### DESIGN OF MACHINE ELEMENTS-I

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### COURSE OBJECTIVES

- To introduce the concepts of design of machine elements.
- To familiarize with fundamental approaches to failure prevention for static and dynamic loading.
- To explain the design procedures of different types of joints.
- To familiarize with various theories related the design of machine elements for different loading conditions.

# SYLLABUS

#### **Unit 1 Design for Static Loads**

**Introduction:** General considerations of design, design process. Selection of Engineering materials - properties – Manufacturing considerations in the design. BIS codes of materials. Preferred numbers. Design for static loads: Modes of failure, Design of components subjected to axial, bending, torsion and combined loads. Theories of failure for static loads.

#### **Unit 2 Design for Dynamic Loads**

Stress concentration –notch sensitivity, Types of fluctuating loads– Design for fluctuating stresses –finite life - Endurance limit - Estimation of Endurance limit – Soderberg, Goodman and Modified Goodman criterion for fatigue failure.

#### Unit 3 Design of Bolted and Welded joints

**Design of Bolted Joints:** Preload of bolts, various stresses induced in the bolts, torque requirement for bolt tightening, eccentrically loaded joints.

**Design of Welded Joints:** Stresses of lap and butt welds, eccentrically loaded welded joints. Joints subjected to bending and torsion.

#### Unit 4 Design of Keys, Cotter and Knuckle Joints

- Keys: Design of sunk, saddle, Kennedy and woodruff keys.
- **Cotter and Knuckle Joints:** Socket and spigot joint, Sleeve and Cotter joints, Gib and Cotter joint, Knuckle joint.

#### Unit 5 Design of Shafts and Couplings

**Shafts:** Design of solid and hollow shafts bending, torsion, axial and combined bending and axial loading.

**Shaft Couplings:** Design of Rigid couplings-Muff, Split muff and Flange couplings-Flexible couplings: bushed pin type.

### COURSE OUTCOMES

A student will be able to

- 1. Summarize the basic concepts of design of machine elements
- 2. Analyse the components subjected to dynamic loading
- 3. Design bolted and welded joints subjected to given loads.
- 4. Design keys, cotter and knuckle joints for various applications.
- 5. Design shafts and shaft couplings for given conditions.

## OUTLINE

- Introduction
- Definition
- General considerations of design
- Design process
- Selection of Engineering materials
- properties
- Manufacturing considerations in the design.
- BIS codes of materials.
- Preferred numbers

#### **Unit 1 Design for Static Loads**

**Introduction:** General considerations of design, design process. Selection of Engineering materials - properties – Manufacturing considerations in the design. BIS codes of materials. Preferred numbers. Design for static loads: Modes of failure, Design of components subjected to axial, bending, torsion and combined loads. Theories of failure for static loads.

#### **Course Outcome 1**

At the end of the unit, student will be able to Summarize the basic concepts of design of machine elements

## **Definition of Design**

- Design is to formulate a plan to satisfy a particular need and to create something with physical reality.
- Realization of a concept or idea into a configuration
- Design is the creation of a plan or convention for the construction of an object, system or measurable human interaction.
- Design is nothing but Decision Making

#### **Engineering Design:**

Engineering design is an iteration process of devising a system to meet a set of desired needs. The design process begins with an identified need, which can be initiated based on any of the following sources:

- Observable deficiency (i.e., A car bumper that gets easily damaged in low speed collisions)
- Product improvement (i.e., A system for improving traction on ice without studs or chains)
- New products (i.e., A more effective alarm clock for reluctant students)
- Change in law (i.e., An average automobile gas mileage of 25 miles per gallon of gasoline)

### Machine Design

- Machine is a combination of several machine elements arranged to work together as a whole to accomplish specific purpose.
- Machine design involves designing the elements and arranging them optimally to obtain some useful work.
- Machine design is defined as "the use of scientific principles, technical information and imagination in the description of a machine or a mechanical system to perform specific functions with maximum economy and efficiency".

### **General Considerations of Design**

- Type of load and stresses caused by the load
- Motion of parts or kinematics of the Machine
- Selection of Materials
- Form and size of the parts
- Frictional resistance and lubrication
- Convenient and Economical features
- Use of standard parts
- Safety of Operation
- Workshop facilities
- Number of Machines to be manufactured
- Cost of Construction
- Assembling

# **DESIGN CONSIDERATIONS**

- > Strength
- > Rigidity
- Reliability
- ➤ Safety
- > Cost
- > Weight
- Ergonomics
- > Aesthetics
- Manufacturing considerations
- > Assembly considerations
- Conformance to standards
- > Friction and wear > Life Vibrations Thermal considerations > Lubrication > Maintenance > Flexibility > Size and shape > Stiffness > Corrosion > Noise > Environmental considerations



# **General Procedure in Machine Design** Need or Aim Synthesis (Mechanisms) Analysis of Forces **Material Selection Design of Elements** (Size and Stresses) Modification **Detailed Drawing** Production



#### **Basic Procedure of Design of Machine elements**

**Specify Functions of Element** 

**Determine the forces acting on Element** 

**Select Suitable Material for Element** 

**Determine Failure Mode of Element** 

Determine Geometric Dimensions of element

Modify Dimensions for Assembly and Manufacture and Check Design at Critical Cross-sections

**Prepare Working Drawing of Element** 

#### **Step 2: Determination of Forces**

In many cases a free-body diagram of forces is constructed to determine the forces acting different parts of the machine.

