CHAPTER 6

Options Contracts – Basic Principles

LEARNING OBJECTIVES

After reading this unit, you will be able to:

- understand the concept, nature and types of option contacts
- know the historical background and uses of option market
- know about the various terminologies used in option trading for example, exercise price, expiration date, option premium, etc.
- understand the role of the market players in option trading
- gain knowledge about the option position like In-the-money, Out-the-money and At-the-money
- understand the difference between option and futures contracts

In finance, an option is a contract whereby one party (the holder or buyer) has the right but not the obligation to exercise a feature of the contract (the option) on or before a future date (the exercise date or expiry). The other party (the writer or seller) has the obligation to honour the specified feature of the contract. Since the option gives the buyer a right and the seller an obligation, the buyer has received something of value. The amount the buyer pays the seller for the option is called the option premium.

Most often the term "option" refers to a type of derivative which gives the holder of the option the right but not the obligation to purchase (a "call option") or sell (a "put option") a specified amount of a security within a specified time span (Specific features of options on securities differ by the type of the underlying instrument involved).

WHAT ARE OPTIONS?

An option is a contract that gives the buyer the right, but not the obligation, to buy or sell an underlying asset at a specific price on or before a certain date. An option, just like a stock or bond, is a security. It is also a binding contract with strictly defined terms and properties.

Say, for example, that you discover a house that you would love to purchase. Unfortunately, you do not have the cash to buy it for another three months. You talk to the owner and negotiate a deal that gives you an option to buy the house in three months for a price of Rs. 200,000. The owner agrees, but for this option, you pay a price of Rs. 3,000. Now, consider two theoretical situations that might arise:

1. It has been discovered that the house is of historical importance and as a result, the market value of the house skyrockets to Rs. 10,00,000. Because the owner sold you the option, he is obligated to sell you the house for Rs. 200,000. In the end, you stand to make a profit of Rs.7,97,000 (Rs.10,00,000 – Rs.2,00,000 – Rs.3,000).
2. While touring the house, you discover that the house is not in proper living conditions. Though you originally thought you had found the house of your dreams, you now consider it worthless. On the upside, because you bought an option, you are under no obligation to go through with the sale. Of course, you still lose the Rs.3,000 price of the option, that is non-refundable.

This example demonstrates two very important points.

First, when you buy an option, you have a right but not an obligation to do something. You can always let the expiration date go by, at which point the option becomes worthless. If this happens, you lose 100% of your investment (option premium), which is the money you used to pay for the option.

Second, an option is merely a contract that deals with an underlying asset. For this reason, options are called derivatives, which mean an option derives its value from something else. In our example, the house is the underlying asset.

HISTORICAL USES OF OPTIONS

Contracts similar to options are believed to have been used since ancient times. For example, Aristotle wrote about Thales, who bought the option to use olive presses during the next harvest. In the real estate market, call options have long been used to assemble large parcels of land from separate owners, e.g. a developer pays for the right to buy several adjacent plots, but is not obligated to buy these plots and might not do so unless he can buy all the plots in the entire parcel. Film or theatrical producers often buy the right, but not the obligation — to dramatize a specific book or script. Lines of credit give the potential borrower the right — but not the obligation — to borrow within a specified time period.

Many choices, or embedded options, have traditionally been included in bond contracts. For example, many bonds are convertible into common stock at the buyer's option, or may be called (bought back) at specified prices at the issuer's option. Mortgage borrowers have long had the option to repay the loan early. Further, privileges were options sold over the counter in nineteenth century America, with both puts and calls on shares offered by specialized dealers. Their exercise price was fixed at the market price on the day the option was bought, and the expiry date was generally three months after purchase. They were not traded in secondary markets.

Some Uses of Options

There are many uses for options. Three possible uses of options are: writing covered calls, using options instead of stock, and obtaining portfolio insurance.

Earn Extra Money by Writing a Covered Call

Suppose you own a share of a particular stock. You might be able to make some extra money by writing a call. Note the difference here. In this case, you are writing (not buying) the call option. Because you already own the underlying security, you are *covered* from infinite losses if the stock price takes off.

We can sketch the payoffs for this strategy. First, we need a few definitions. Let *S^** be the ultimate value of the stock price on the maturity date. Let *k* be the exercise price. The call price will be denoted by *c*. The price of a zero coupon bond that matures on the same day as the option is denoted by *B*. The straight line is the payoffs from holding the stock long. The kinked solid line is the payoffs from shorting (writing) the call option. The dashed line is the net payoff. This is referred to as a *hedge position*. Note if the stock price stays below the exercise price, then you are clearly better off. If the stock price rises dramatically, then you do not capitalize on all the gain. The region below the dashed line denotes the gain from writing the covered call.

Using Options Instead of Stock

Suppose you have a choice of two investment strategies. The first is to invest $100 in a stock. The second strategy involves investing $90 in 6 month T-bills and $10 in 6 month calls. So we will want to buy *10/c* calls. That is, if the call is priced at $5, then you are able to buy two calls.

The payoffs for this strategy are outlined below. Note that because we own two calls, the payoffs are two for one. That is for every dollar the stock price is above the exercise price, we make two dollars on the call. The diagram shows the payoffs from strategy 2. The slope of the call payoff is 2. The T-bill payout is flat. As soon as the stock price goes past the exercise price, the portfolio value rises rapidly. A comparison of the two investment strategies is outlined. Note that the option substitution strategy does not do as well if the stock price does not move that much.

Portfolio Insurance

One can obtain insurance on a portfolio of stocks by buying a put. So for every dollar that the stock portfolio drops, you make back by holding the put. So portfolio managers can cover the downside by taking out *portfolio insurance*. In the diagram, the solid lines represent the payoffs from the long stock position and long put position. The dashed line is the total return or the *hedge position*. Note that the cost of covering the downside is lower returns if the stock rises in value.

BASIC TYPES OF OPTION

There are two basic types of options—call options and put options.

(i) *Call option:* A call option gives the holder the right but not the obligation to buy an asset by a certain date for a certain price.

(ii) *Put option:* A put option gives the holder the right but not the obligation to sell an asset by a certain date for a certain price.

The price of options is decided between the buyers and sellers on the trading screens of the exchanges in a transparent manner. The investor can see the best five orders by price and quantity. The investor can place a market limit order, stop loss order, etc. The investor can modify or delete his pending orders. The whole process is similar to that of trading in shares.

In simple words, a call option gives the holder the right to buy an asset at a certain price within or at the end of a specific period of time. Calls are similar to having a long position on a stock. Buyers of calls hope that the stock will increase substantially before the option expires.

Similarly, a put option gives the holder the right to sell an asset at a certain price within or at the end of a specific period of time. Puts are similar to having a short position on a stock. Buyers of put options hope that the stock will decrease substantially before the option expires.

An investor with a long equity call or put position may exercise that contract at any time before the contract expires, up to and including the Friday (in the Indian stock market) before its expiration. To do so, the investor must notify his brokerage firm of intent to exercise in a manner, and by the deadline specified by that particular firm.

Any investor with an open short position in a call or put option may nullify the obligations inherent in that short (or written) contract by making an offsetting closing purchase transaction of a similar option (same series) in the marketplace. This transaction must be made before the assignment is received, regardless of whether you have been notified by your brokerage firm to this effect or not.

Categories of Options

There are three main categories of options: *European, American and Bermudan.*

The distinction between American and European options has nothing to do with geographic location. European options can be exercised only at expiration time. American options can be exercised at any moment prior to maturity (expiration). A third form of exercise, which is occasionally used with OTC (over the counter) options, is Bermudan exercise. A Bermuda option can be exercised on a few specific dates prior to expiration. The name 'Bermuda' was chosen perhaps because Bermuda is half way between America and Europe.

There are hundreds of different types of options which differ in their payoff structures, path-dependence, and payoff trigger and termination conditions. Pricing some of these options represent a complex mathematical problem. Let us briefly discuss the various other types of options.

Other Types of Options

Various other types of options are listed below:

1. *Real options:* A **real option** is a choice that an investor has when investing in the real economy (i.e. in the production of goods or services, rather than in financial contracts). This option may be something as simple as the opportunity to expand production, or to change production inputs. Real options are an increasingly influential tool in corporate finance. The liquidity of this kind of exchange-traded options is relatively lower.

2. *Traded options – (Exchange-Traded Options):* Traded Options are, Exchange-traded derivatives, as the name implies. As for other classes of exchange traded derivatives, they have: standardized contracts; quick systematic pricing and are settled through a clearing house (ensuring fulfillment).

