is transferred from lower temp reservoir to higher temp reservoir thus acts as a heat pump or refrigerator.

⊕ Development of Carnot cycle is practically impossible becoz.,

→ All process in nature are irreversible.

→ It is not possible to replace the covers diathermic & adiabatic continuously.

→ Isothermal heat addition & rejection requires infinite time ie, piston has to move infinitely slow where adiabatic process (expansion & compression) must be completed very quickly. Two kinds of motions cannot be obtained.

→ Friction less motion of piston is not possible.

→ The curves isothermal & adiabatic when drawn on P-V diagram due to the small diff. in their slope they will not converge easily & requires a large stroke. Thus V_s will be very high with less enclosed area i.e., work development.

Thermodynamic temp Scale :-

we know that η of reversible carnot heat engine is a function of temp only.

The absolute temp scale is also known as Kelvin scale. It depends on heat absorbed & heat rejected & is independent of peculiar characteristics of working substance. In defining Kelvin scale the triple point of water ($0^\circ C = 273.16\,K$) is taken as standard reference point.

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \quad (\text{or}) \quad T = 273.16 \frac{Q}{Q_t} \; . \left[\frac{Q}{Q_{tp}} = \frac{T}{T_{tp}}\right]$$

Thus the absolute TD temp scale has a definite zero point of water which can be attained by imaging a series of reversible engines contending from source T_1 to a lower temp. If enough engines are placed in series to make the total work output equal to Q_1, then the heat rejected by last engine is zero. However 2nd law do not allow to develop such an engine.

When the heat rejected approaches zero the temp of heat rejection also approaches a zero limit. Thus it appears a definite zero point exists an absolute temp scale but the point can't be reached without violation of second law.

① An engine operating on Carnot cycle works within temp limit of 600 K & 300 K. If engine receives 2000 KJ of heat. Calculate workdone & thermal efficiency of engine.

Sol:-

$T_1 = 600$ K

$T_2 = 300$ K

$Q_H = 2000$ KJ

$$\eta_{carnot} = \frac{T_H - T_L}{T_H} = \frac{W}{Q_H}$$

$$\Rightarrow \frac{600-300}{600} = \frac{W}{2000}$$

$$Q_H - Q_C = W$$

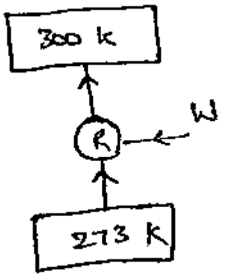
$$W = 1000 \text{ KJ}$$

$$2000 - Q_L = 1000$$

$$\eta_{th} = 0.5 = 50\%$$

$$Q_L = 1000 \text{ KJ}.$$

② Determine power required to run refrigerator that transfers 2000 KJ/min heat from a cooled space at 0°C to the surrounding atmosphere at 27°C. The refrigerator operates on reverse Carnot cycle.



Sol: CoP $\eta_{ref} = \frac{T_L}{T_H - T_L} = \frac{Q_L}{W}$

$$CoP = \frac{273}{300-273} = 10.111$$

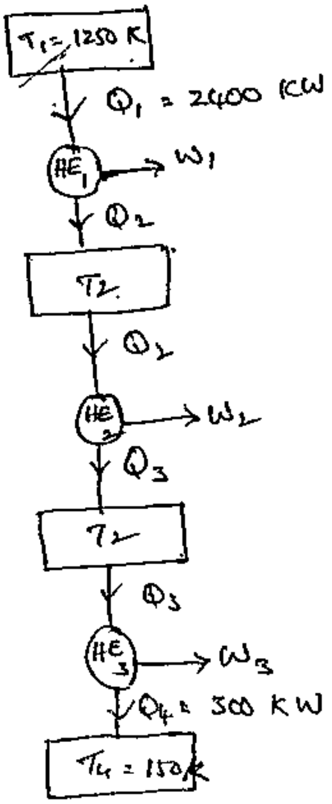
$$10.111 = \frac{Q_L}{Q_H - Q_L} \quad [\because Q_L = 2000 \text{ kJ/min}]$$

$$Q_H = 2197.8 \text{ kJ/min}$$

$$W = Q_H - Q_L = 2197.8 - 2000 = 197.8 \text{ kJ/min}$$

$$\boxed{W = 3.297 \text{ kW}}$$

③ HE₁, HE₂, HE₃ are connected in series as shown. They are working with same thermal efficiency. Calculate W_1, W_2, W_3. (Carnot engine).



Sol: Given

$\eta_1 = \eta_2 = \eta_3$

HE₁ :-

$$\eta_1 = \frac{Q_H - Q_L}{Q_H} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

$$= 1 - \frac{Q_2}{Q_1}$$

$$1 - \eta = \frac{Q_2}{Q_1} \longrightarrow ①$$

HE₂ :-

$$\eta_2 = \frac{Q_2 - Q_3}{Q_2} = 1 - \frac{Q_3}{Q_2}$$

$$1 - \eta_2 = \frac{Q_3}{Q_2} \longrightarrow ②$$

HE_3 :-

$$\eta_3 = \frac{Q_3 - Q_4}{Q_3} = 1 - \frac{Q_4}{Q_3}$$

$$1 - \eta_3 = \frac{Q_4}{Q_3} \longrightarrow ③$$

Multiplying ①, ② & ③, we get.

$$(1-\eta)^3 = \frac{\cancel{Q_2}}{Q_1} \times \frac{\cancel{Q_3}}{\cancel{Q_2}} \times \frac{Q_4}{\cancel{Q_3}} = \frac{Q_4}{Q_1}$$

$$(1-\eta)^3 = \frac{300}{2400}$$

$$1 - \eta = 0.5$$

$$\eta = 0.5 = 50\%$$

$\underline{HE_1}$:- $\eta_1 = \frac{Q_1 - Q_2}{Q_1} = \frac{W_1}{Q_1}$

$W_1 = \eta_1 \times Q_1 = \underline{2400 \times 0.5 = 1200 \text{ kW}}$

$\underline{HE_2}$:- $\eta_2 = \frac{W_2}{Q_2}$ $\qquad \frac{Q_1 - Q_2}{Q_1} = 0.5$

$\therefore W_2 = \eta_2 \times Q_2$ $\qquad 1 - \frac{Q_2}{Q_1} = 0.5 \Rightarrow 1 - \frac{Q_2}{2400} = 0.5$

$= 0.5 \times 1200$ $\qquad \underline{Q_2 = 1200 \text{ kW}}$

$\underline{W_2 = 600 \text{ kW}}$

$\underline{HE_3}$:- $\eta_3 = \frac{W_3}{Q_3}$ $\qquad 1 - \frac{Q_4}{Q_3} = 0.5$

$W_3 = \eta_3 \times Q_3$ $\qquad 1 - \frac{300}{Q_3} = 0.5 \Rightarrow \underline{Q_3 = 600 \text{ kW}}$

$= 0.5 \times 600$

$\underline{W_3 = 300 \text{ kW}}$.

④ 2 Reversible heat engines A & B are connected in series with 'A' rejecting heat directly to 'B' through an intermediate reservoir. Engine 'A' receives 200 KJ of heat from a reservoir at 421°C & engine 'B' is in thermal communication with a sink at 4.4°C. If work output of 'A' is twice of 'B', calculate

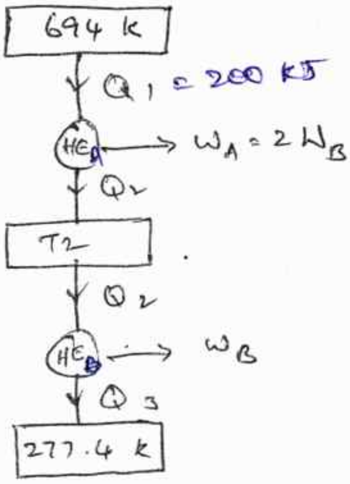
(1) Intermediate temp b/w A & B.

(2) η of each engine

(3) Heat rejected to cold sink.

Draw the operating system for the above data.