#### **Forces are:**

- 1. Force due to Frictional Resistance
- 2. Inertia Force due to change in linear or angular velocity
- 3. Centrifugal force due to change in direction of velocity
- 4. Static force due to deadweight of machine part
- 5. External force due to energy, power transmission
- 6. Force due to thermal gradient or variation in temperature











### **Selection of Engineering Materials**

- a) Metals
- b) Non-metals

Metals:

Ferrous – Which contains iron as the major constituent

Ex. Steel, Cast Iron

Non-ferrous – materials don't contains Iron.

Ex. Copper, Aluminium

Non-Metals:

- (i) Ceramic materials oxides, carbides and nitrides of various metals. Ex. Glass, Brick, Concrete, Cement etc.
- (ii) Organic materials Polymeric materials composed of carbon compounds. Ex: Paper, fuel, rubber, paints, etc.

#### Factors to be considered for the selection of materials

- 1. Availability
- 2. Cost
- 3. Physical properties
- 4. Mechanical Properties
- 5. Manufacturing process
- Physical properties:
- Color
- > Shape
- > Size

### Density

Electrical conductivity Thermal conductivity

### **Manufacturing Processes**

- Primary Process
- Machining Process
- Surface Finishing Process
- Joining Process
- Process effecting change in properties like heat treatment, cold & hot working

# Manufacturing Considerations in Design

- Design considerations in Casting
- Design considerations in Machining
- Design considerations in Forging

### **Design Considerations in Casting**

- Design the part so that the shape is cast easily
- Select a casting process and material suitable for the part, size, mechanical properties, etc.
- Locate the parting line of the mold in the part.
- Locate and design the gates to allow uniform feeding of the mold cavity with molten material.
- Select an appropriate runner geometry for the system.
- Locate mold features such as sprue, screens and risers, as appropriate
- Make sure proper controls and good practices are in place.

### Contd...

#### • Avoid corners, Angles and section thickness



Fig. 3.1 (a) Incorrect (Part in Tension) (b) Correct (Part in Compression)

Fig. 3.3 Provision of Fillet Radius





### **Design Considerations of Machining**

- Avoid Machining
- Specify Liberal Tolerances
- Avoid sharp corners
- Use stock dimensions
- Design rigid parts
- Avoid shoulders and undercuts
- Avoid hard Materials





# **Stress Concentration**

Presented by Ajaya Kumar Reddy K Assistant Professor Department of Mechanical Engineering

### **Unit 2 Design for Dynamic Loads**

Stress concentration –notch sensitivity, Types of fluctuating loads– Design for fluctuating stresses – finite life - Endurance limit - Estimation of Endurance limit – Soderberg, Goodman and Modified Goodman criterion for fatigue failure.

### **Course Outcome 2**

At the end of the unit, student will be able to Analyse the components subjected to dynamic loading

# Introduction

- 1. Stress Concentration, Notch Sensitivity
- 2. Fluctuating stresses
- 3. Design for fluctuating stresses
- 4. Finite life
- 5. Endurance Limit
- 6. Soderberg, Goodman lines & Modified Goodman criterion for fatigue failure.

### **Stress Concentration**

- Whenever a machine component changes the shape of its crosssection, the simple stress distribution no longer holds good and the neighbourhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called stress concentration.
- It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc.


#### **Concept of Stress Concentration**

#### **Stress** Concentration



$$K = \frac{\sigma_{\text{max}}}{\sigma_0}$$
$$\sigma_0 = \frac{P}{(W-d)t}$$

## Theoretical stress concentration factor, $K_t$

$$K_t = rac{\sigma_{\max}}{\sigma_0}$$
  $K_{ts} = rac{ au_{\max}}{ au_0}$ 

Nominal stress, max stress with no discontinuity



## Contd....

#### **Stress Concentration :**

It is defined as the localization of high stress due to irregularities present in the component and abrupt changes of the cross section

•  $K_t = \frac{Highest \ value \ of \ actual \ stress \ near \ discontinuity}{normal \ stress \ obtained \ by \ elementary \ equations \ for \ minimum \ cross-section}$ 

or

•  $K_t = \frac{Maximum stress}{nominal stress}$ 

## Contd....

### Causes

- Abrupt Change of c/s
- Poor surface finish
- Localized loading
- Variation in the material properties
- Methods of Reducing Stress Concentration
- •Avoiding sharp corners
- •Providing fillets
- •Use of multiple holes instead of single hole.
- •Undercutting the shoulder part

## Contd.....

#### • Additional Notches and Holes in Tension Member



#### • Fillet Radius, Undercutting and Notch for Member in Bending:



## Contd.....

#### Drilling Additional Holes for Shaft:



#### **Reduction of Stress Concentration in Threaded Members:**



## **Stress Concentration Factors**

Stress concentration factors are determined by

- Mathematical Model
- Photo-elasticity method
- Brittle Coatings
- Electrical strain guages
- Finite element method

## Photoelasticity

A plane polarized light is passed thru a photelastic material (all transparent plastics) resulting in a colorful fringe pattern indicating the intensity of the stress.





## Coatings

Brittle Coating – a specially prepared lacquers are usually applied by spraying on the actual part. After air drying, the part is subjected to stress. A pattern of small cracks appear on the surface. Data could be used to locate strain gages for precise measurement of the stress. The method is sensitive to temperature and humidity.



## **Electrical Strain Guages**

The method is the most popular and widely accepted for strain measurements and stress analysis. The strain gauge consists of a grid of strain-sensitive metal foil bonded to a plastic backing material. When the gauge is subjected to a mechanical deformation, its electrical resistance changes proportionally. The change in voltage is converted to strain and the stress is calculated from the strain.