3. *Vanilla and exotic options:* Generally speaking, a vanilla option is a 'simple' or well understood option, whereas an exotic option is more complex, or less easily understood (hybrid options). European options and American options on stock and bonds are usually considered to be "plain vanilla". Asian options, lookback options, barrier options are often considered to be exotic, especially if the underlying instrument is more complex than simple equity or debt.

- Buyers and sellers of exchange-traded options do not usually interact directly—the futures and options exchange acts as intermediary. The seller guarantees the exchange that he can fulfill his obligation if the buyer chooses to execute.
- The *risk* for the option holder is limited: he cannot lose more than the premium paid as he can "abandon the option". His potential gain is theoretically unlimited; see strike price.
- The maximum loss for the writer of a put option is equal to the strike price. In general, the risk for the writer of a call option is unlimited. However, an option writer who owns the underlying instrument has created a covered position; he can always meet his obligations by using the actual underlying. Where the seller does not own the underlying on which he has written the option, he is called a "naked writer", and has created a "naked position".
- Options can be *in-the-money, at-the-money* or *out-of-the-money*. The "in-the-money" option has a positive intrinsic value, options in "at-the-money" or "out-of-the-money" have an intrinsic value of zero. Additional to the intrinsic value an option has a time value, which decreases the closer the option is to its expiry date.

OPTIONS TERMINOLOGY

1. *Buyer of an option:* The option buyer is the person who acquires the rights conveyed by the option: the right to purchase the underlying futures contract if the option is a call or the right to sell the underlying futures contract if the option is a put. In other words, the buyer of an option is the one who by paying the option premium, buys the right but not the obligation to exercise his option on the seller/writer.
2. *Writer of an option:* The option seller (also known as the option writer or option grantor) is the party that conveys the option rights to the option buyer. In other words, the writer of a call/put option is the one who receives the option premium and is thereby obliged to sell/buy the asset if the buyer exercises on him.
3. *Option Class:* All calls and puts on a given underlying security or index represent an "option class." In other words, all calls and puts on XYZ stock are one class of options, while all calls and puts on ZYX index are another class.
4. *Option Series:* All options of a given type (calls or puts) with the same strike price and expiration date are classified as an "option series." For example, all XYZ June 110 calls would be an individual series, while all XYZ June 110 puts would be another series.
5. *Contract Size of Equity Options:* The contract size of an option refers to the amount of the underlying asset covered by the options contract. For each unadjusted equity call

or put option, 100 shares of stock (usually, but this may differ from stocks to stocks) will change hands when one contract is exercised by its owner. These 100 shares of underlying stock are also referred to as the contract's "unit of trade."

6. *Contract Size of Index Options:* The contract size of a cash-settled index option is determined by its multiplier. The multiplier determines the aggregate value of each point of the difference between the exercise price of the option and the exercise settlement value of the underlying interest. For example, a multiplier of 100 means that for each point by which a cash-settled option is in the money upon exercise, there is a $100 increase in the cash settlement amount.
7. *Option price:* Option price is the price which the option buyer pays to the option seller. It is also referred to as the option premium.
8. *Expiration date:* The date specified in the options contract is known as the expiration date, the exercise date, the strike date or the maturity.
9. *Strike Price(K):* Also known as the "exercise price," this is the stated price at which the buyer of a call has the right to purchase a specific futures contract or at which the buyer of a put has the right to sell a specific futures contract. The exchanges decide the strike price at which call and put options are traded. Generally, to simplify matters, the exchanges specify the strike price interval for different levels of underlying prices, meaning the difference between one strike price and the next strike price over and below it. For example, the strike price interval for Bharat Heavy Electricals is Rs10. This means that there would be strike prices available with an interval of Rs10. Typically, the investor can see options on Bharat Heavy Electricals with strike prices of Rs150, Rs. 160, Rs. 170, Rs. 180, Rs. 190 etc.

   Following (Table-6.1) are the strike price intervals specified by exchanges.

Table 6.1:- Strike price intervals for Options

Price level of Underlying	Strike Price Interval (in Rs.)
Less than or equal to 50	2.5
Above 50 to 250	5.0
Above 250 to 500	10.0
Above 500 to 1000	20.0
Above 1000 to 2500	30.0
Above 2500	50.0

As the price of underlying moves up or down, the exchanges introduce more strike prices in keeping with the strike price interval rules. At any point in time, there are at least five strike prices (one near the stock price, two above the stock price and two below the stock price) available for trading in one-, two- and three-month contracts.

10. ***American options:*** American options are options that can be exercised at any time upto the expiration date. Most exchange-traded options are American.

11. ***European options:*** European options are options that can be exercised only on the expiration date itself. European options are easier to analyze than American options, and properties of an American option are frequently deduced from those of its European counterpart.

12. ***Index options:*** These options have the index as the underlying. Some options are European while others are American. Like index futures contracts, index options contracts are also cash settled.

13. ***Stock options:*** Stock options are options on individual stocks. Options currently trade on over 500 stocks in the United States. A contract gives the holder the right to buy or sell shares at the specified price.

14. ***Option Premium:*** The "price" an option buyer pays and an option writer receives is known as the premium. Premiums are arrived at through open competition between buyers and sellers according to the rules of the exchange where the options are traded. A basic knowledge of the factors that influence option premiums is important for anyone considering options trading. The premium cost can significantly affect whether the investor realize a profit or incur a loss.

    The premium is the price at which an option trades, and is paid by the buyer to the writer (seller) of the contract. The premium paid by the buyer is non-refundable payment for the rights inherent in the long contract. The writer (seller) of an option contract keeps the premium received, whether assigned or not, and is in turn obligated to fulfill the short contract's obligations if assignment is received. The two components of an option's total premium are intrinsic value and time value.

15. ***Moneyness:*** In finance, moneyness is a measure of the degree to which a derivative is likely to have positive monetary value at its expiration, in the risk-neutral measure.

    Three are three positions in an options:- In-the-money; At-the-money; and Out-of-the-money.

    (a) *In-the-money option:* An in-the-money (ITM) option is an option that would lead to a positive cashflow to the holder if it were exercised immediately. A call option on the index is said to be in-the-money when the current index stands at a level higher than the strike price (i.e. spot price >strike price). If the index is much higher than the strike price, the call is said to be deep ITM. In the case of a put, the put is ITM if the index is below the strike price.

    (b) *Out-of-the-money option:* An out-of-the-money (OTM) option is an option that would lead to a negative cashflow if it were exercised immediately. A call option on the index is out-of-the-money when the current index stands at a level which is less than the strike price (i.e. spot price < strike price). If the index is much lower than the strike price, the call is said to be deep OTM. In the case of a put, the put is OTM if the index is above the strike price.

    An out-of-the-money option currently has no intrinsic value —e.g. a call option is out-the-money if the strike price ("the strike") is higher than the current underlying price. An in-the-money option conversely does have intrinsic value. The strike price of an in-the-money call option is lower than the current underlying price.

    (c) *At-the-money option:* An at-the-money (ATM) option is an option that would lead to zero cashflow if it were exercised immediately. An option on the index is at-the-money when the current index equals the strike price (i.e. spot price = strike price). In other words, an option is at-the-money if the strike price, i.e., the price the option holder must pay to exercise the option, is the same as the current price of the underlying security on which the option is written.

    For example, suppose the current stock price of SBI is Rs.1,000. A call or put option with a strike of Rs.1,000 is at-the-money. A call option with a strike of Rs.800 is in-the-money (1000 - 800 = 200 > 0). A put option with a strike at Rs.800 is out-of-the-money (800 - 1000 = -200 < 0). Conversely, a call option with a Rs.1200 strike is out-of-the-money and a put option with a Rs.1,200 strike is in-the-money.

16. ***Intrinsic value of an option:*** The option premium can be broken down into two components – intrinsic value and time value. The intrinsic value of a call is the amount the option is ITM, if it is ITM. If the call is OTM, its intrinsic value is zero. Putting it another way, the intrinsic value of a call is Max [0, $(S_t - K)$] which means the intrinsic value of a call is the greater of 0 or (St- K). Similarly, the intrinsic value of a put is, Max [0, $(K - S_t)$] i.e. the greater value of 0 or $(K - S_t)$. . S_t is the spot price at time t; K is the strike price.