Sol: Given



$$W_A = 2 \times W_B$$

HE_A :- $$\eta_A = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\frac{T_1 - T_2}{T_1} = \frac{W_A}{Q_1}$$

$$W_A = Q_1\left[\frac{T_1 - T_2}{T_1}\right] \rightarrow ①$$

HE_B :- $$\eta_B = \frac{T_2 - T_3}{T_2} = \frac{Q_2 - Q_3}{Q_2} = \frac{W_B}{Q_2}$$

$$W_B = Q_2\left[\frac{T_2 - T_3}{T_2}\right] \rightarrow ②$$

We know that $W_A = 2W_B$

$$Q_1\left[\frac{T_1 - T_2}{T_1}\right] = 2Q_2\left[\frac{T_2 - T_3}{T_2}\right]$$

$$\frac{\cancel{Q_2}\cancel{T_1}}{\cancel{T_2}} \times \frac{T_1 - T_2}{\cancel{T_1}} = 2\cancel{Q_2}\left(\frac{T_2 - T_3}{\cancel{T_2}}\right)$$

$$T_1 - T_2 = 2T_2 - 2T_3$$

$$T_1 + 2T_3 = 3T_2$$

$Q_1 = T_1$; $Q_2 = T_2$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\boxed{Q_1 = \frac{Q_2 T_1}{T_2}}$$

$694 + 554.8 = T_2$

$$\boxed{T_2 = 416.26\ K}$$ $\therefore \eta = \frac{T_1 - T_2}{T_1}$

HE$_A$:-

$\eta_A = \frac{694 - 416.26}{694} = 40\ \%$

$\eta_A = \frac{W_A}{Q_1} \Rightarrow W_A = 200 \times 0.4 = 80$ KJ

HE$_B$:- $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$Q_2 = \frac{Q_1}{T_1} \times T_2 = \frac{200}{694} \times 416.26$

$Q_2 = 119.96$ KW

$\eta_B = \frac{Q_2 - Q_3}{Q_2} = \frac{T_2 - T_3}{T_2} = \frac{416.26 - 277.4}{416.26} = 33.3\ \%$

$\eta_B = \frac{W_B}{Q_2} \Rightarrow W_B = \eta_B \times Q_2$
$= 0.333 \times 119.96 = 39.946$ KJ

$\therefore Q_3 = Q_2 - W_B$

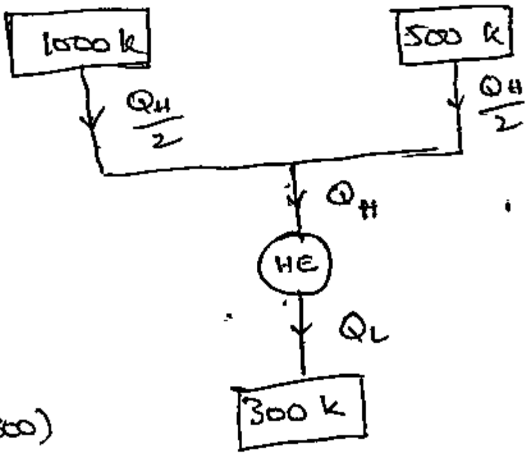
$Q_3 = 120 - 39.6 = 80.4$ KJ.

(5) Consider 3 hypothetical heat engines A, B & C, each operating b/w 1000 K & 300 K. when each engine involves itself with a heat interaction of 1000 kJ with high temp reservoir, it is claimed that while A develops a work of 600 kJ, B & C develops 700 & 800 kJ. Use carnot statement & identify the engines A, B, & C are reversible, irreversible or impossible.

Sol:- $T_1 = 1000$ K $\quad W_A = 600$ kJ
$T_2 = 300$ K $\quad W_B = 700$ kJ
$Q_1 = 1000$ kJ $\quad W_C = 800$ kJ

All the engines are operating b/w same two temp reservoirs, 8 A heat engine receives half of its heat supply at 1000 k and half at 500 k while rejecting at a sink 300 k. What is the maximum thermal efficiency of the heat engine?

Sol: Given,



$T_A = 1000$ k

$T_B = 500$ k

$T_C = 300$ k

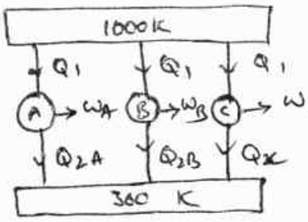
$$\text{Max. } \eta_{th} = 1 - \left[\frac{T_A T_B + T_B T_C}{2 T_A T_B}\right]$$

$$= 1 - \frac{(1000 \times 300) + (500 \times 300)}{2 \times 1000 \times 500}$$

$$= 1 - 0.45 = 0.55$$

$\therefore \eta_{th\ max} = 55\,\%$

$$\therefore \eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{1000} = 0.7$$



Consider engine Ⓐ

$$\eta_A = \frac{W_A}{Q_1} = \frac{600}{1000} = 0.6$$

As $\eta_A < \eta_{max}$, the heat engine is irreversible.

engine Ⓑ :- $\eta_B = \frac{W_B}{Q_1} = \frac{700}{1000} = 0.7$

$\eta_{max} = \eta_B = 0.7$, So it is Reversible.

engine Ⓒ :- $\eta_C = \frac{W_C}{Q_1} = \frac{800}{1000} = 0.8$

$\eta_{max} < \eta_C$, So it is impossible.

Ⓔ A direct heat engine A & a reversed heat engine B operate b/w 177°C & 27°C. The COP of B as a heat pump is 2.5. A drives B. The magnitude of heat interaction of A & B with the reservoir at 27°C are 200 & 50 KJ. The combined work output of A & B is 20 KJ. Identify whether the heat engine A is reversible or irreversible

Sol: $T_1 = 450$ K

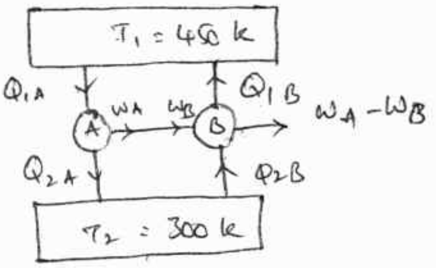
$T_2 = 300$ K

$Q_{2A} = 200$ KJ

$Q_{2B} = 50$ KJ

$(COP)_B = 2.5$

$W_{net} = 20$ KJ



$$(COP)_{HP,B} = \frac{Q_{1B}}{W_B} = 2.5$$

$$\Rightarrow Q_{1B} = W_B \times 2.5 \rightarrow ①$$

Also $W_B = Q_{1B} - Q_{2B} \Rightarrow Q_{1B} = W_B + Q_{2B} \rightarrow ②$

Equate ① & ②, we get $W_B \times 2.5 = W_B + Q_{2B}$

$$W_B \times 2.5 = W_B + 50$$

$$W_B = 33.3 \text{ KJ}$$

From data $W_A = W_B + W_{output}$

$W_A = 33.3 + 20 = 53.3$ kJ

Applying I law to engine A,

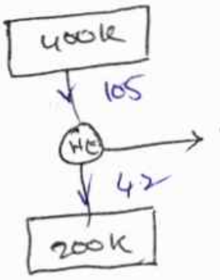
$$Q_{1A} = Q_{2A} + W_A = 200 + 53.3 = 253.3 \text{ kJ}$$

$$\eta_A = \frac{W_A}{Q_1 A} = \frac{53.3}{253.5} = \underline{0.21}$$

but $\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{450} = 0.33$.

As $\eta_A < \eta_{max}$, engine A is irreversible.

⑦ An inventor claims to ~~have~~ developed an engine that takes in 105 MJ at a temp of 400 K, rejects 42 MJ to a temp of 200 K. & delivers 15 KWH of mechanical work. Would you advice investing money to put this engine in the market.