## Finite Element method

- Pre-processor
- Solution
- Post-processor





## **Stress Concentration: Hole**





Discontinuities of cross section may result in high localized or *concentrated* stresses.

$$K = \frac{\sigma_{\max}}{\sigma_{\text{ave}}}$$

## Stress Concentration: Fillet





#### Fatigue:

When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **\*fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc



- (i) Regions of discontinuity, such as oil holes, keyways, screw threads, etc.
- (ii) Regions of irregularities in machining operations, such as scratches on the surface, stamp mark, inspection marks, etc.
- (iii) Internal cracks due to defects in materials like blow holes

# Endurance limit by rotating beam method





Fig. 6.2. Time-stress diagrams.

## S-N Curve or Wohler diagram



## **ENDURANCE LIMIT**

- It may be defined as the safe maximum stress which can be applied to the machine part working under actual condition.
- It is defined as maximum value of completely reversed bending stress which a polished specimen can withstand without failure for infinite number of cycles.
  FACTORS AFFECTING ENDURANCE STRENGTH
- Load factor  $(K_I)$
- Surface finish factor(K<sub>SF</sub>)
- Size factor (K<sub>SZ</sub>)
- Reliability factor(K<sub>R</sub>)
- Miscellaneous factors(K) ns.

#### **NOTCH SENSITIVITY (q)**

This is defined as the degree to which the actual stress concentration effect compares with theoretical stress concentration effect.

## Effect of Load factor (K<sub>L</sub>)

 Shigley and Mischke proposed following exponential formulae to calculate load factor

For 2.79 mm  $\le d < 51$  mm  $K_b = 1.24 d^{-0.107}$ 

For 51 mm  $< d \le 254$  mm  $K_b = 0.859 - 0.000\ 873\ d$ 

> .: Endurance limit for reversed bending load, Endurance limit for reversed axial load,

and endurance limit for reversed torsional or shear load,

 $\sigma_{eb} = \sigma_e K_b = \sigma_e$   $\sigma_{ea} = \sigma_e K_a$  $\tau_e = \sigma_e K_s$ 

Load factor : K<sub>a</sub>=0.8 k<sub>s</sub>=0.55 for ductile 0.8 for brittle

## Surface Finish Factor (K<sub>SF</sub>)

Shigley and Mischke have suggested an exponential relation for surface

$$K_a = a(S_{ut})^b$$
 [if  $K_a > 1$ , set  $K_a = 1$ ]



Surface finish	а	b	
Ground	1.58	-0.085	
Machined or cold-drawn	4.51	-0.265	
Hot-rolled	57.7	-0.718	
As forged	272	-0.995	

et 
$$K_{sur}$$
 = Surface finish factor.

.: Endurance limit,

L

$$\sigma_{e1} = \sigma_{eb} \cdot K_{sur} = \sigma_{e} \cdot K_{b} \cdot K_{sur} = \sigma_{e} \cdot K_{sur} \qquad \dots (:: K_{b} = 1)$$

...(For reversed bending load)

...(For reversed torsional or shear load)

te : The surface finish factor for non-ferrous metals may be taken as unity.

 $= \sigma_{ea} K_{sur} = \sigma_{e} K_{a} K_{sur}$ 

 $= \tau_c K_{sur} = \sigma_c K_c K_{sur}$ 

## Size Factor (K<sub>SZ</sub>)

Let  $K_{sz} = \text{Size factor.}$   $\therefore$  Endurance limit,  $\sigma_{e2} = \sigma_{e1} \times K_{sz}$  ...(Considering surface finish factor also)  $= \sigma_{eb} K_{sur} K_{sz} = \sigma_{e} K_{b} K_{sur} K_{sz} = \sigma_{e} K_{sur} K_{sz}$  ( $\because K_{b} = 1$ )  $= \sigma_{ea} K_{sur} K_{sz} = \sigma_{e} K_{a} K_{sur} K_{sz}$  ...(For reversed axial load)  $= \tau_{e} K_{sur} K_{sz} = \sigma_{e} K_{s} K_{sur} K_{sz}$  ... (For reversed torsional or shear load)

Notes: 1. The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm.

 When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85.

 When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

# Relation between endurance strength and ultimate stress



## Reliability factor(K<sub>R</sub>)

- The reliability factor is depends upon the reliability requirement of the mechanical component.
- The reliability correction factor accounts for the scatter and uncertainty of material properties (endurance limit).

Reliability	50	90	95	99	99.9	99.99	99.999
Factor	1	0.897	0.868	0.814	0.753	0.702	0.659

### Miscellaneous factors(K) ns.

 For the reversed bending load, endurance limit,

 $\sigma'_{e} = \sigma_{eb} K_{sur} K_{sz} K_{r} K_{r} K_{r}$ 

2. For the reversed axial load, endurance limit,

 $\sigma'_{e} = \sigma_{ea}K_{sur}K_{sz}K_{r}K_{t}K_{t}$ 

 For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e K_{sur} K_{sz} K_r K_i K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

## Modifying factor for stress concentration $k_d = 1/k_f$

## **Fatigue stress concentration factor**

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor.