17. ***Time value of an option:*** The time value of an option is the difference between its option premium and its intrinsic value. Both calls and puts have time value. An option that is OTM or ATM has only time value. Usually, the maximum time value exists when the option is ATM. The longer the time to expiration, the greater is an

Difference between Options and Futures

Distinction	*Options*	*Futures*
1. Right and Obligation	An option gives the buyer (holder) the **right** but not the **obligation** while the seller has an obligation to comply with the contract. It represents a **right** to one partly and an **obligation** on the other.	In case of futures, there is an **obligation** on the part of the buyer and the seller. It represents an **obligation** to both a buyer and a seller.
2. Premium and Margins	Buyer of the option contract pays **premium** to the seller of the contract for obtaining the right. It is non-refundable whether the option is exercised or not.	Both buyer and seller have to deposit upfront **margin** with the exchange. Different types of margins like initial margin, maintenance margin and variation margins, etc are to be deposited by the traders.
3. Pay-off (Profit/Loss)	The buyers (holders) of option contracts have the possibility of unlimited profit but their losses are restricted to the premium paid. Sellers (writers) of the option contracts have the possibility of limited profit only to the extent of the premium received but they are exposed to the possibility of unlimited losses.	In case of rise in futures prices, the buyer gains and *vice verse*. The position is opposite in case of the seller of the futures contract. Thus, both buyers and sellers face the possibility of unlimited gain or loss.
4. Common Users in the Derivative Market	Options are preferential derivative contracts for the hedgers in the derivative market to minimise risk. Normally they are the covered call writers.	Futures are the preferential contracts for the speculators in the derivative market to maximise profit. Normally they are the call holders.
5. Pricing Methods/ Models	Binomial option pricing model and Black-Scholes models are used.	Cost-of-carry model, Expectation model, Normal Backwardation model and CAPM are used.

8.5 VALUE OF AN OPTION

option entry

The buyer of the option pays premium to the seller (or writer) of the option. In return, the writer of the call option is obligated to deliver the underlying asset to the buyer of the option if the call is exercised. In case of put option, the holder of the option will buy the underlying asset if the put is exercised.

The price of an option can be decomposed into two parts: Intrinsic value and Time value (Extrinsic value). Thus,

$$\text{Option Value} = \text{Intrinsic Value} + \text{Time Value (Extrinsic Value)}$$

'Intrinsic value of business is the PV of all expected future cash bon flow discounted at the appropriate discount rate.

8.5.1 Intrinsic Value

Intrinsic Value of the option that can be realised if the option is exercised. Therefore, only in-the-money (ITM) options have intrinsic value. In other words, the value that can be made by exercising the option is known as intrinsic value. As a matter of trade practice, the owner (the holder) of the option will never choose to loss money by exercising his option. Therefore, an option can never have a negative value, i.e., > 0.

The intrinsic value of an option is the difference between its exercise price (X) and the current spot price of the underlying asset (S_T).

Symbolically, the intrinsic value of an option is given as follows:

Intrinsic Value of a Call Option (C_T): $\text{Max}[(S_T - X), 0]$ OR $(S_T - X)^+$

Intrinsic Value of Put Option (P_T): $[\text{Max}(X - S_T), 0]$ OR $(X - S_T)^+$

The intrinsic value is realised only when the option is exercised. The following table makes the concept clear:

Let Spot Price of the underlying asset on exercise date = S_T

Exercise or Strike Price = X

Market Condition	Call Option	Put Option
$S_T > X$	In-the-money Intrinsic value > 0	Out-of-the-money Intrinsic value = 0
$S_T < X$	Out-of-the money Intrinsic value = 0	In-the-money Intrinsic value > 0
$S_T = X$	At-the-money Intrinsic Value = 0	At-the-money Intrinsic Value = 0

Graphic Presentation: The intrinsic value of a call option and put option can be shown in the following graphs:

specification
Participants of option market
option pay off
margin requirements
open interest
Exercise
Liquidity

Call Option Intrinsic Value

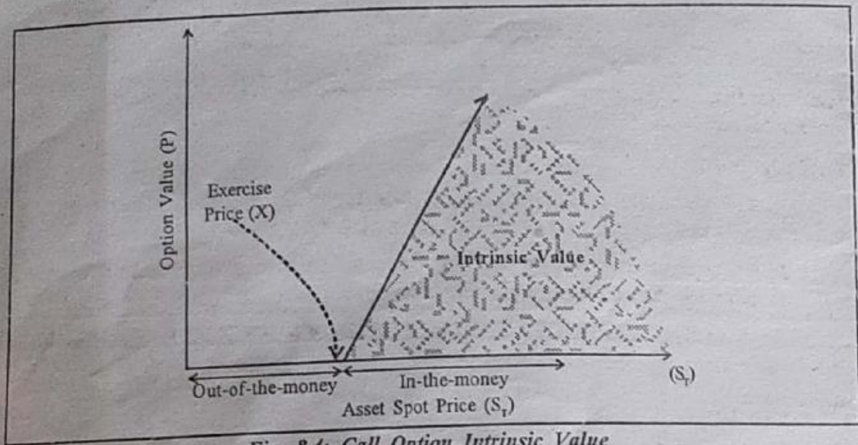


Fig. 8.4: Call Option Intrinsic Value

Put Option Intrinsic Value

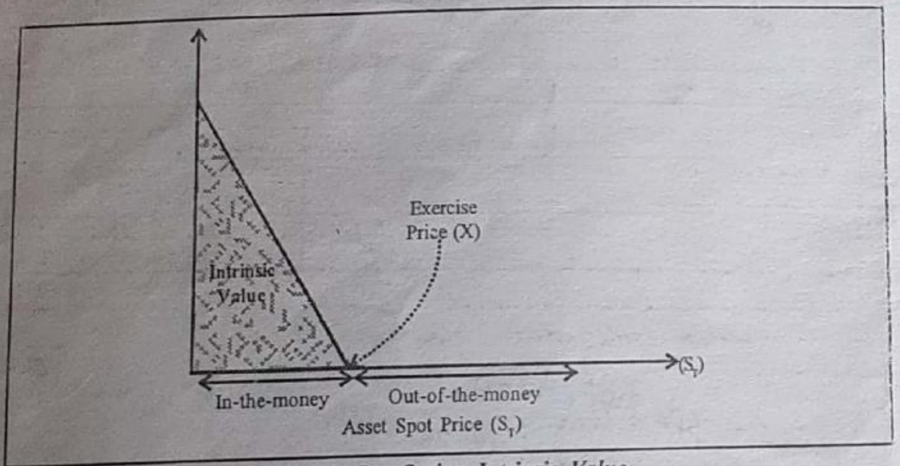


Fig 8.5: Put Option Intrinsic Value

As seen in the graph, the intrinsic value of a call option is positive when underlying asset's spot price (S_T) exceeds the option's strike price (X). Similarly, the intrinsic value of a put option is positive when option's strike price (X) exceeds the spot price of the underlying asset. However, the fact that the option cannot have negative value and also works in the owner's favour.

Option value can be determined via a predictive formula such as **Black-Scholes** or using a numerical method such as the **Binomial model**. These models will be discussed later part of this chapter.

Example 8.1: Intrinsic value of a call option.

Spot Price (S_T)	110	120	130	140	150	160	165	167	170	175	180
Exercise Price (X)	150	150	150	150	150	150	150	150	150	150	150
Intrinsic Value (C_T)	0	0	0	0	0	10	15	17	20	25	30

As spot price of the underlying asset increases, the intrinsic value of the option also increases.

Example 8.2: Intrinsic value of a put option.

Exercise Price (S_T)	150	150	150	150	150	150	150	150	150	150	150	150
Spot Price (X)	180	175	170	165	160	155	150	145	140	135	130	125
Intrinsic Value (C_T)	0	0	0	0	0	0	0	5	10	15	20	25

As spot price of the underlying asset decreases, the option price (intrinsic value) increases.

8.5.2 Time Value of an Option

When an option trading at more than the intrinsic value, the difference is known as Extrinsic value, or more commonly known as *Time Value*.

Consider the following call options:

Case (a): Underlying: P. Ltd. Stock
Underlying Price: ₹ 150 per share
Type: Call option
Style: American
Exercise Price: ₹ 145
Expiry Date: 31st December

Let assume the current spot price of this stock is ₹ 160 on 30th September. So this particular call option is traded at ₹ 15 (i.e., ₹ 160 – ₹ 145).

On 30th September, we understand that the option has the following values:

(i) Intrinsic value – ₹ 150 – ₹ 145 = ₹ 5

(ii) Actual value on trading = ₹ 160 – ₹ 145 = ₹ 15

Here the difference between these two value, i.e., ₹ 15 – ₹ 5 = ₹ 10 is the Time value.

Point to Note:

The difference between the value of an option (American) at the particular time on or before the expiration and its intrinsic value is known as TIME VALUE of an option.

Mathematically,

Time Value = Option Price – Intrinsic Value

- Option Price is the option premium.