Sol: Amt of heat given by source to the engine $= Q_1 = 105$ MJ

$Q_L = 42$ MJ

$\left(\frac{kJ}{s} \times 3600\,s\right)$

work delivered $= W_{net} = 15$ KWH $= 54$ MJ | $\frac{15 \times 3600}{1000} = 54$ MJ

$$\eta_{HE} = \frac{WD}{HS} = \frac{W}{Q_1} = \frac{54}{105} = 0.5142$$

$\eta_{carnot\ engine}$ $\eta_c = 1 - \frac{T_2}{T_1} = 1 - \frac{200}{400} = 0.5$

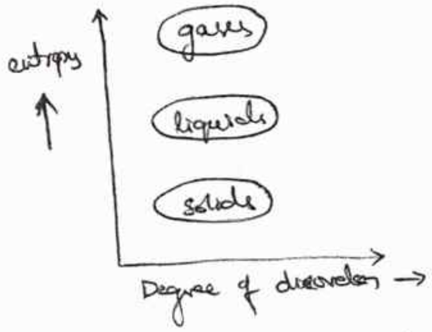
A/c to Carnot principle no engine has high η that Carnot engine

Here $\eta_{HE} > \eta_{carnot}$

∴ It is not advisable to invest money to put this engine into market.

UNIT - IV

Entropy is a measure of molecular disorder.



Consider a tank containing gas. The molecules of gas are in random motion & they move in different directions with different velocities colloinding with each other.

Let us heat the gas, random motion of gas molecules further increases which leads to more irregularity (or) more disorderness. This degree of disorderness existing in a system is known as entropy.

The units of entropy $s = \frac{Q}{T} = KJ/kg\,K$

* I law of TD $\Rightarrow$ $\boxed{Q = \Delta U + W}$

II law of TD $\Rightarrow$ $\boxed{Q = T\,ds}$

Uses of Study of Entropy :-

② ⇒ Since entropy is a property, it can be used to determine state of a substance.

⑦ ⇒ Entropy represents maximum amount of work obtainable per degree drop in temperature.

⑥ ⇒ Entropy is generally assumed as zero at $-273.15°C$ ie at $0\,K$

③ ⇒ Since it depends on mass, entropy is an extensive property.

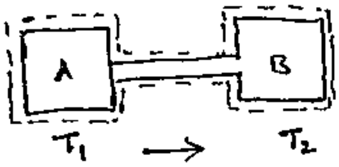
④ ⇒ Entropy / unit mass = sp. entropy (intensive property)

⑧ ⇒ A change in entropy shows that heat energy has been transfered to or from a system & the temp level at which that heat is transfered.

⑩ ⇒ $\oint \frac{dQ}{T} \leq 0$ is called Clausius Inequality.

Applications :-

Heat transfer through finite temp difference :-



let A & B are two bodies with temp's T_1 & T_2 resp.

change in entropy of body A $= \Delta S_A = \frac{-Q}{T_1}$

$B = \Delta S_B = \frac{Q}{T_2}$

For ~~total~~ isolated system $\Delta S = \Delta S_A + \Delta S_B$

$$\Delta S = -\frac{Q}{T_1} + \frac{Q}{T_2} \Rightarrow \boxed{\Delta S = Q\left[\frac{T_1 - T_2}{T_1 T_2}\right]}$$

case(i): if $T_1 > T_2$ then $\Delta S > 0$ i.e., irreversible process

$T_1 < T_2$ then $\Delta S < 0$ i.e., impossible

$T_1 = T_2$ then $\Delta S = 0$ i.e., reversible process

⊛ I law of TD → Internal energy (extensive property)

⊛ II law of TD → Entropy (extensive property)

Entropy :- (S)

→ It is a measure of molecular disorder

① → It is a thermodynamic property

⑤ → It is a point function i.e., its value does not change when the state of the system changes.

⑥ → Therefore entropy change of a system is zero, if state of system does not change during the process.

Ex:- Nozzles, compressor, turbines, pumps etc.

CLASIUS INEQUALITY :-

It is defined as,

"whenever a system executes a complete cycle, the cyclic integral $\oint \frac{\delta Q}{T}$ is less than or in the limit equal to zero."

Consider an engine operating b/w two fixed temp reservoirs. Let dQ_1 units of heat be supplied at temp T_1 & dQ_2 units of heat be rejected at temp T_2 during a cycle. Then thermal efficiency of this engine,

$$\eta_E = \frac{dQ_1 - dQ_2}{dQ_1}$$

thermal efficiency of a reversible engine operating b/w same temp is given by $\eta_R = \frac{T_1 - T_2}{T_1}$

We know that $\eta_E \ngtr \eta_R$ or in limiting case $\eta_E \leq \eta_R$ since no engine can be more efficient than a reversible engine (Carnot's theorem)

$$\therefore \quad \frac{dQ_1 - dQ_2}{dQ_1} \le \frac{T_1 - T_2}{T_1}$$

$$1 - \frac{dQ_2}{dQ_1} \le 1 - \frac{T_2}{T_1}$$

$$\frac{dQ_2}{dQ_1} \le \frac{T_2}{T_1}$$

$$\Rightarrow \frac{dQ_2}{T_2} - \frac{dQ_1}{T_1} \le 0$$

Hence, in the limit $\boxed{\oint \frac{dQ}{T} \le 0}$ where the equality applies in case of reversible cycle & inequality applies in case of irreversible cycle.

<u>Meaning</u> & <u>concept</u> of <u>Entropy</u> :-

Entropy literally means 'transformation'. "It is a thermodynamic state' property of matter which is held constant if no heat is supplied in a reversible adiabatic process."

Entropy increases as heat is supplied to a system & decreases as heat is rejected from a system. But it remains constant if no heat is supplied nor rejected. That is why adiabatic process is called 'Isentropic' process.

If dQ = a small change in heat transfer at T

T = absolute temp

$d\phi$ = change in entropy

Then $d\phi = \frac{dQ}{T}$ or $\phi = \int \frac{dQ}{T}$

For a reversible cycle, $\boxed{\oint \frac{dQ}{T} = 0}$

Further we have $\eta_{carnot} = \frac{\Delta T}{T} = \frac{W}{Q}$

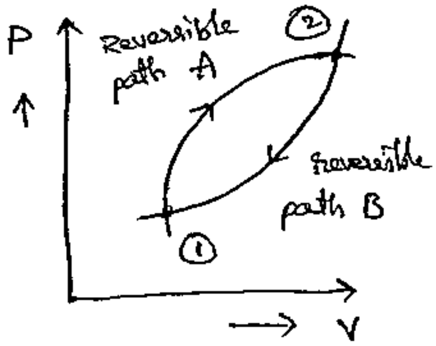
If $\Delta T = 1$, then $W = \frac{Q}{T}$ (or) $dw = \frac{dQ}{T}$

But $\frac{dQ}{T} = d\phi$

$\therefore d\phi = dw$

~~Hence~~ entropy is regarded as a measure of rate of availability of heat for transformation into work.

Entropy — A Property :-



Let us consider two equilibrium states ① & ② connected by two reversible paths A & B as shown.

Now, if path B is reversed then the two paths make the cycle a reversible cycle.

Then $\oint \frac{dQ}{T} = 0$

we have $\oint \frac{dQ}{T} = \int_1^2 \left(\frac{dQ}{T}\right)_A + \int_2^1 \left(\frac{dQ}{T}\right)_B = 0$

$$\int_1^2 \left(\frac{dQ}{T}\right)_A = -\int_2^1 \left(\frac{dQ}{T}\right)_B$$

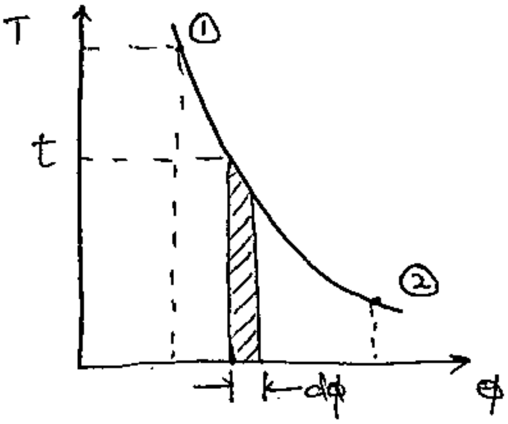
$$\int_{1A}^2 \left(\frac{dQ}{T}\right) = \int_2^1 B\left(\frac{dQ}{T}\right)$$

From the above it is clear that the expression $\int \frac{dQ}{T}$ is independent of the path. It is a point function & hence represents a property of the system. This property is called entropy.

$$d\phi = \left(\frac{dQ}{T}\right)_{rev} \quad \& \quad \int_1^2 d\phi = \phi_2 - \phi_1$$

Temperature entropy (T-φ) diagram :-

A reversible TD process may be plotted on Temp Vs Entropy diagram.