 $K_f = -$ Endurance limit with stress concentration

#### **NOTCH SENSITIVITY (q)**

This is defined as the degree to which the actual stress concentration effect compares with theoretical stress concentration effect. The notch sensitivity factor q is defined as

$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

$$q = \frac{K_f - 1}{K_t - 1}$$
  
$$K_f = 1 + q (K_t - 1)$$
  
$$K_{fs} = 1 + q (K_{ts} - 1)$$

#### REVERSED STRESSES— DESIGN FOR FINITE AND INFINITE LIFE

There are two types of problems in fatigue design—

- (i) Components subjected to completely reversed stresses
  - Design for infinite life
  - Design for finite life.

(ii) Components subjected to fluctuating stresses.

## Design for infinite life

- When the component is to be designed for infinite life, the endurance limit becomes the criterion of failure.
- The amplitude stress induced in such components should be lower than the endurance limit in order to withstand the infinite number of cycles.
- Such components are designed with the help of the following equations:





## Design for finite life

• When the designed is based on finite life S-N curve is used. It consists of a straight line AB drawn from  $(0.9\sigma_{ut})$  at  $10_3$  cycles to  $\sigma_e$  at  $10^6$  cycles on log-log paper.



## Design procedure for finite life problems

• Locate the point A with co-ordinates  $[3, log_{10}(0.9\sigma_{ut})]$ , since  $log_{10}(10^3)=3$ 

#### • Locate the point B with co-ordinates [6, $log_{10}(\sigma_e)$ ], since

 $\log_{10}(10^6)=6$ 

• Join  $\overline{AB}$ , Which is used as a criterion of failure for finite life problems

- Depending upon the life N of the component. Draw a vertical line passing through log10(N) on the abscissa. This line intersects AB, at point F.
- Draw a line  $\overline{FE}$  parallel to the abscissa. The ordinate at point E, i.e.  $\log 10 (\sigma_f)$  gives the fatigue strength corresponding to *N* cycles.

**Q.** A rotating bar made of steel 45C8 ( $S_{ut} = 630 \text{ N/mm}^2$ ) is subjected to a completely reversed bending stress. The corrected endurance limit of the bar is 315 N/mm<sup>2</sup>. Calculate the fatigue strength of the bar for a life of 90,000 cycles.



# Design of Keys

Presented by Ajaya Kumar Reddy K Assistant Professor Department of Mechanical Unit 4 Design of Keys, Cotter and Knuckle Joints Keys: Design of sunk, saddle, Kennedy and woodruff keys.

**Cotter and Knuckle Joints:** Socket and spigot joint, Sleeve and Cotter joints, Gib and Cotter joint, Knuckle joint.

**Course Outcome 4:** 

At the end of the topic, student will be able to design keys, cotter and knuckle joints for various applications.

## Key

A key is a fastening inserted into the keyway of two mating parts, it is used to transmit from a shaft to a hub or vice versa.

- Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses.
- It is inserted parallel to the axis of the shaft in a groove or slot which called "keyway".

#### Functions of Keys:

- To transmit power from a shaft to hub or vice versa
- To prevent relative rotational motion between the shaft and the joined machine element like gear or pulley.



## Types of Keys

Shaft keys come in a wide variety of types and shapes and can be divided into four categories and subcategories.

- Sunk Keys
  - Rectangular & square keys
  - Parallel keys
  - Gib head keys
  - Feather key (sliding clearance with keys)
  - Woodruff key
- Saddle keys
  - Flat & Hollow saddle keys
- Tangent keys
- Round/Circular keys

## Sunk Keys

- The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley.
- The types of sunk keys are as follows
  - 1. Rectangular sunk key
  - 2. Parallel keys
  - 3. Gib head keys
  - 4. Feather key (sliding clearance with keys)
  - 5. Woodruff key
#### Rectangular sunk key

A rectangular sunk key is shown in Fig. The usual proportions of this key are :

Width of key,  $\mathbf{w} = \mathbf{d} \ / \ \mathbf{4}$  ; and

thickness of key,  $\mathbf{t} = 2\mathbf{w} \ / \ \mathbf{3} = \mathbf{d} \ / \ \mathbf{6}$ 

where d = Diameter of the shaft or diameter of the hole in the hub. The key has taper 1 in 100 on the top side only.



#### Parallel sunk key

It is a taperless Rectangular or square sunk key is shown in Fig. Uses: Hub is required to slide along the shaft.



### Square Key

The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal.

$$\mathbf{w} = \mathbf{t} = \mathbf{d} / \mathbf{4}$$



## Gib Head Key

It is a rectangular sunk key with a head at one end known as gib head.

- It is usually provided to facilitate the removal of key. A gib head key is shown in Fig.
- The usual proportions of the gib head key are :

Width, w = d/4;

and thickness at large end,  $t = \frac{2w}{3} = \frac{d}{6}$ 



## Feather Key

- It is fitted either to the shaft or Hub
- A key attached to one member of a pair and which permits relative axial movement is known as *feather key*. *It is a special type of parallel key which transmits a turning* moment and also permits axial movement.
- It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.



## Cont..



## Wood Ruff key

The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig

- Advantages: Accommodates itself to any taper in the hub
  - Its extra depth in the shaft, prevents any tendency to turnover in the keyway
- Disadvantages: Depth of the keyway weakens the shaft

It cannot be used as a feather key



#### Saddle keys

#### -Hollow saddle key

#### Flat saddle key/

t = w/3 = d/12

#### Tangent Key

 The tangent keys are fitted in pair at right angles as shown in Fig. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts





## **Round Keys**

The round keys, as shown in Fig., are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.



# **Splines**

#### Shaft + Integrated key = Splines

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.