The following examples make clear of the computational aspect of Time value:

Example on Call Option

Option	Strike Price	Option Premium	Spot Price	Intrinsic Value	Time Value
Call	130	5	128	2	3
Call	125	2	125	0	2
Call	115	3	120	0	3

Example on Put Option

Option	Strike Price	Option Premium	Spot Price	Intrinsic Value	Time Value
Put	150	4	152	2	2
Put	160	8	165	5	3
Put	140	20	155	15	5

Note: In the above examples, we find that the intrinsic value plus the time value equals the total premium of the option.

8.5.3 Factors Influencing Time Value of an Option

There are certain factors which are influencing the time value of an option. These are:

1. **The Expected Volatility in the Futures Spot Price of Underlying Instrument**

   We know that:

   Option Time Value = Option Price – Option Intrinsic Value

   Intrinsic Value = (Spot Price – Strike Price) for Call Option

   Intrinsic Value = (Strike Price – Spot Price) for Call Option

   Thus, spot price of the underlying instrument is a determinant of intrinsic value. So, the expected volatility in the future spot price influences the time value of an option. However, volatility in the prices of the underlying instrument can stimulate option demand and enhance its value.

2. **Time to Expiration (Remaining Life Span of the Option)**

   The second influencing factor on time value of an option is the length of the period remaining to the expiry date of an option. It may be noted that the more time that remains in the life span of an option, the time value tends to be higher.

Numerically, time value depends on the time until the expiration date and volatility of the underlying instrument's price.

8.6 OPTIONS POSITIONS

Options are two types — calls and puts. Calls give the buyer the right but not the obligation to buy an asset at a prespecified price on, or before, a specified date in the future. Puts give the buyer the right but not obligation to sell an asset at a prespecified price on, or before, a specified date in the future. Thus, there are two sides to every option contracts. As there are two types of options, so there are four basic option positions.

Call Options:

1. Call options give the holder (buyer) the right (but not the obligation) to buy the underlying asset at the exercise/strike price any time until the expiry date.
2. Call options obligate the writer (seller) to sell the underlying asset at the exercise/strike price any time until the expiry date.

Thus, the option writer has the exact opposite position in comparison to the option holder, this is because they are on opposite sides of the same contract.

Put Options:

3. Put options give the holder (buyer) the right (but not the obligation) to sell the underlying asset at the exercise/strike price any time until the expiry date.
4. Put options obligate the writer (seller) to buy the underlying asset at the exercise/strike price any time until the expiry date.

Thus, the writer has the exact opposite in comparison to the holder, this is because these are on opposite sides of the same contract.

Summary of the Four Basic Options Positions		
	Holder	Writer
	(The buyer of the right)	(The seller of the right)
Call Options	Right buy (1)	Obligation to sell (2)
Put Options	Right to sell (3)	Obligation to buy (4)

In the language of option derivatives, these four positions are called as:

1. A long position in a call option – Buyer of call option
2. A short position in a call option – Writer of call option
3. A long position in a put option – Buyer of put option
4. A short position in a put option – Writer seller of put option

Summary of Transaction through Option (European Style)

Option Position	Now	At Expiration Let X = Strike Price, S_T = Market Price of the underlying asset
Buyer (Holder) of a call (Long call) → Right to buy	The option holder pays premium and acquires the right to buy the underlying asset at the strike price on the expiration date.	• If $S_T > X$, the option holder exercises the option. Then Gross pay-off = $S_T - X$ and Net Profit = $S_T - X$ – Premium. • If $S_T < X$, the option holder does not exercise the option, then Net Profit = Zero – Premium.
Buyer (Holder) of a put (Long put) → Right to sell	The option holder pays the premium and acquires the right to sell the underlying asset at the strike price on the expiration date.	• If $S_T < X$, the option holder exercises the option. Then Gross pay-off = $X - S_T$ and Net Profit = $X - S_T$ – Premium. • If $S_T > X$, the option holder does not exercise the option, then Net Profit = Zero – Premium.
Seller (Writer) of a call (Short call) → Obliged to sell	The option writer receives the premium and commits to delivery the underlying asset at the strike price on the expiry date, if the holder exercies the option and demands the delivery.	• If $S_T > X$, the option holder exercises the option, then Net Loss = $(X - S_T)$ + Premium. • If $S_T < X$, the holder does not exercise the option. Then, Net Profit = Premium.
Seller (Writer) of a put (Short put) → Obliged to buy	The option writer receives the premium and commits to buy and take delivery of the underlying assets at the exercise price on the expiration date if the holder exercises the option and give delivery of the underlyings.	• If $S_T < X$, the option holder exercises the option. Then Net Loss = $(S_T - X)$ + Premium. • If $S_T > X$, the holder does not exercise the option. Then, Net Profit = Premium.

8.7 PAY-OFF PROFILES OF OPTION POSITIONS

There are two sides to every option contract. In case of call (right to buy) option, the holder exercises his right only when the future outcome is favourable (in the money) to him. The writer of a call option, being the owner of the asset, obliged to sell to the option holder in such situation. Therefore, the buyer of the call option (holder) must pay upfront a price, called premium to the option seller (writer). The writer of an option receives cash upfront but has potential liabilities later.

In case of put (right to sell) option, the holder exercises his right only when the future outcome is favourable (in the money) to him. The writer of a put option, being the buyer of the asset, is obliged to buy in such situation. Therefore, the buyer of the put option, being the holder of the right, must pay upfront a price, called premium to the option writer. The writer of an option receives cash upfront but he has potential liabilities later.

Now we illustrate the pay-off of the following four positions:

1. pay-off of the call option buyer
2. pay-off of the call option seller
3. pay-off of the put option buyer
4. pay-off of the put option seller

Besides, pay-off diagrams are also drawn to show how option will pay out prior to expiry and at the time just before expiration.

The pay-off of a call option or put option depends on the following factors:

(i) Spot price of underlying asset just before the expiration.

(ii) Exercise price

(iii) The option value, i.e., option premium.

I. Pay-off of a Call Option Buyer (Long Call) at Expiration

Consider the following basics of a call option:

Underlying : Reliance share

Type of option : Call option

Style of option : European

Position : Long (Buyer)

Exercise Price = ₹ 150 per share

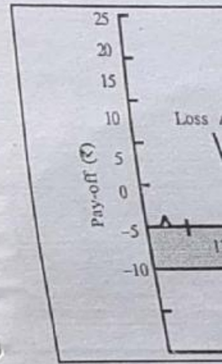
Option premium = ₹ 5 per share

Spot price at expiration say ₹ 130, ₹ 140, ₹ 150, ₹ 160, ₹ 170, ₹ 180 per share.

Now we calculate the pay-off of the call option holder in the following table:

Table 8.1: Pay-off of the Call Option Holder at Expiration

	₹	₹	₹	₹	₹	₹	₹
(a) Spot price (S_T)	130	140	150	155	160	170	180
(b) Exercise price (X)	150	150	150	150	150	150	150
(c) Option exercise	No	No	–	Yes	Yes	Yes	Yes
(d) Pay-off (S_T – X)	–	–	0	5	10	20	30
(e) Call premium (paid)	–5	–5	–5	–5	–5	–5	–5
(f) Profit/Loss : (d) + (e) (Net pay-off)	–5	–5	–5	0	5	15	25



II. Pay-off of

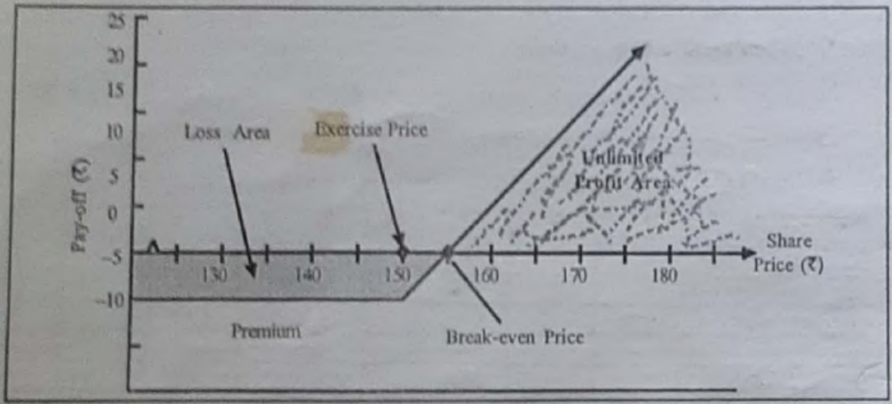


Fig. 8.6: Pay-off of the Call Option Buyer

II. Pay-off of a Call Option Seller (Short Call) at Expiration

Consider the following basics of a call option:

Underlying: Reliance stock

Type of option: Call option

Style of option: European

Position: Short (Seller)

Exercise price = ₹ 150 per share

Option premium = ₹ 5 per share

Spot price at expiration say: ₹ 130, ₹ 140, ₹ 150, ₹ 160, ₹ 170, ₹ 180 per share.