Consider an elemental increase in entropy dφ due to a small amount of heat transfer dQ at an absolute temp T.

$$d\phi = \left(\frac{dQ}{T}\right)_{rev}$$

$$(dQ)_{rev} = T \times d\phi \quad \text{(or) simply} \quad \boxed{dQ = T \times d\phi}$$

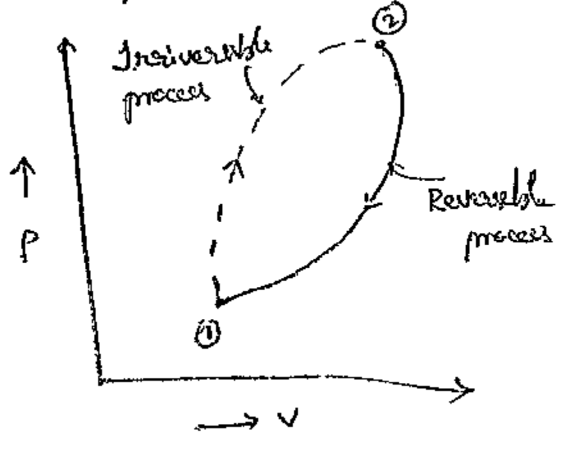
$$[dQ = T\,ds]$$

Thus area under a curve for a reversible process plotted on T-φ diagram, gives the quantity of heat transfered during the process.

Principle of Entropy Increase :-

The entropy of an isolated system increases in all real processes & is conserved (constant) in reversible process. *

In order to prove this let us consider an irreversible process 1-2 along path A shown in fig.



Let the cycle be completed by a reversible process 2-1 along path B, so that process 1-A-2 & 2-B-1 together form an irreversible cycle. From the Clausius Inequality, $\oint \frac{\delta Q}{T} < 0$ for irreversible process

$$\text{(or)} \quad \underset{\text{path-A}}{\int_1^2 \left(\frac{\delta Q}{T}\right)_{irre}} + \underset{\text{path-B}}{\int_2^1 \left(\frac{\delta Q}{T}\right)_{rev}} < 0$$

$$\text{But} \quad \underset{\text{path-B}}{\int_2^1 \left(\frac{\delta Q}{T}\right)_{rev}} = \underset{\text{path-B}}{\int_2^1 ds}$$

But since path-B is reversible, the limits of the integral can be reversed & therefore

$$\int_2^1 \left(\frac{\delta Q}{T}\right)_{rev} = -\int_1^2 ds$$

$$\therefore \underset{\text{path-A}}{\int_1^2 \left(\frac{\delta Q}{T}\right)_{irrev}} - \underset{\text{path-B}}{\int_1^2 ds} < 0$$

$$\text{(or)} \quad \int_1^2 ds > \int_1^2 \left(\frac{\delta Q}{T}\right)_{irrev}$$

$$\text{(or)} \quad ds > \left(\frac{\delta Q}{T}\right)_{irrev} \qquad \text{But } ds = \left(\frac{\delta Q}{T}\right)_{rev}$$

Hence in general, we may write $ds \geq \frac{\delta Q}{T}$

$$\text{or} \quad \Delta S = S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

where equality & inequality signs shows the reversibility & irreversibility respectively.

For an adiabatic process whether reversible or irreversible $\delta Q = 0$

hence $\Delta S \geq 0$.

This means that entropy is zero in reversible adiabatic & greater than zero in irreversible adiabatic.

Although entropy changes b/w two fixed states is the same for any path, reversible or irreversible, the heat transfer to the system is less (or more, if heat is transferred from the system) for irreversible path than for the reversible one.

This is obvious from the fact that entropy increases in irreversible process & is conserved in reversible process.

① A qty of 400 KJ of heat is supplied per second to a heat engine at a fixed temp of 580 K. The heat rejection takes place at 280 K. The following results are obtained.

ⓐ 320 KJ/s are rejected ⓑ 193 KJ/s are rejected

ⓒ 80 KJ/s are rejected

Classify which of results report a reversible cycle or irreversible cycle or impossible results.

Sol Applying Clausius Inequality to the cycle in each case, we have

ⓐ $\oint \frac{dQ}{T} = \frac{400}{580} - \frac{320}{280} = -0.4532 < 0$

Since $\oint \frac{dQ}{T} < 0$, cycle is irreversible.

ⓑ $\oint \frac{dQ}{T} = \frac{400}{580} - \frac{193}{280} = 0$ $\because \oint \frac{dQ}{T} = 0$, cycle is reversible

ⓒ $\oint \frac{dQ}{T} = \frac{400}{580} - \frac{80}{280} = 0.4039 > 0$ $\because \oint \frac{dQ}{T} > 0$, cycle is impossible

This shows Clausius inequality.

② One kg of air is heated reversibly from an initial state of 1 atm & 15°C to 1 atm & 75°C. For reversible heating of air at const. pressure, $\frac{dQ}{dT} = 1.2$ KJ/K. Determine the change in entropy.

Sol. For a closed system undergoing a reversible process, change of entropy, by definition $ds = \frac{dQ}{T}$

$$\frac{dQ}{dT} = 1.2 \text{ KJ/K (given)}$$

$$dQ = 1.2 \; dT$$

$$\therefore \; ds = \frac{dQ}{T} = \frac{1.2 \, dT}{T}, \quad \text{Integrating}$$

$$\int_1^2 ds = 1.2 \int \frac{dT}{T}$$

$$(s)_1^2 = 1.2 \left[\log_e T\right]_{T_1}^{T_2}$$

$$s_2 - s_1 = ds = 1.2 \left[\log_e T_2 - \log_e T_1\right]$$

$$ds = 1.2 \log_e \frac{T_2}{T_1}$$

$T_1 = 15 + 273 = 288$ K

$T_2 = 75 + 273 = 348$ K

$$ds = 1.2 \log_e \frac{348}{288}$$

$$= 0.227 \text{ KJ/kg K.} \quad (0.23)$$

AVAILABLE ENERGY :-

Heat energy derived from combustion of fossil fuels or from nuclear fission is said to be low grade energy.

On the other hand, mechanical (shaft) work, electrical energy, water power, tidal power etc are rated as high grade energy.

The bulk of high grade energy is obtained from sources of low grade energy. But the complete conversion of low grade energy i.e., 'heat' into high grade energy i.e., 'shaft work' is impossible a/c to II law of TD.

"That part of low grade energy which is available for conversion into high grade energy is called "available energy."

Available energy is also called as Exergy.

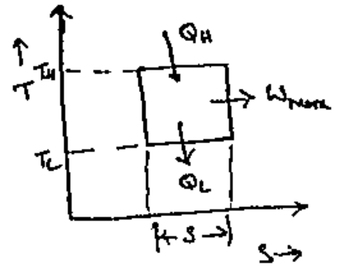
Obviously the other part of energy, a/c to II law, must be rejected. It is known as Unavailable energy, which is also known as Unavailable Anergy.

In a thermodynamically reversible cycle engine, max. work o/p obtainable from a certain heat input is called available energy (A.E.). The minimum energy that has to be rejected to the sink is called unavailable energy (U.E). If Q_1 is heat input,

$Q_1 = A.E + U.E$ $\boxed{\text{Energy} = \text{Exergy} + \text{Anergy}}$

(or) $W_{max} = A.E$

$= Q_1 - U.E$



If T_1 & T_2 are temp's of source & sink,

$$\eta_{rev} = 1 - \frac{T_2}{T_1}$$

For a given T_1, the efficiency of reversible engine η_{rev}, will increase with the decrease of T_2. The lowest practicable value of temp of heat rejection is the temp of surrounding, T_0.