## **Forces acting on a Sunk Key**

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

**1.** Forces  $(F_1)$  due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

**2.** Forces  $(F_2)$  due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.



- Considering shearing failure
- F = Area resisting shearing × Shear stress =  $l \times w \times \tau$

Torque transmitted by the shaft  $\mathbf{T}=F X \frac{d}{2} = \mathbf{I} \times \mathbf{w} \times \tau X \frac{d}{2}$ Considering crushing failure

F = Area resisting crushing × Shear stress =  $1 \times \frac{\tau}{2} \times \sigma_c$ 

Torque transmitted by the shaft  $\mathbf{T} = F X \frac{d}{2} = \mathbf{I} \times \frac{t}{2} \times \sigma_c X \frac{d}{2}$ 

• The key is equally strong in shearing and crushing, if

$$\mathbf{l} \times \mathbf{w} \times \tau X \frac{d}{2} = \mathbf{l} \times \frac{t}{2} \times \sigma_c X \frac{d}{2}$$
$$\frac{w}{t} = \frac{\sigma_c}{2\tau}$$

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore from above equation, we have w = t. In other words, a square key is equally strong in shearing and crushing. Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.

• Given d=50 mm,  $\tau$ =42 Mpa &  $\sigma_c = 70$  Mpa W=d/4=12.5mm~14mm T=d/6=8.33 mm ~10mm Torque transmitted by shaft T= $\frac{\pi}{16}d^3\tau = 1.03 \times 10^6$  N-mm

Considering Shearing failure

$$T=l X \le X \frac{d}{2} X \tau$$
$$l=$$

Considering Crushing failure

$$T=l X \frac{t}{2} X \frac{d}{2} X \sigma_c$$
$$l=$$

A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.

• Given: P=15000w, N=960 rpm, d=40mm, I = 75 mm.  $\tau$  = 56 Mpa &  $\sigma_c$  = 112 Mpa

As  $\sigma_c = 2 \tau$  it is a square key, for which w=t=d/4=10 mm

According to Moore

$$T = \frac{P \times 60}{2 \pi N} = \frac{15 \times 10^3 \times 60}{2 \pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) = 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2\times40}\right) = 0.8125$$

:. Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56 \ (40)^3 \ 0.8125 = 571 \ 844 \ N$$

shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\ 000\ N$$

Shear strength of the key  $= \frac{840\ 000}{571\ 844} = 1.47$ 

Normal strength of the shaft 571 844

## **Cotter Joints**



## INTRODUCTION

- A cotter is a flat wedge shaped piece of rectangular cross-section and its width is tapered (either on one side or both sides) from one end to another for an easy adjustment.
- The taper varies from 1 in 48 to 1 in 24 and it may be increased up to 1 in 8, if a locking device is provided. The locking device may be a taper pin or a set screw used on the lower end of the cotter.
- The cotter is usually made of mild steel or wrought iron.
- A cotter joint is a temporary fastening and is used to connect rigidly two co-axial rods or bars which are subjected to axial tensile or compressive forces.
- It is usually used in connecting a piston rod to the crosshead of a reciprocating steam engine, a piston rod and its extension as a tailor pump rod, strap end of connecting rod etc.

# **Types of Cotter Joints**

Following are the three commonly used cotter joints to connect two rods by a cotter:

Socket and spigot cotter joint,
 Sleeve and cotter joint, and

3. Gib and cotter joint

## Socket and spigot Cotter joint





- P = Load carried by the rods,
- d = Diameter of the rods,
- $d_1 =$ Outside diameter of socket,
- d2 = Diameter of spigot or inside diameter of socket,
- $d_1 =$ Outside diameter of spigot collar,
- t, = Thickness of spigot collar,
- $d_4$  = Diameter of socket collar,
- c = Thickness of socket collar,
- b = Mean width of cotter,
- t =Thickness of cotter,
- l = Length of cotter,
- a = Distance from the end of the slot to the end of rod,
- $\sigma_{i}$  = Permissible tensile stress for the rods material,
- $\tau$  = Permissible shear stress for the cotter material, and
- $\sigma_c$  = Permissible crushing stress for the cotter material.

#### 1) Failure of the rod in tension



#### Cont....

(Tensile failure of Rod). The rod may fail in tension due to the tensile load P. Area resisting tearing=  $(\pi/4)^*d^2$ Tearing strength of rods= $(\pi/4)^*(d^{2})^*(\sigma_t)$ 

Equating this to load (P), we have  $P=[(\pi/4)^*(d)^2]^*\sigma_t$ 

#### From this equation, diameter of the rod (d) may be calculated

# 2) Failure of the Spigot in Tension across the weakest section (or slot)



#### Since the weakest section of the spigot is that section which has a slot in it for the cotter, therefore

- Area resisting tearing  $A_t = \frac{\pi}{4} d_2^2 d_2 t$
- Tearing strength of the rods,  $P_t = \left[\frac{\pi}{4}d_2^2 d_2t\right]\sigma_t$
- To avoid failure tearing strength ≥ Applied load,  $P_t \ge P$ In limiting condition  $P = P_t$

$$P = \left[\frac{\pi}{4}d_2^2 - d_2t\right]\sigma_t$$

In above equation both  $d_2 \& t$  are unknowns, take an assumption

 $t = d_2/4$ 

 $d_2$  may be calculated



Cont....