Now we calculate the pay-off of the call option seller writer in the following table:

Table 8.2: Pay-off of the Call Option Writer (Seller) at Expiration

	₹	₹	₹	₹	₹	₹	₹
(a) Strike price (X)	150	150	150	150	150	150	150
(b) Current spot price (S_T)	130	140	150	155	160	170	180
(c) Option exercise by holder	No	No	No	Yes	Yes	Yes	Yes
(d) Option pay-off	–	–	0	–5	–10	–20	–30
(e) Option premium (received)	+5	+5	+5	+5	+5	+5	+5
(f) Profit or Loss : (d) + (e) (Net pay-off)	5	5	5	0	–5	–15	–25

Pay-off Diagram:

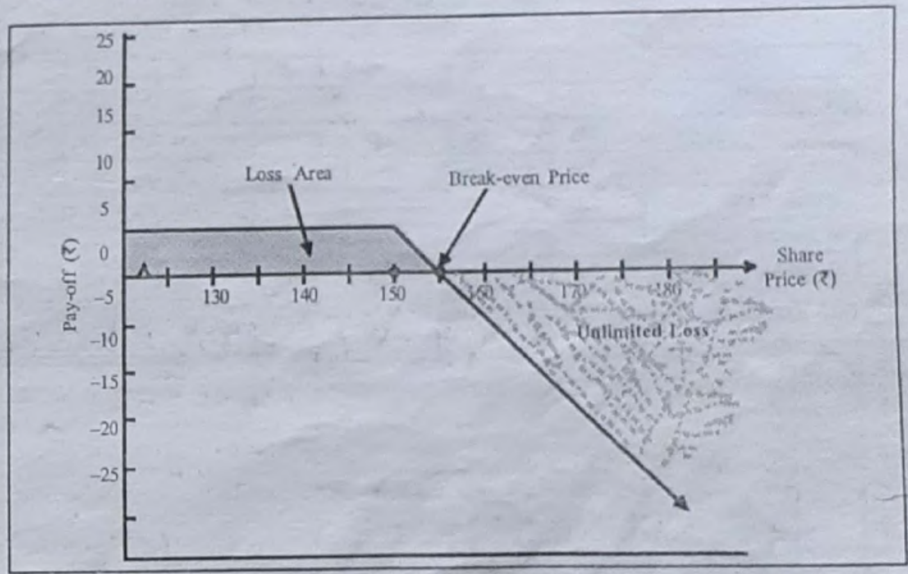


Fig. 8.7: Pay-off of the Call Option Seller

III. Pay-off of a Put Option Buyer (Long Put) at Expiration

Consider the following basics of a put option:

Underlying: Reliance stock

Type of option: Put option

Style of option: European

Position: Long (Buyer)

Exercise price = ₹ 150 per share

Option premium = ₹ 10 per share

Spot price at expiration say: ₹ 120, ₹ 130, ₹ 140, ₹ 150, ₹ 160, ₹ 170 per share.

Now we calculate the pay-off of the put option buyer in the following table:

Table 8.3: Pay-off of the Put Option Buyer (Long Put) at Expiration

	₹	₹	₹	₹	₹	₹	₹
(a) Exercise price (X)	150	150	150	150	150	150	150
(b) Spot price (S_T)	120	130	140	150	160	170	180
(c) Exercise or Not	Yes	Yes	Yes	No	No	No	No
(d) Pay-off ($X - S_T$)	30	20	10	–	–	–	–
(e) Premium paid	–10	–10	–10	–10	–10	–10	–10
(f) Net pay-off	20	10	0	–10	–10	–10	–10

Pay-off Diagram:

pricing of put option before expiration

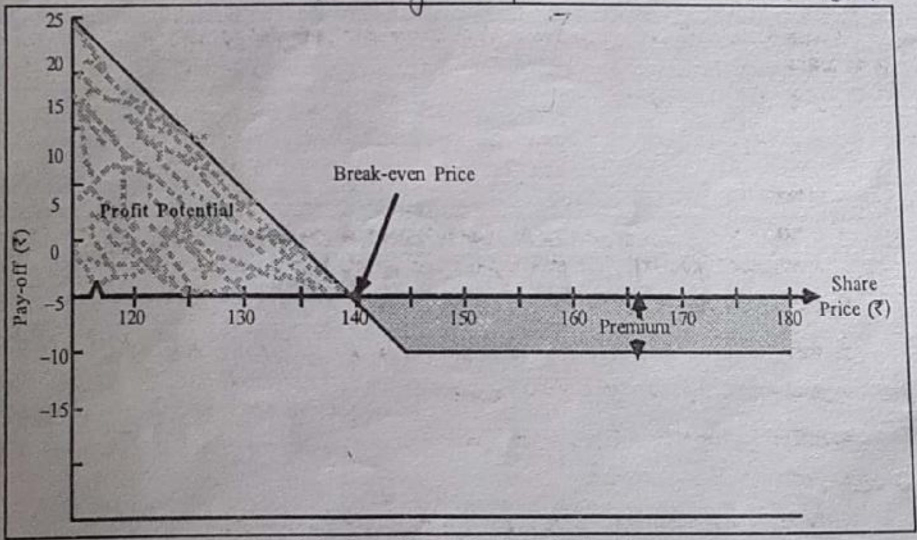


Fig. 8.8: Pay-off of the Put Option Buyer

IV. Pay-off of a Put Option Seller (Short Put) at Expiration

Consider the following basics of a put option:

Underlying: Reliance stock

Type of option: Put option

Style of option: European

Position: Long (Seller)

Exercise price = ₹ 150 per share

Option premium = ₹ 10 per share

Spot price at expiration say: ₹ 120, ₹ 130, ₹ 140, ₹ 150, ₹ 160, ₹ 170, ₹ 180 per share.

Now we calculate the pay-off of the put option seller in the following table:

Table 8.4: Pay-off of the Put Option Seller (Short Put) at Expiration

	₹	₹	₹	₹	₹	₹	₹
(a) Spot price (S_T)	120	130	140	150	160	170	180
(b) Exercise price (X)	150	150	150	150	150	150	150
(c) Exercise or Not	Yes	Yes	Yes	No	No	No	No
(d) Pay-off = (a) – (b)	–30	–20	–10	0	–	–	–
(e) Option premium (received)	+10	+10	+10	+10	+10	+10	+10
(f) Net pay-off (Profit loss)	–20	–10	0	+10	+10	+10	+10

Pay-off Diagram:

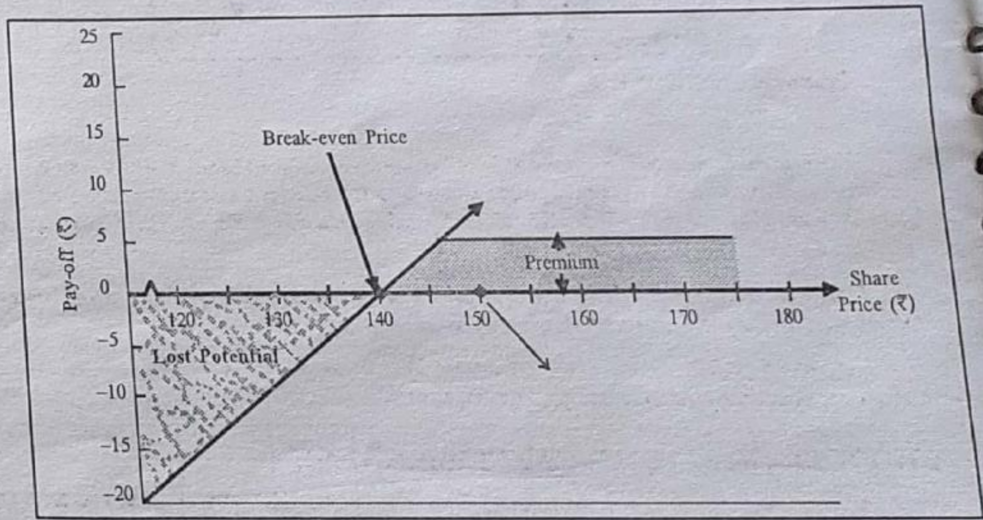


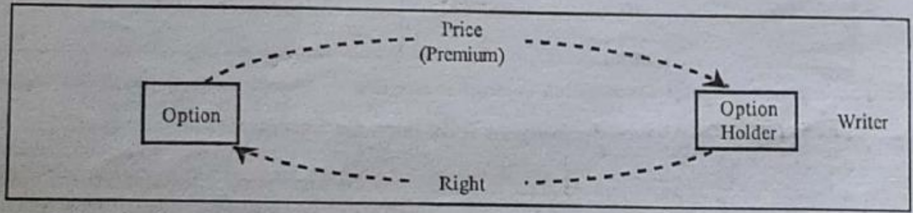
Fig. 8.9: Pay-off of the Put Option Seller

CHAPTER SUMMARY

An option is a special type of contract between two parties where one party grants the other party the right to buy or sell a specific asset or commodity (or instrument) at a specified price within a specific time period. There are only two basic types of options, i.e., ***call option*** and ***put option***.