Then, $\eta_{max} = 1 - \frac{T_0}{T_1} = \frac{W_{max}}{Q_1}$

$\therefore W_{max} = Q_1\left[1 - \frac{T_0}{T_1}\right]$

we know,
$W_{max} = AE$
$Q_1 = Q$ (say)
$T_1 = T$

$$AE = Q\left[1 - \frac{T_0}{T_1}\right]$$

$$= Q - T_0\left(\frac{Q}{T}\right)$$

$$\boxed{AE = Q - T_0 \cdot ds}$$

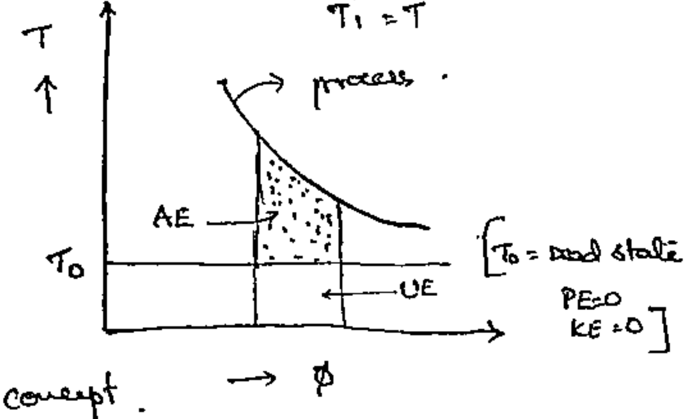


Figure shows the availability concept.

T_0 is the dead state which is a level below which the system cannot by itself go down, like atm. temp, zero altitude (PE=0), zero velocity (KE=0).

① Calculate AE & UE of a system that absorbs 15000 KJ of heat from heat source at 500 K, while environment temp is 290 K.

Sol: $AE = Q_H - \frac{Q_H T_L}{T_H} = 15000 - \frac{15000 \times 290}{500} = 6300$ kJ

$UE = 8700$ kJ ; $W_{max} = Q_H\left[1 - \frac{T_0}{T_1}\right]$

(or)

$Q_H = T_H ds$

$ds = \frac{Q_H}{T_H} = \frac{15000}{500} = 30$ KJ/K

$UE = T_L ds = 290 \times 30 = 8700$ kJ

(or)

$\eta = \frac{T_H - T_L}{T_H} = \frac{W_{max}}{Q_H}$

$W_{max} = \frac{(T_H - T_L) Q_H}{T_H}$

$= 6300$ KJ

② In a carnot cycle heat is supplied at 350°c & rejected at 27°c. Working fluid is water which absorbs heat and evaporates liquid at 350°c to steam at 350°c. The associated entropy change is 1.44 kJ/kg K.

~~Sol~~: cycle operates with mass flow rate of 1 kg of water.

$T_H = 623\ K$ Calculate ① Heat supplied, workdone

$T_L = 300\ K$ ② cycle operates in steady flow with power output of 20 kW. Determine mass flowrate of the system.

Sol:

$$\eta = \frac{T_H - T_L}{T_H} = \frac{623 - 300}{623} = 52\%$$

$ds = 1.44\ KJ/kg\,K$

$Q_H = T_H\,ds = 632 \times 1.44 = 897.12\ KJ$

$Q_L = T_L\,ds = 300 \times 1.44 = 432\ KJ$

$W = Q_H - Q_L = 465\ KJ/kg = AE$

Power = Ma~~ximum~~ss flow rate kg/s × work kJ/kg

$$\text{Mass flow rate} = \frac{\text{Power } kJ/s}{\text{work } kJ/kg} = \frac{20}{465} = \underline{\underline{0.043\ kg/sec}}$$

③ Air at 1 bar & 30°C is heated in reversible manner at constant pressure until its temp reaches to 205°C.

① calculate amount of heat added per kg of air.

② Available energy if sink temp is 4°C.

Sol:

$T_1 = 30^\circ C = 303\ K$

$T_2 = \cancel{200}\,205^\circ C = \cancel{573}\ K\ 478$

$T_0 = 4^\circ C = 277\ K$

$m = 1\ kg$

$(P = C)$

$\therefore Q = m\,C_p\,(T_2 - T_1)$

$= 1 \times 1.005\,(\overset{478}{\cancel{573}} - 303) = 175.87\ kJ/kg$

$$\text{Available Energy} = Q\left[1 - \frac{T_0}{T}\right] = 175.87\left[1 - \frac{277}{478}\right] = 73.85\ kJ/kg.$$

④ 0.2 kg of air initially at 575 k temp receives 300 kJ of heat reversibly at const. pressure. Determine the available & unavailable energy's of the heat added. Cp of air = 1.005 kJ/kg; surrounding temp = 300 k.

Sol: $T_1 = 575$ k

$Q = 300$ kJ

$C_p = 1.005$ kJ/kg k

$dS = \frac{dQ}{T}$

$= m\, c_p \ln\left(\frac{T_2}{T_1}\right)$

$= 0.2 \times 1.005 \ln\left(\frac{2067.5}{575}\right)$

$= 0.2572$ kJ/kg k $= 0.258$

unavailable energy $= T_a \times ds \Rightarrow UE = \frac{575}{2067.5} \times 0.2572$ $= 77.16$

$= 531.81$ kJ/kg

$dQ = m c_p \Delta T$

$300 = 0.2 \times 1.005 \times (T_2 - 575)$

$T_2 = 2067.5$ k

$= 300\left(1 - \left(\frac{2067.5}{575}\right)\right)$

$AE = 216.565$ kJ/hr $= 25646$

$= 147.89$ kJ/hr

Entropy change during a Process:-

$dQ = du + dw$

$= du + pdv$

entropy $ds = \frac{dQ}{T} \Rightarrow T.ds = dQ$

$$\boxed{T.ds = du + pdv} \rightarrow ①$$

we know

$h = u + pV$

$dh = du + d(pV)$

$dh = du + pdv + Vdp$ $\qquad [T ds = du + pdV]$

$dh - vdp = du + pdv$

$$\boxed{T.ds = dh - Vdp} \rightarrow ②$$

① Constant volume Process :-

$$dQ = m c_v dT$$

$$ds = \frac{dQ}{T} = m c_v \frac{dT}{T}$$ Integrate on both sides,

$$\int_1^2 ds = \int_1^2 m c_v \frac{dT}{T}$$

$$S_2 - S_1 = m c_v (\ln T)_1^2$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\boxed{S_2 - S_1 = m c_v \ln\left(\frac{T_2}{T_1}\right) \text{ (or) } = m c_v \ln\left(\frac{P_2}{P_1}\right)}$$

② Constant Pressure process :-

$$dQ = m c_p dT$$

$$ds = \frac{dQ}{T} = m c_p \frac{dT}{T}$$ Integrate,

$$\int_1^2 ds = \int_1^2 m c_p \frac{dT}{T}$$

$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

$$\boxed{S_2 - S_1 = m c_p \ln\left(\frac{T_2}{T_1}\right) \text{ or } m c_p \ln \frac{V_2}{V_1}}$$

③ Isothermal process :-

$$dQ = du + dw$$ [For isothermal $du = 0$]

$$= du + pdv$$

$$P_1V_1 = P_2V_2$$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$dQ = pdv$$

$$Pv = RT$$

$$ds = \frac{dQ}{T} = \frac{pdv}{T}$$

$$P = \frac{RT}{v}$$ [where $T = C$]

$$ds = \frac{R}{v} dv$$, Integrate

$$\int_1^2 ds = \int_1^2 \frac{R}{v} dv \Rightarrow \boxed{ds = R \ln\left(\frac{v_2}{v_1}\right) \text{ (or) } R \ln\left(\frac{P_1}{P_2}\right)}$$

④ Reversible Adiabatic process :-

$$dQ = 0$$

$$ds = \frac{dQ}{T} = 0 \Rightarrow S_2 - S_1 = 0 \Rightarrow S_2 = S_1$$

Entropy is Constant. This is also known as Isentropic Process.

⑤ Polytropic Process :- $\left[pv^n = C\right]$

case(i):- $ds = \frac{dQ}{T}$ $\quad dQ = du + dw$

$= c_v dT + p dv$

$pv = RT$

$p = \frac{RT}{v}$

$ds = c_v \frac{dT}{T} + p \frac{dv}{T}$, Integrate

$$\int_1^2 ds = \int_1^2 c_v \frac{dT}{T} + \int_1^2 p \frac{dv}{T}$$

$$\boxed{s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)} \rightarrow ①$$

This is in terms of temp. & volume.