#### 3) Failure of the rod or cotter in crushing



#### Cont....

Area that resists crushing of a rod or cotter A<sub>c</sub> = d<sub>2</sub>t
Crushing strength of the rods, P<sub>c</sub> = d<sub>2</sub>tσ<sub>c</sub>
To avoid failure crushing strength ≥ Applied load, P<sub>c</sub> ≥ P
In limiting condition P= P<sub>c</sub> P = d<sub>2</sub>tσ<sub>c</sub>

From above equation induced crushing strength may be verified, i.e., induced stress should be less than allowable stress. Then finalise  $d_2 \& t$ 

#### 4) Failure of the socket in tension across the slot



#### Cont....

- Area resisting tearing A<sub>t</sub> = \$\frac{\pi}{4}\$ [d\_1^2 d\_2^2] (d\_1 d\_2)t\$
  Tearing strength of the rods, P<sub>t</sub> = \$\begin{bmatrix} \pi\_4\$ [d\_1^2 d\_2^2] (d\_1 d\_2)t\$ \$\pi\_t\$ \$\sigma\_t\$ To avoid failure tea\frac{\pi}{4}\$ ring strength \$\ge\$ Applied load\$
- $P_t \ge P$

In limiting condition  $P = P_t$ 

$$P = \left\{ \frac{\pi}{4} \left[ d_1^2 - d_2^2 \right] - \left( d_1 - d_2 \right) t \right\} \sigma_t$$

From above equation outside diameter of socket may be calculated



## 5) Failure of the cotter in shear



#### Cont....

- Area resisting shearing A<sub>s</sub> =2 b t
  Shearing strength of the rods, P<sub>s</sub> = 2 b t τ
  To avoid failure shearing strength ≥ Applied load P<sub>s</sub> ≥ P
- In limiting condition  $P = P_s$  $P = 2 b t \tau$

From above equation mean width of cotter may be calculated



## 6) Failure of socket collar in crushing

**Crushing Failure of Socket End or Socket Collar** 



From this Eq. diameter d<sub>4</sub> can be obtained.

### Cont....

• Area resisting shearing  $A_c = (d_4 - d_2)$  t

- Crushing strength of the rods,  $P_c = (d_4 d_2) t \sigma_c$
- To avoid failure shearing strength  $\geq$  Applied load

$$P_{\mathcal{C}} \ge \mathbf{F}$$

In limiting condition  $P = P_c$ 

$$P = (d_4 - d_2) \operatorname{t\sigma}_c$$

From above equation diameter of socket collar may be calculated



## 7) Failure of socket end in shearing



#### Cont....

Area resisting shearing  $A_s = (d_4 - d_2) c$ Shearing strength of the rods,  $P_s = (d_4 - d_2) c \tau$ To avoid failure shearing strength  $\geq$  Applied load  $P_s \geq P$ 

• In limiting condition  $P = P_C$  $P = (d_A - d_2) c \tau$ 

From above equation 'c' may be calculated



## 8) Failure of rod end in shear



#### Cont....

• Area resisting shearing  $A_s = 2 a d_2$ • Shearing strength of the rod end,  $P_s = 2 a d_2 \tau$ • To avoid failure shearing strength  $\geq$  Applied load  $P_s \geq P$ • In limiting condition  $P = P_c$  $P = 2 a d_2 \tau$ 

From above equation 'a' may be calculated

#### 9) Failure of spigot collar in crushing

• Area resisting shearing  $A_s = \frac{\pi}{4} (d_3^2 - d_2^2)$ 

 $P_{c} \geq P$ 

- •Crushing strength of the rods,  $P_c = \frac{\pi}{4} (d_3^2 d_2^2) \sigma_c$
- To avoid failure shearing strength  $\geq$  Applied load

In limiting condition  $P = P_c$  $P = \frac{\pi}{4} (d_3^2 - d_2^2) \sigma_c$ 

From above equation diameter of spigot collar may be calculated


#### 10) Failure of spigot collar in shearing



### Cont....

- Area resisting shearing  $A_s = \pi d_2 t_1$
- •Shearing strength of the rods,  $P_s = \pi d_2 t_1 \tau$
- To avoid failure shearing strength  $\geq$  Applied load  $P_s \geq P$
- In limiting condition  $P = P_s$  $P = \pi d_2 t_1 \tau$

From above equation thickness of spigot collar may be calculated



Failure of cotter in bending

The maximum bending moment occurs at the centre of the cotter and is given by



 $M_{MAX=}P/2[1/3*(d_4-d_2)/2+d_2/2]-P/2[d_2/4]$ 

 $M_{MAX=}P/2[(d_4-d_2)/6+d_2/2-d_2/4]$ 

 $M_{MAX=}P/2[(d_4-d_2)/6+d_2/4]$ 

We know that section modulus of the cotter ,  $Z{=}t^{*}b^{2}{/}6$ 

Bending stress induced in the cotter,

 $\sigma_b = (M_{MAX}/Z)$ 

#### Failure of cotter in bending

Bending stress induced in the cotter,  $\sigma_{b} = \left( M_{MAX}/Z \right)$ 

Bending stress induced in the cotter,  $\sigma_b = P/2[(d_4-d_2)/6+d_2/4]/(t*b^2/6)$ 

Bending stress induced in the cotter,  $\sigma_b = P[(d_4)+0.5 * d_2]/(2* t*b^2)$ 

This bending stress induced in the cotter should be less than the allowable bending stress of the cotter.

# Design of Couplings

Presented by Ajaya Kumar Reddy K Assistant Professor Department of Mechanical Engineering

#### **Unit 5 Design of Shafts and Couplings**

Shafts: Design of solid and hollow shafts bending, torsion, axial and combined bending and axial loading.