9.1 AN INTRODUCTION TO OPTION PRICING

A call option gives the holder the right to buy an asset (say stock) at a fixed (strike) price while the put option gives the holder the right to sell at a fixed (strike) price. Let us consider a stock option. A call option holder pays option premium to the writer of the option. In return, the option writer is obliged to sell the shares if the option is exercised by the option holder. If the stock price (S_t) at the time of exercise is less than the exercise price (X), the option holder will not exercise the option. In this case the liability of the option writer is nil. On the other hand, if the stock price (S_t) is higher than the exercise price (X), the option holder will exercise the option. The reverse situation is in case of put option. Thus, in option trading :



It is very clear that the option holder pays certain price which is otherwise known as option premium to the option writer to obtain the right to trade, i.e., to buy or sell.

Option prices (or Premiums) have two components; prices are determined by six factors; and the way prices change are measured by six indicators (called Greeks).

There are various models used to determine the option price.

To understand the option pricing one has to conceptualise each element of the following figure:

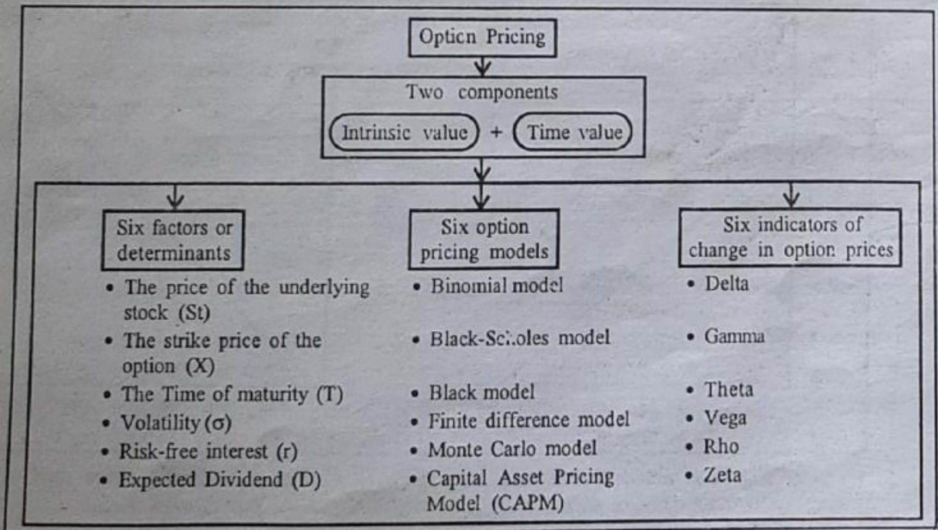


Fig. 9.1: The Fundamentals of Option Pricing

In this chapter, we will discuss the six determinants of option pricing.

9.2 DETERMINANTS OF OPTION PRICE

There are six factors influencing the value of an option. These are:

1. Current price of the underlying security (S_t) at maturity.
2. Strike price (X).
3. Time to maturity (T).
4. Volatility of the underlying security price (σ).
5. Risk-free interest rate (r).
6. The expected dividends during the life of the asset (D).

These factors influence the value of an option depending upon the type (call option or put option) and style (American or European) of an option. We shall now discuss these determinants.

9.2.1 Current Price of the Underlying Security (S_t) at Maturity

We know that pay-off an option is calculated by comparing the current price (S_t) of the underlying asset with its strike price (X). Thus:

Option	Call option	Put option
Pay-off	$(S_t - X)$	$(X - S_t)$

In case of increase in the value of underlying asset, the value of call option will increase and the value of put option will decrease. In case of decrease in the value of underlying asset, the value of call option will decrease and the value of put option will increase.

Thus, the higher the asset price, the higher is the chance that it will rise above the exercise price and therefore, the higher the premium for call option. On the other hand, the higher the asset price, the lower is the chance that it will fall below the strike price and therefore, the premium for a put option would be lower.

Example:

Call Option

Current Stock (S_t)	Strike Price (X)	Pay-off $(S_t - X)$
150	150	0
155	150	5
160	150	10
165	150	15
170	150	20
175	150	25

Put Option

Strike Price (X)	Current stock Price (S_t)	Pay-off $(X - S_t)$
175	150	25
175	155	20
175	160	15
175	165	10
175	170	5
175	175	0

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These above stated data can be presented is the following Figures 9.2 and 9.3 to show the manner in which call option and put option prices depend on the price of the current price of the underlyings.

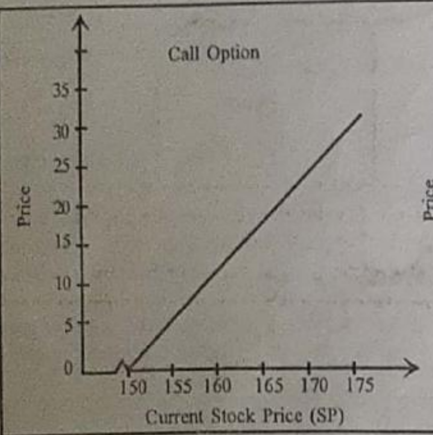


Fig. 9.2: Call Option Price and Current Price of the Underlying

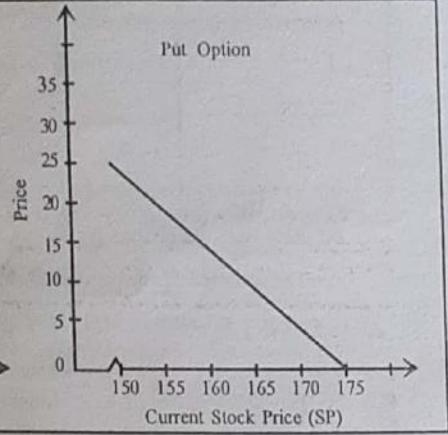


Fig. 9.3: Put Option Price and Current Price of the Underlying

Remarks:

1. Increase in the current price of underlying asset results higher pay-off for a call option. Thus, a call option becomes more valuable as the price of the underlying increases. The reverse situation will be for a put option.
2. Decrease in the current price of underlying asset results lower pay-off for a call option and higher pay-off for a put option. Thus, a call option becomes less valuable and a put option becomes more valuable as the price of the underlying decreases.

9.2.2 Strike (Exercise) Price of an Option (X)

A call option will be exercised only when the price of the underlying asset at maturity (S_t) exceeds the strike price (X). A put option will be exercised only when the strike price (X) exceeds the price of the underlying asset at maturity (S_t). The pay-off in both options may be summarised in the following table:

Type of Option	Option Buyer (Holder)	Option Seller (Writer)
Call Option	Max $[0, (S_t - X)]$	Min $[0, (X - S_t)]$
Put Option	Max $[0, (X - S_t)]$	Min $[0, (S_t - X)]$

Keeping all other variables (determinants) constant, if we change strike price, what is the impact on pay-off *vis-à-vis* value of on option? Consider the following example.

Example:

Call Option

Current Stock (S_t)	Strike Price Price at Maturity (X)	Pay-off ($S_t - X$)
180	155	25
180	160	20
180	165	15
180	170	10
180	175	5

Put Option

Current Stock (S_t)	Current Price Price at Maturity (X)	Pay-off ($X - S_t$)
150	155	5
150	160	10
150	105	15
150	170	20
150	175	25

The strike price of an option contract is just like an insured value of the underlying asset.

In case of call option, the lower the strike price, the higher would be the pay-off. This means the value of the call will be higher and ultimately the option premium will be higher. Logically, if the strike price of an option contract is lower, then the asset price will easily exceed this level. Accordingly, the option seller (writer) will demand for higher premium because the probability of exercising the option by the option holder is very less. The situation is the reverse for a put option.

From the above, it is very clear that there is a negative correlation between strike price and option value in case of call option and positive correlation between call option and positive correlation between strike price and option value in case of put option.

The following diagram depicts the above features.