$\therefore \frac{pv}{T} = \text{constant} \Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow \frac{T_2}{T_1} = \frac{P_2V_2}{P_1V_1}$

$\therefore$ Eq ① can be written as,

case(ii):- $s_2 - s_1 = c_v \ln\left[\frac{P_2V_2}{P_1V_1}\right] + R \ln\left(\frac{V_2}{V_1}\right)$

$= c_v \ln\left(\frac{P_2}{P_1}\right) + c_v \ln\left(\frac{V_2}{V_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)$

$= c_v \ln\left(\frac{P_2}{P_1}\right) + (c_v + R) \ln\left(\frac{V_2}{V_1}\right)$ $\quad \left[\because c_p = c_v + R\right]$

$$\boxed{s_2 - s_1 = c_v \ln\left(\frac{P_2}{P_1}\right) + c_p \ln\left(\frac{V_2}{V_1}\right)} \rightarrow ②$$

case(iii):- $\frac{pv}{T} = \text{const}$ $\quad \therefore \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow \frac{V_2}{V_1} = \frac{P_1T_2}{P_2T_1}$

$\therefore$ Eq ① can be written as,

$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_1T_2}{P_2T_1}\right)$

$= c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_1}{P_2}\right) + R \ln\left(\frac{T_2}{T_1}\right)$

$= (c_v + R) \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_1}{P_2}\right)$

$$\boxed{s_2 - s_1 = c_p \ln \frac{T_2}{T_1} + R \ln\left(\frac{P_1}{P_2}\right)} \rightarrow ③$$

case IV:- In terms of n & γ

$P = \frac{RT}{V}$

$$dQ = \left[\frac{\gamma - n}{\gamma - 1}\right] dW$$

$$ds = \frac{dQ}{T} = \left[\frac{\gamma - n}{\gamma - 1}\right] \frac{dW}{T} = \left[\frac{\gamma - n}{\gamma - 1}\right] \frac{p\,dv}{T}$$

$$ds = \left[\frac{\gamma - n}{\gamma - 1}\right] R \frac{dv}{V}$$, Integrate ...

$$\int_1^2 ds = \int_1^2 \left[\frac{\gamma - n}{\gamma - 1}\right] R \frac{dv}{V}$$

$$\boxed{S_2 - S_1 = \frac{\gamma - n}{\gamma - 1} R \ln\left(\frac{V_2}{V_1}\right)} \rightarrow ④$$

$\frac{C_p}{C_v} = \gamma$

$\gamma C_v = C_p$

But $\frac{R}{\gamma - 1} = C_v$

$$\therefore S_2 - S_1 = (\gamma - n)\, C_v \ln\left(\frac{V_2}{V_1}\right)$$

$$= \gamma . C_v - n C_v \ln\left(V_2/V_1\right)$$

$$\boxed{S_2 - S_1 = C_p - n . C_v \ln(V_2/V_1)} \rightarrow ⑤$$ [in terms of vol]

$pV^n = $ const.

$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/n}$, then eq ⑤ becomes,

$$S_2 - S_1 = (C_p - n . C_v) \ln\left(\frac{P_1}{P_2}\right)^{1/n}$$

$$\boxed{S_2 - S_1 = (C_p' - n . C_v) \frac{1}{n} \ln\left(\frac{P_1}{P_2}\right)} \rightarrow ⑥$$ [in terms of pressure]

we know,

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{n-1/n}$, take log on both sides,

$$\ln\left(\frac{T_2}{T_1}\right) = \left(\frac{n-1}{n}\right) \ln\left(\frac{P_2}{P_1}\right)$$

$$\therefore \ln\left(\frac{P_2}{P_1}\right) = \left(\frac{n}{n-1}\right)\ln\left(\frac{T_2}{T_1}\right) \Rightarrow \ln\left(\frac{P_1}{P_2}\right) = -\left(\frac{n}{n-1}\right)\ln\left(\frac{T_2}{T_1}\right)$$

∴ Eq. ⑥ can be written as,

$$s_2 - s_1 = (C_p - n.C_v)\cdot\frac{1}{\gamma}\cdot -\left(\frac{K}{n-1}\right)\ln\left(\frac{T_2}{T_1}\right)$$

$$= \frac{n\,C_v - C_p}{n-1}\cdot\ln\left(\frac{T_2}{T_1}\right) \qquad \bigg| \quad \frac{C_p}{C_v} = \gamma$$

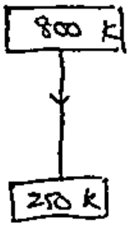
$$s_2 - s_1 = \frac{C_v\left(n - \frac{C_p}{C_v}\right)}{n-1}\ln\left(\frac{T_2}{T_1}\right)$$

$$\boxed{\therefore S_2 - S_1 = \left(\frac{n-\gamma}{n-1}\right) C_v \ln\left(\frac{T_2}{T_1}\right)} \rightarrow ⑦$$

① Heat flows from a reservoir at 800 K to another reservoir at 250 K. If entropy change of the heat reservoir is −4 KJ/kg K. Determine the entropy change of cold reservoir.

sol:-

$$\frac{dQ}{T} = (ds)_H$$



$$\frac{dQ}{800} = -4 \Rightarrow dQ = -3200\ \text{KJ}$$

Same amount of heat is supplied to cold body.

For cold body,

$$\frac{dQ}{T} = (ds)_c \quad \Rightarrow \frac{+3200}{250} = ds$$

$$(ds)_c = 12.8\ \text{KJ/kg K}.$$

② Heat flows from a hot reservoir at 800K to another reservoir at 250 K. If the entropy change of overall process is 4.25 KJ/kg K. Make calculations for heats flowing out of high temp reservoir.

Sol/ change of entropy of hot & cold body = 4.25 KJ/kg k

$$\left(\frac{dQ}{T}\right)_h + \left(\frac{dQ}{T}\right)_c = 4.25$$

$$-\frac{dQ}{800} + \frac{dQ}{250} = 4.25$$

$$Q = 1545.45 \text{ KJ}.$$

③ Water is heated at const pressure of 0.7 MPa, boiling point is 164.97 c. The initial temp of water is 0°C. Latent heat of evaporation is 2066.3 KJ/kg. Find the increase in entropy of water if final state is steam at 164.97°C. Cp of water = 4.2 KJ/kg k.

Sol/ From (1-2) $(m = 1)$

$$S_2 - S_1 = m C_p \ln \frac{T_2}{T_1}$$

$$= 4.2 \times \cancel{1} \times \ln\left(\frac{437.97}{273}\right)$$

$$= 1.985 \text{ KJ/kg k}$$

From (2-3)

$$S_3 - S_2 = \frac{Q}{T} = \frac{\text{latent heat}}{T} = \frac{2066.3}{437.97} = 4.717 \text{ KJ/kg K}$$

Total change in entropy

$$S_2 - S_1 + S_3 - S_2 = 1.985 + 4.717$$

$$= 6.7029 \text{ KJ/kg K}$$

④ 2 kg of water at 80°C is mixed adiabatically with 3kg of water at 30°C at a constant pressure process of 1 atm. Find the increase in entropy of total mass of water due to mixing process.

80):- Heat lost by 2 kg of water at 80°C = Heat gained by 3 kg of water at 30°C.

$$\therefore \quad 2\, c_p (80 - t) = 3\, c_p (t - 30) \qquad [Q = mc_p \Delta T]$$

$$t = \underline{50}\ ^\circ C = 323\ K \qquad [c_{p\,water} = 4.2\ KJ/kg\,k]$$

$$(\Delta S)_1 = \int_{T_1}^{T_2} \frac{\delta Q}{T} = \int_{353}^{323} \frac{m\, c_p\, dT}{T}$$

$$= 2 \times 4.2 \ln \left(\frac{T_2}{T_1}\right)_{353}^{323} = 8.4 \ln \left(\frac{323}{353}\right)$$

$$= -0.746 \ KJ/kg\,k$$

Entropy change for 3 kg of water,

$$(\Delta S)_2 = \int_{T_1}^{T_2} \frac{\delta Q}{T} = \int_{303}^{323} \frac{m\, c_p\, dT}{T}$$

$$= 3 \times 4.2 \times \ln \left(\frac{323}{303}\right) = 0.8054 \ KJ/kg\,k$$

Entropy increase of total mass of water due to mixing.

$$= (\Delta S)_1 + (\Delta S)_2$$

$$= -0.746 + 0.8054$$

$$= 0.059 \ KJ/kg\,k$$

Thermodynamic Potential :-

The energy when expressed in terms of parameters like pressure, vol, temp,. U, H, S etc are called Thermodynamic potentials.