**Shaft Couplings:** Design of Rigid couplings-Muff, Split muff and Flange couplings-Flexible couplings: bushed pin type.

#### **Course Outcome 5**

At the end of the topic, student will be able to **Design of** shaft couplings for given conditions.

### Couplings

Coupling is a device used to connect two shafts together at their ends for the purpose of transmitting power



Motor

# **Uses of coupling**

- To provide connection of shafts of units made separately
- To allow misalignment of the shafts or to introduce mechanical flexibility.
- To reduce the transmission of shock loads
- To introduce protection against overloads.
- To alter the vibration characteristics

# **Types of Couplings**

1) Rigid Couplings2) Flexible Couplings



#### **Rigid coupling**



#### **Flexible coupling**



# **Rigid Couplings**

• It is used to connect two shafts which are perfectly aligned.

#### Types of **<u>Rigid Couplings</u>** are

- 1) Sleeve or Muff coupling
- 2) Clamp or Split-Muff coupling
- 3) Flange Coupling

# **Flexible Couplings**

- It is used to connect two shafts having both lateral and angular misalignment.
- Types of **Flexible Couplings** are
- 1) Bushed pin type coupling,
- 2) Universal coupling, and
- 3) Oldham Coupling

# **1. Sleeve or Muff-Coupling**

- It is the simplest type, made of cast iron.
- It consists of sleeve and is fitted over the ends of the twos hafts by means of gib head key
- Power is transmitted by means of a key and a sleeve

# **Design of Muff Coupling**

The following parts are to be designed

- Shaft
- Sleeve

• Key



# 1) Design of Shaft

- Based on the loading the design of shaft is to be done as per earlier discussion
- Diameter of the shaft (d) is obtained

# 2) Design of Sleeve

Let T = Torque to be transmitted by the coupling, and  $\tau_{c=}$  Permissible shear stress for the material of the sleeve which is cast iron =14 Mpa

$$T = \frac{\pi}{16} X \tau_{c} \left( \frac{D^{4} - d^{4}}{D} \right) = \frac{\pi}{16} X \tau_{c} X D^{3} \left( 1 - k^{4} \right)$$

## 3) Design of Key

- Length of the key is l=L/2 = 3.5d/2
- After fixing the length of the Key the in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

T=F X 
$$\frac{d}{2}$$
= l ×  $\frac{t}{2}$  ×  $\sigma_c$  X  $\frac{d}{2}$   
T=F X  $\frac{d}{2}$ = l × w ×  $\tau$  X  $\frac{d}{2}$ 

# **Design of Split Muff Coupling**

- In this coupling, sleeve or muff is made in two halves, which are split along the plane passing through the axes of the shafts and are bolted together .
- The halves of the muff are made of cast iron.



### Cont....

- The shaft ends are made to a butt each other
- A single key is fitted directly in the keyways of both the shafts.
- One-half of the muff is fixed from below and the other half is placed from above.
- Both the halves are held together by means of mild steel studs or bolts and nuts.
- The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings.

Applications: This coupling may be used for heavy duty and moderate speeds.

### a) Design of Muff and Key

• The usual proportions of the muff for the clamp or compression coupling are :

Diameter of the muff or sleeve, D = 2d + 13 mmLength of the muff or sleeve, L = 3.5 dwhere d = Diameter of the shaft.

Let T = Torque to be transmitted by the coupling, and  $\tau_{c=}$  Permissible shear stress for the material of the sleeve which is cast iron =14 Mpa

$$T = \frac{\pi}{16} X \tau_{c} \left( \frac{D^{4} - d^{4}}{D} \right) = \frac{\pi}{16} X \tau_{c} X D^{3} \left( 1 - k^{4} \right)$$

- Length of the key is l=L/2 = 3.5d/2
- After fixing the length of the Key the in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

T=F X 
$$\frac{d}{2}$$
= l ×  $\frac{t}{2}$  ×  $\sigma_c$  X  $\frac{d}{2}$   
T=F X  $\frac{d}{2}$ = l × w ×  $\tau$  X  $\frac{d}{2}$ 

### b) Design of Clamp

• The force exerted by each bolt,

$$F = (\pi/4) (d_b^2) \sigma_t$$

• Force exerted by the bolts on each side of the shaft

$$F = (\pi/4) (d_b^2) \sigma_t x (n/2)$$

• Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$
  
between each shaft and muff.

Frictional force between each shaft and muff,

•  $F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L = \mu \times \frac{\pi^2}{8} \times (d_b^2) \sigma_t \times n$ 

• The torque that can be transmitted by the coupling,  $T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8} (d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d$ 

# **Flange Coupling**

- A flange coupling usually applies to a coupling having two separate cast iron flanges.
- Each flange is mounted on the shaft end and keyed to it.
- The faces are turned up at right angle to the axis of the shaft Flange coupling are
- 1.Unprotected type flange coupling
- 2. Protected type flange coupling
- 3. Marine type flange coupling



# 1. Unprotected type flange coupling

- Each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts.
- Generally, three, four or six bolts are used.
- The keys are staggered at right angle along the circumference of the shafts in order to divide the weakening effect caused by keyways.