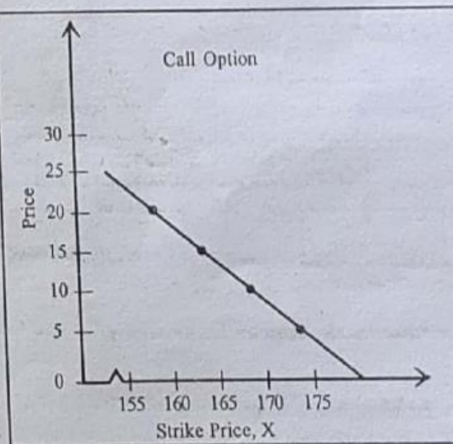


Fig. 9.4: Strike Price and Call Option Value

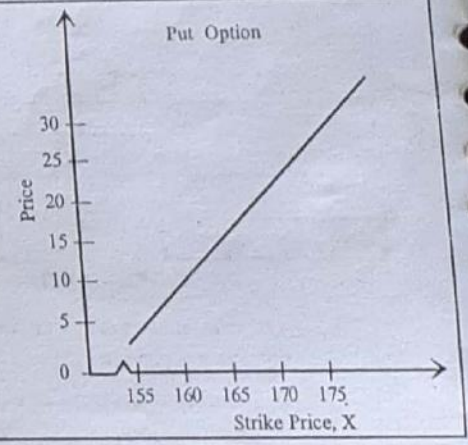


Fig. 9.5: Strike Price and Put Option Value

Remark: Keeping all other factor constant, call option is less valuable as the strike price increases and *vice versa*. Similarly, put option is more valuable as the strike price increases and *vice versa*.

9.2.3 Time to Maturity (T)

Time to maturity is one of the important factor which influences the option value. Generally, the longer the time taken for maturity, the higher is the option value. Because, the longer the life of an option, the greater the chances of price fluctuations and *vice versa*.

Option value is the aggregate of its intrinsic value and time value. As an option's expiration date approaches, its time value diminishes and ultimately becomes zero.

Remark: Longer the time to expiration, higher is the value of call as well as put.

9.2.4 Volatility of the Underlying Security (σ)

Volatility (i.e., price variability) of the underlying asset influences the value of an option. The fluctuation in the price of an asset brings risk to the investors (the option holders). Options are used to manage such risk. For example, the call option protects the option holder against upward movement of the price and a put option protects against downward movement of price. But the degree of variability (i.e., volatility) in the prices of the underlying assumes risk contents in the dealings. This volatility is measured by Standard Deviation (σ). Both call options and put options become more valuable when volatility of the underlying asset increase.

Remark: Higher the volatility, higher is the option value (Premium).

9.2.5 Risk-free Interest Rate (r)

The risk interest rate affects the price of an option indirectly.

There are two-fold effect of risk-free interest on option contract. These are:

Cause	Effects
Increase in the interest rate in an economy (country as a whole).	(i) The price of underlyings (stock) will rise. (ii) The present value of future cash flow received by the holder of the option decreases.

In case of a call option, the exercise price is fixed at the time of contract. The holder pays this contract price (exercise price/strike price) at a future date when he exercises his option.

Similarly, in case of a put option, the option holder receives the exercise price at a future date when he exercises his option.

The present value of the exercise price (future value) depends on the interest rate (r) and the time until the expiration of the option (T).

Consider the following example:

Exercise Price	Risk-free Interest Rate	Time to Expiration	Present Value of Exercise Price
240	10%	1 year	$\frac{240}{1.10} = 218.18$
240	11%	1 year	$\frac{240}{1.10} = 216.22$
240	12%	1 year	$\frac{240}{1.12} = 214.29$

From the above example, it is very clear that if the interest rate rises, the present value declines. This situation will be favo-rable to the call holder (the partly who pays) and unfavourable to the put holder (the party who receives). Thus, the call option price increases as the risk-free interest rate increases and *vice versa*. Similarly, the put option price declines as the risk-free interest rate increases. See the following figures:

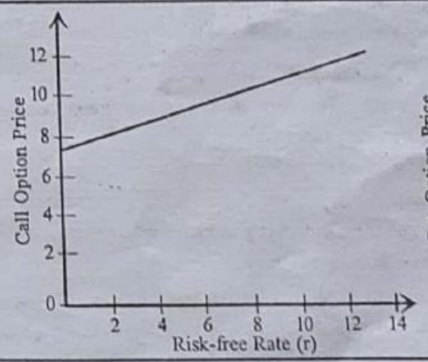


Fig 9.6: Effect of Changes in Risk-free Interest on Call Option Price

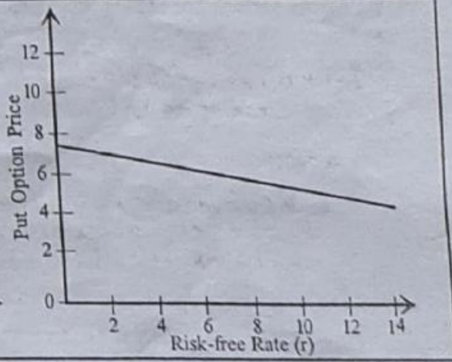


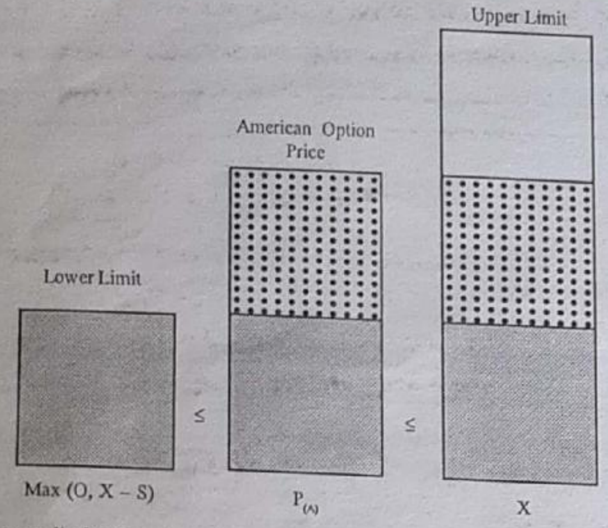
Fig. 9.7: Effect of Changes in Risk-free Interest on Put Option Price

9.2.6 Dividends Expected During the Life of the Asset (D)

The most common popular type of call option is the option on stocks.

Sometimes, stocks can be sold just before the declaration of dividend, i.e., dividend has become due to the shareholder but has not been received by him. In such circumstances, if the market price includes the accrued dividend, then the price will be higher to the extent of such dividend. In this case, the buyer will receive such dividend as and when the dividend will be paid by the company. Thus, when the stock price includes dividend, the price is called **cum-dividend price**.

Contrary to this, if the market price of stock does not include such dividend, then the buyer pays only the normal market price and ultimately the seller receives the accrued dividend as it is due to him. Thus, when the stock price does not include the amount of dividend, then such price is called **ex-dividend price**.



Thus, the upper limit of an American put option can be expressed as:

$$\text{Max}(O, X - S) \le P_{(A)} \le X$$

European Put Option: Like American put option, we may now combine the lower limit and upper limit of an European option and summarise the bounds as follows:

$$\text{Max}(O, Xe^{-rt} - S) \le P_E \le Xe^{-rt}$$

9.4 PUT-CALL PARITY

Meaning: Put-call Parity is a financial relationship between the price of a put option and a call option on the same underlying instrument with identical strike price and expiry.

Application: To derive the **put-call parity** relationship, the assumption is that the options are not exercised before the date of maturity. Thus, the **put-call parity** is a concept related to European call and put options.

Concept: The put-call parity is an option pricing concept that requires the values of call and put options to be in ***equilibrium*** to prevent arbitrage.