Gibbs Function :-

It is denoted by (G). It is defined as the diff b/w enthalpy & product of temp & entropy.

$$\boxed{G = H - TS}$$

Helmholtz's Function :-

It is defined as diff b/w I.E & product of temp & entropy. It is denoted by 'F'.

$$\boxed{F = U - Ts}$$

Maxwell's Equations :-

Out of 8 properties (P, V, T, U, H, S, G & F), any one property can be expressed as a function of any other two properties. The relation b/w properties of pure substance are known as "Maxwell relations."

① Internal Energy :-

A/c to I law of TD, $Q = U + W$ $\quad \left[ds = \frac{dQ}{T} \Rightarrow Tds = dQ\right]$

$$dQ = du + dw$$

$$Tds = du + pdv$$

$$\boxed{du = Tds - pdv} \rightarrow ①$$

② Enthalpy :-

$$H = U + pV \qquad [du = Tds - pdv]$$

$$dH = du + d(pV)$$

$$= Tds - \cancel{pdv} + \cancel{pdv} + Vdp$$

$$\boxed{dH = Tds + Vdp} \rightarrow ②$$

③ Gibb's Function :- $G = H - Ts$

$$dG = dH - d(Ts)$$

$$= \cancel{Tds} + Vdp - \cancel{Tds} - SdT$$

$$\boxed{dG = Vdp - SdT} \rightarrow ③$$

④ <u>Helmholtz Function</u> :-

$F = U - TS \Rightarrow dF = dU - d(TS)$ $\quad (du = Tds - pdv)$

$= T\cancel{ds} - pdv - T\cancel{ds} - SdT$

$$\boxed{dF = -pdv - SdT} \rightarrow ④$$

The above ④ eq's are known as Maxwell's Relations. These are in the form of $\boxed{dz = Mdx + Ndy}$

The differential eq. must satisfy condition of

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \text{ for exactness Test.}$$

<u>Applying exactness test for Maxwell's relation</u> :-

① $du = Tds - pdv$

comparing with standard differential eqn.

$z = u,\ M = T,\ x = S,\ N = -P,\ y = V$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \Rightarrow \boxed{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V} \rightarrow ⑤$$

② $dH = Tds + Vdp$, Apply std. diff. eqn.

$z = H,\ M = T,\ x = S,\ N = V,\ y = P$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \Rightarrow \boxed{\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P} \rightarrow ⑥$$

<u>Sp. heats</u>

$$\boxed{C_p = T\left(\frac{\partial S}{\partial T}\right)_P}$$

$$\boxed{C_V = T\left(\frac{\partial S}{\partial T}\right)_V}$$

③ $dG = Vdp - SdT$

$z = G,\ M = V,\ N = -S,\ x = P;\ y = T$

$$\boxed{\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial p}\right)_T} \longrightarrow ⑦$$

④ $dF = -pdV - SdT$ $\quad z = F,\ M = -P,\ N = -S,\ x = V,\ y = T$

$$\boxed{+\left(\frac{\partial P}{\partial T}\right)_V = +\left(\frac{\partial S}{\partial V}\right)_T} \rightarrow ⑧$$

① The HE receives heat at the rate of 500 kW, from a source temp of 1200 K. It rejects waste heat at 300 K. In this process it delivers a power o/p of 180 KW. Determine
① η of engine ② Ideal power o/p ③ Irreversibility.

Sol: ① $\eta = \frac{T_H - T_L}{T_H} = \frac{1200-300}{1200} = \underline{75\%}$

~~② Ideal power o/p = heat i/p = 500 kW~~

② W_{actual} $\eta = \frac{Q_H - Q_L}{Q_H} = \frac{W}{Q_H}$

Ideal power $w = 0.75 \times 500 = \underline{375}$ KW

③ Irreversibility = Ideal power − Actual power

$= 375 - 180$

$= \underline{195}$ KW.

② Using Maxwell Relations derive two Tds equations.

① Consider entropy to be a function of temp & vol.

$$s = f(T, V)$$

Sol $$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

Multiplying above eq. with T on both sides,

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_v dT + T\left(\frac{\partial s}{\partial v}\right)_T dv \qquad \left[\because c_v = T\left(\frac{\partial s}{\partial T}\right)_v\right]$$

$$\boxed{Tds = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_v dv} \qquad \left[\text{From MW relation } \left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v\right]$$

② Consider entropy is a function of temp & pressure, then

$$s = f(T, P)$$

Sol:
$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

Multiply with T, $\left[\ c_p = T\left(\frac{\partial s}{\partial T}\right)_p\ \right]$

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_p dT + T\left(\frac{\partial s}{\partial p}\right)_T dp$$

From M.W.R $\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$, substitute

$$\boxed{Tds = c_p\, dT - T\left(\frac{\partial v}{\partial T}\right)_p dp} \rightarrow ②$$

③ Consider combustion of fuel-air mixture in an IC engine which generates hot gases at 2773 K in cylinder. The gas expands a/c to polytropic process $pV^{1.38} = C$. volume ratio is 9. & surrounding air temp is 293 K. Determine ⓐ work transfer ② change in I.E ③ Heat Transfer. Assume $R = 0.26$ kJ/kgK
$c_v = 0.82$ kJ/kgK.

Sol:
$T_H = 2773$ K
$T_L = 293$ K

① work transfer $= \frac{P_1V_1 - P_2V_2}{n-1} = \frac{mRT_1 - mRT_2}{1.38-1}$

Given $\frac{V_2}{V_1} = 9$

In polytropic process

$$\left(\frac{V_2}{V_1}\right)^{n-1} = \frac{T_1}{T_2} = \left(\frac{\text{[illegible]}}{9}\right)^{\cancel{1.8-1}\ 1.38-1} = 2.3024$$

$$T_2 = \frac{2773}{2.3\cancel{024}} = 1203 \text{ K}$$

$T_2 = 1203$ K
$T_1 = 2773$ K

$$\text{work} = \frac{mR(T_1 - T_2)}{n-1}$$
$$= \frac{0.26(2773-1203)}{1.38-1} = \underline{\underline{1074 \text{ kJ}}}$$

② $\Delta U = m\, c_v (T_2' - T_1)$
$= 1 \times 0.82(1203 - 2773) = \underline{\underline{-1287 \text{ kJ}}}$.

③ $Q = \Delta U + W$

$= -1287 + 1074 = -213$ kJ

Heat transfer $Q = 213$ kJ (rejected).

④ A cylinder contains 0.115 m³ of gas at 1 bar & 90°C. The gas is compressed to a vol. of 0.0288 m³. The final pressure of gas is 5.67 bar. Calculate

(i) Mass of gas (ii) Value of index of compression

(iii) Increase in internal energy of the gas (Already Completed)

(iv) Heat transfer during compression.

Assume $\gamma = 1.4$, $R = 0.3$ kJ/kg K, $C_v = 0.75$ kJ/kg K.

Sol:
$V_1 = 0.115\ m^3$
$P_1 = 1$ bar $= 100$ kP
$V_2 = 0.0288\ m^3$
$P_2 = 5.67 \times 10^2$ kP
$T_1 = 90°C = 363$ K.