#### Cont...

- The usual proportions for an unprotected type cast iron flange couplings, are as follows :
- If d is the diameter of the shaft or inner diameter of the hub, then

Outside diameter of hub, D = 2 d

Length of hub, L = 1.5 d

Pitch circle diameter of bolts,  $D_1 = 3d$ 

Outside diameter of flange,  $D_2 = D_1 + (D_1 - D) = 2 D_1 - D = 4 d$ Thickness of flange,  $t_f = 0.5 d$ 

Number of bolts = 3, for d upto 40 mm

- = 4, for d upto 100 mm
- = 6, for d upto 180 mm

# 2. Protected type flange coupling.

- In a protected type flange coupling, as shown in Fig., the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.
- The thickness of the protective circumferential flange (tp ) is taken as 0.25 d.
   The other proportions of the coupling are same as for unprotected type flange coupling.



# **3. Marine type flange coupling.**

- In a marine type flange coupling, the flanges are forged integral with the shafts as shown in Fig. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft.
- The other proportions for the marine type flange coupling are taken as follows : Thickness of flange = d / 3
  Taper of bolt = 1 in 20 to 1 in 40

Pitch circle diameter of bolts,

 $D_1 = 1.6 d$ 

Outside diameter of flange,

 $D_2 = 2.2 d$ 



#### Design of Flange Coupling

Let d = Diameter of shaft or inner diameter of hub,

- **D** = Outer diameter of hub,
- d = Nominal or outside diameter of bolt,
- **D**<sub>1</sub> = **Diameter of bolt circle**,
- **n** = Number of bolts,

t<sub>f</sub> = Thickness of flange,

 $\tau_s, \tau_b$ , and  $\tau_k$  = Allowable shear stress for shaft, bolt and key material respectively

 $\tau_{\rm c}$  = Allowable shear stress for the flange material i.e. cast iron,

 $\sigma_{cb,} \sigma_{ck}$  = Allowable crushing stress for bolt and key material respectively. The flange coupling is designed as discussed below :

### 1) Design of Hub

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$\mathrm{T} = \frac{\pi}{16} \mathrm{x} \tau_{\mathrm{c}} \left( \frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (L) is taken as 1.5 d.

2) Design for Key

$$T = F X \frac{d}{2} = \mathbf{l} \times \frac{t}{2} \times \sigma_c X \frac{d}{2}$$
$$T = F X \frac{d}{2} = \mathbf{l} \times \mathbf{w} \times \tau X \frac{d}{2}$$

#### 3) Design for flange

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the torque transmitted,

T =Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi \mathbf{x} \mathbf{D} \times \boldsymbol{t}_f \times \boldsymbol{\tau}_c \ X \ \frac{D}{2} = \frac{\pi D^2}{2} \times \boldsymbol{t}_f \times \boldsymbol{\tau}_c$$

#### 4) Design for Bolts

• The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D) is taken as 3 d. We know that

Load on each bolt = 
$$\frac{\pi}{4} \ge d_1^2 \ge \tau_b$$
  
Total load on all the bolts =  $\frac{\pi}{4} \ge d_1^2 \ge \tau_b \ge n$   
and torque transmitted T =  $\frac{\pi}{4} \ge d_1^2 \ge \tau_b \ge n \ge \frac{D_1}{2}$ 

From this equation, the diameter of bolt  $(d_1)$  may be obtained. Now the diameter of bolt may be checked in crushing.

Torque, 
$$\mathbf{T} = n\mathbf{xd}_1 \times \mathbf{t}_f \times \sigma_{cb} X \frac{D_1}{2}$$

# **Flexible Coupling**

- Flexible coupling is used to join the abutting ends of shafts which are not in exact alignment.
- Permits an axial misalignment of the shaft without undue absorption of the power which the shaft are transmitting.
- Types of Flexible Coupling:
  - 1) Bushed -pin type flexible coupling
  - 2) Oldham's Coupling
  - 3) Universal Coupling

### 1) Bushed –pin type flexible coupling

- A bushed-pin flexible coupling, as shown in Fig, is a modification of the rigid type of flange coupling.
- The coupling bolts are known as pins.
- The rubber or leather bushes are used over the pins.
- The two halves of the coupling are dissimilar in construction.
- A clearance of 5 mm is left between the face of the two halves of the coupling.
- There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.



Bearing load acting on each pin,

 $W = p_b \times d_2 \times l$ 

Total bearing load on the bush or pins  $\mathbf{T} = W \mathbf{x} \mathbf{n} = \mathbf{p}_{\mathbf{b}} \times \mathbf{d}_{\mathbf{2}} \times \mathbf{l} \mathbf{x} \mathbf{n}$ 

The torque transmitted by the coupling,

$$\mathbf{T} = W \mathbf{x} \mathbf{n} \mathbf{X} \frac{D_1}{2} = \mathbf{p}_{\mathbf{b}} \times \mathbf{d}_2 \times \mathbf{l} \mathbf{x} \mathbf{n} \mathbf{x} \frac{D_1}{2}$$

Direct shear stress due to pure torsion in the coupling halves,

Assuming a uniform distribution of the load W along the bush, the maximum bending moment on the pin,

$$M = W(\frac{l}{2} + 5mm)$$

Bending stress,

$$\boldsymbol{\sigma_b} = \frac{M}{Z} = \frac{W(\frac{t}{2} + 5mm)}{\frac{\pi}{4} X d_1^3}$$

1

Maximum principal stress

$$= \frac{1}{2} \bigg[ \sigma + \sqrt{\sigma^2 + 4\tau^2} \bigg]$$

the maximum shear stress on the pin

$$= \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

The value of maximum principal stress varies from 28 to 42 MPa.



#### Cont..

 $\boldsymbol{\tau} = \frac{\boldsymbol{\tau}}{\frac{\pi}{4} \ge \boldsymbol{d}_{1}^{2}}$ 