(i) we have $P_1V_1 = mRT_1$

$$m = \frac{P_1V_1}{RT_1}$$

$m = 0.1056$ kg

(ii) $PV^n = C$ i.e., $P_1V_1^n = P_2V_2^n$

$$100 \times (0.115)^n = 567 \times (0.0288)^n$$

$$n = 1.253.$$

(iii) $\Delta U = mC_v(T_2 - T_1)$

$= 0.1056 \times 0.75\,(515.27 - 363)$

$\Delta U = 12.06$ kJ

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1}$$

$$\frac{363}{T_2} = \left[\frac{0.0288}{0.115}\right]^{1.253-1}$$

$T_2 = 515.27$ K

(iv) $Q = \Delta U + W$

$= 12.06 - 19.089$ kJ

$Q = -7.029$ kJ

$$W = \frac{P_1V_1 - P_2V_2}{n-1}$$

$= -19.089$ kJ

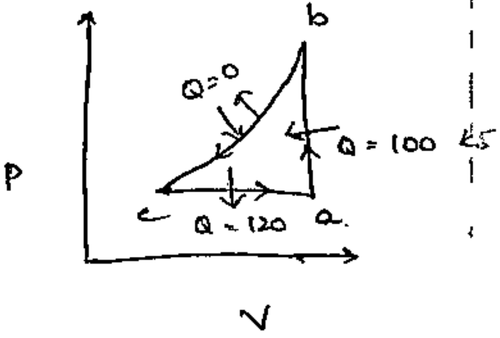
Heat Transfer $Q = 7$ kJ

⑤ Calculate internal energy & work transfer for the following p-v diagram.

$U_a = 150$ kJ

work done$_{(b-c)} = -70$ kJ (on the gas during process)

$U_b = ?$, $U_c = ?$ $W_{ca} = ?$



Sol:- process (a-b),

$\dot{Q} = 100$ kJ

$\Delta U = U_b - U_a = U_b - 150$ kJ

$W = 0$ ($\because$ const. vol. process)

$Q = \Delta U + W$

$100 = U_b - 150 - 0 \Rightarrow U_b = 250$ kJ

process (b-c), $Q = 0$

$\Delta U = U_c - U_b = U_c - 250$

$Q = \Delta U + W$

$0 = U_c - 250 - 70 \Rightarrow U_c = 320$ kJ

process (c-a) :-

$Q = -120$ kJ

$\Delta U = U_a - U_c = 150 - 320 = -170$

$Q = \Delta U + W$

$W = Q - \Delta U = -120 + 170 = 50$ kJ

$\underline{W = 50 \text{ kJ}}$

Third law of Thermodynamics:-

NERNST

Third law of TD was formulated by W.H. Nernst & Max Planck. It states that the entropy of a perfect crystalline substance is zero (or constant) at the absolute zero temp & it represents the maximum degree of order. → 0 K, −273°C

"Entropy is zero at absolute temperature". −459.67 F

This constant value cannot depend on any other parameters characterizing the closed system such as pressure or applied magnetic field. At absolute zero the system must be in a state with the minimum possible energy. Entropy is related to the number of accessible microstates, and there is typically one unique state with minimum energy. In such a case, the entropy at absolute zero will be exactly zero.

Available Energy:-

$$AE = Q \times \left[1 - \frac{T_0}{T_1}\right]$$

$$W_{max} = Q_1 \left[1 - \frac{T_0}{T_1}\right]$$

$$dW_{max} = dQ_1 - \frac{T_0}{T_1} dQ_1$$

Integrate on both sides,

$$\int_1^2 dW_{max} = \int_1^2 dQ_1 - \int_1^2 T_0 \left(\frac{dQ_1}{T_1}\right)$$

$$W_{max} = \int_1^2 dQ_1 - T_0 \int_1^2 (S)$$

$$\boxed{W_{max} = AE = Q_{1-2} - T_0 (S_2 - S_1)}$$

$$\left[\begin{array}{l} W = h - TS \\ AE = \text{"} \end{array}\right]$$

Availability in non-flow system:-

$$AE = W_{max} = (u_1 + P_0 v_1 - T_0 S_1) - (u_0 + P_0 V_0 - T_0 S_0)$$

$$W_{max} = a_1 - a_0$$

Availability in steady flow process:-

$$AE = (h_1 - h_0) - T_0 (S_1 - S_0)$$

$$AE = W_{max} = (h_1 - T_0 S_1) - (h_0 - T_0 S_0)$$

Irreversibility:- Idealized WD − actual WD

$$I = W_{max} - W$$

This is also called as degradation (or) dissipation.

Gouy Stodola Theorem:-

It states that the rate of loss of AE in a process is directly proportional to the rate of entropy generation ($\dot{S}_G$)
entropy

$$\boxed{\dot{I} \propto \dot{S}_G}$$

$$I = W_{lost} = T_0 \, \Delta S_{universe} = T_0 S_G$$

$$\therefore \dot{I} = T_0 \dot{S}_G \rightarrow$$ known as Gouy-Stodola eq.

∴ A thermodynamically efficient process would involve minimum exergy loss with minimum rate of entropy generation.

⑥ Calculate the decrease in AE when 25 kg of water at 95°C mix with 35 kg of water at 35°C. The pressure being taken as constant & the temp of surroundings being 15°C. $C_p = 4.2$ kJ/kgK

Sol/ The AE of a system of mass m, C_p, & T is given by.

$$AE = m C_p \int_{T_0}^{T_1} \left(1 - \frac{T_0}{T}\right) dT \qquad Q_1 - T_0\left(\frac{Q}{T}\right)$$

$$(AE)_{25} = 25 \times 4.2 \int_{273+15}^{273+95} \left(1 - \frac{288}{T}\right) dT \qquad Q_1\left(1 - \frac{T_0}{T_1}\right) dT \quad mC_p \int \left(1 - \frac{T_0}{T}\right) dT$$

$$= 105 \left[(368 - 288) - 288 \ln\left(\frac{368}{288}\right)\right] = 987.49 \text{ kJ}.$$

$$(AE)_{35} = 35 \times 4.2 \int_{288}^{308} \left(1 - \frac{288}{T}\right) dT$$

$$= 97.59 \text{ kJ}$$

Total AE before mixing

$$(AE)_{total} = 987.49 + 97.59 = 1085.08 \text{ kJ}$$

After mixing if t_f is final temp, then

$$25 \times 4.2 (95 - t_f) = 35 \times 4.2 (t_f - 35)$$

$$t_f = 60^\circ C$$

∴ Total mass after mixing = 35+25 = 60 kgs

$(AE)_{60}$ = AE of 60 kg of water at 60°c

$= 60 \times 4.2 \int_{288}^{333} \left(1 - \frac{288}{T}\right) dT$

$= 803.27$ KJ

(AE) = 2016.8

AE = 1532.4

Decr = 484.4KJ

∴ Decrease in AE due to mixing,

$= (AE)_{\text{before mixing}} - (AE)_{\text{After mixing}}$.

$= 1085.08 - 803.27$

$= 281.81$ KJ

⑦. One kg of ice at −5°c is exposed to the atmosphere which is at 25°c. The ice melts & comes into thermal equilibrium. Find the entropy increase of the universe.

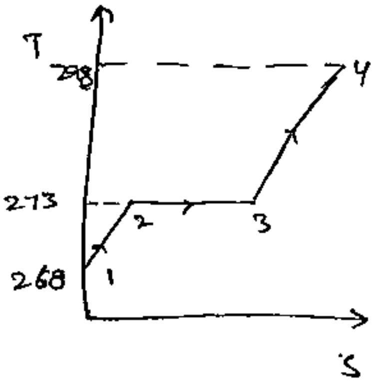
Sol: Given $m = 1$ kg.

$T_1 = -5°c = 268$ K

$T_4 = 25°c = 298$ K

Take $C_{p\,ice} = 2.093$ KJ/kg K

$C_{pw} = 4.187$ KJ/kg K



Latent heat of fusion = 334.5 KJ/kg.

$\Delta S_{Total} = \Delta S_{(1-2)} + \Delta S_{(2-3)} + \Delta S_{(3-4)}$

$Q = m h_{fg}$.

$\Delta S_{2-3} = \frac{Q}{T}$

$= m C_{p\,ice} \ln \frac{T_2}{T_1} + \frac{m h_{fg}}{T_2} + m C_{pw} \ln \frac{T_4}{T_3}$

$= 1 \times 2.093 \ln \frac{273}{268} + \frac{1 \times 334.5}{273} + 1 \times 4.187 \ln \frac{298}{273}$.

$\Delta S_{Total} = 0.038689 + 1.2252 +$ ~~0.296~~ 0.3668

$\Delta S_{Total} =$ ~~[illegible]~~ KJ/kg K.
1.63