Relationship: Put-Call Parity: A close relationship among the price of a call, price of a put and the price of the underlying stock. Thus:

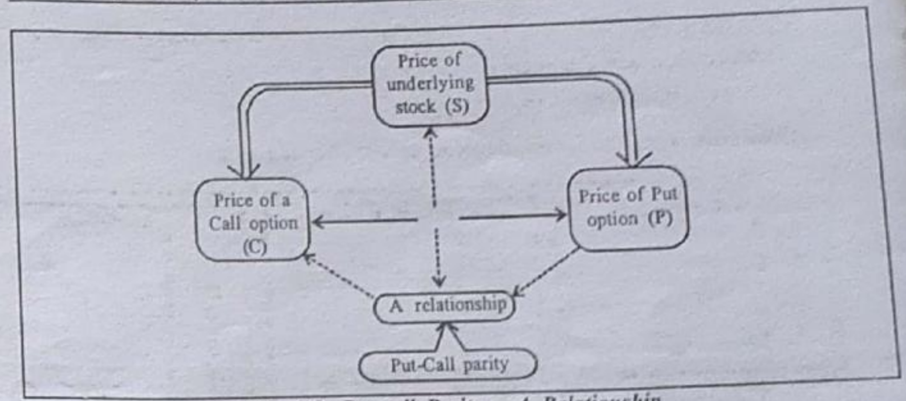


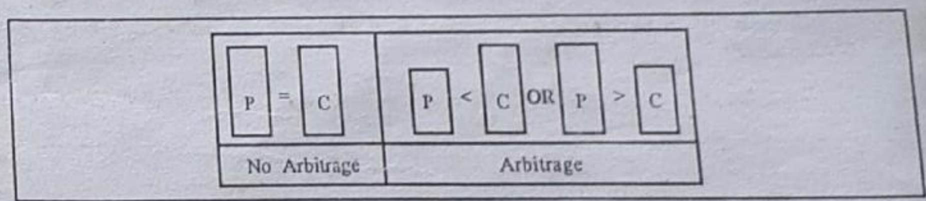
Fig. 9.9: Put-call Parity – A Relationship

As a matter of fact, there is a close relationship among the prices of put options, call options and their underlyings. A change in the price of the underlying stock affects the price of both call and put options that are written on the stock. Put-Call parity defines this relationship.

It states that the premium (price) of a call option implies a certain fair price for the corresponding put option having the same strike price and expiration date, and the *vice versa*.

Prevent Arbitrage

Arbitrage opportunities would prevail if there is a divergence between the value of calls and puts. Arbitrageurs would come into make profitable, riskless trades until the put-call parity is restored. Consider these cases:



Here P = Price of the put option, C = Price of the call option.

The *put-call parity* implies that if the price of a put option, price of a call option and price of the underlying stock are in the equilibrium, there is no opportunity for arbitrage. This relationship is strictly for European style options.

Put-Call Parity Formula

To understand the put-call parity relationship, let's consider the following two portfolios:

Portfolio A: It consists of a *European call option* and cash. The cash component is equal to the number of shares covered by the call option multiplied by the call's strike price.

Where S = Stock price = ₹ 270

X = Strike price = ₹ 265

r = 10%

$t = 6 \text{ months} = \frac{6}{12} = 0.5$

Now the lower bound of call option will be:

$$S - Xe^{-rt} = 270 - 265\ e^{-(.10 \times .5)}$$

$$= 270 - 265\ e^{-0.05}$$

$$= 270 - 265 \times 0.9512$$

$$= 270 - 252$$

$$= 18$$

Problem 2

A one-year call option with an exercise price of ₹ 180 is available at a premium of ₹ 18. Further a one-year put with an exercise price is available with an exercise price of ₹ 165 at a premium of ₹ 9.

Set up a portfolio combining both put and call. Also calculate the pay-off if the share price after one year is (a) ₹ 174 (b) ₹ 135 or (c) ₹ 225.

Solution:

(a) At share price ₹ 174, neither call option, not put option will be exercised.

Thus, profit/loss = – ₹ 18 – ₹ 9 = – ₹ 27.

(b) At share price ₹ 135, call option will not be exercised, but put option can be exercised.

Thus, profit/loss = (₹ 165 – ₹ 135) – ₹ 27

= ₹ (30 – 27) = ₹ 3

(c) At share price ₹ 225, call will be exercised but not put.

Thus, profit/loss = (₹ 225 – ₹ 180) – ₹ 27

= ₹ (45 – 27) = + ₹ 18

Problem 3

The current market price of a share is ₹ 155. The volatility of the share is measured as 20%. The risk free interest rate is currently 8% per annum. There is a call option and put option on the share. Time to expiration is 6 months with exercise price ₹ 150. Calculate put option price using put-call parity of call price is ₹12.

Solution:

By using the put-call parity relation model, we have

$$P = C - S + Xe^{-rt}$$

Where P = Put price

C = Call price = 12

S = Current stock price = ₹ 160

r = Rate of interest continuously compounded

t = Time to expiration = 6 months

Now we can calculate

$$P = 12 - 155 + 150\ e^{(.08 \times \frac{6}{12})}$$

$$= 12 - 155 + 150 \times 0.9608$$

$$= 12 - 155 + 144.12$$

$$= 1.12$$

Problem 4

Calculate the call option price from the following data by using put-call parity theory:

Present value of Exercise price = ₹ 128

Value of put option = ₹ 10

Current stock price = ₹ 125

Solution:

Put-call parity relation is:

$$C + PV(X) = P + S$$

Where C = Call Price

P = Put Price

X = Exercise Price

S = Stock Price

Putting the values in the above equation, we get,

$C + 128 = 10 + 135$ or $C = 135 - 128 = 7$

10.1 INTRODUCTION

In finance, the *Binomial Option Pricing Model (BOPM)* provides a generalisable numerical method for the valuation of options. The original version of this Binomial model was developed by John Cox, Stephen Ross and Mark Rubinstein in 1979. This model is also known as C-R-R model.

This model is a "discrete-time" model, because it breaks down the total time to expiration into potentially a very large number of time intervals or steps. In other words, the total time of the option is divided into discrete bits or time steps. These steps form a tree like format. At each step, it is assumed that the price of the underlying asset say stock will move *up or down* by an amount calculated using volatility and time to expiration. Thus, this model is based on binomial approach. This approach assumes that the price of the stock at every point of time may have only two possible states, i.e., move up or move down.

Initially, a tree of stock prices is drawn showing the possible stock prices at every point of time in a forward form (left to right). This tree is commonly known as Binomial Tree. Fig. 10.1 shows a binomial tree.

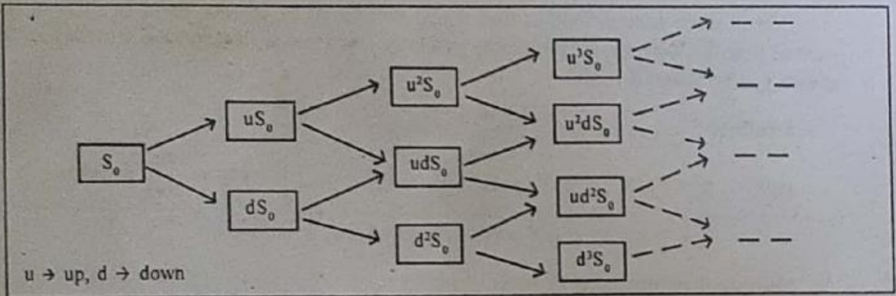


Fig. 10.1: The Binomial Process and Binomial Tree

Points to Note:

(i) This Binomial tree produces a binomial distribution of underlying stock prices.

(ii) The Binomial tree represents all the possible paths that the stock price could take during the life of the option.

From the above concept, we may define a BOPM tree as follows:

"A BOPM tree is an option pricing model, obeys a binomial generating process, in which the underlying stock can assume one of only two possible discrete values in the next time period, for each value that it can take in the proceeding time period."

The tree must be systematically structured and ended at the expiration of the option. The most important concept of this model is that all the terminal option prices for the final possible stock prices are known as they simply equal to their intrinsic value. Thus, the option values at the final point of times are calculated first.

Next option prices at each step of the tree are calculated working back from expiration to present. The option prices at each step are used to derive the option prices at next step of the tree. For calculating the option prices, this BOPM uses "Risk Neutral Valuation" method.

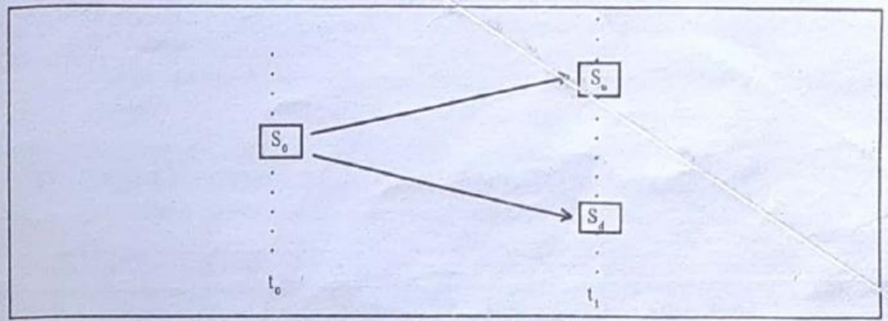


Fig. 10.2: Conceptual Framework of Binomial Tree

Example 1. The current market price of a stock is ₹ 160. It is expected that the price may either move up by 10% or move down by 5% by the end of the month.

Here S_0 = 160 up factor (u) = 10%, down factor (d) = 5%.

Now we represent the above facts in the following binomial tree.

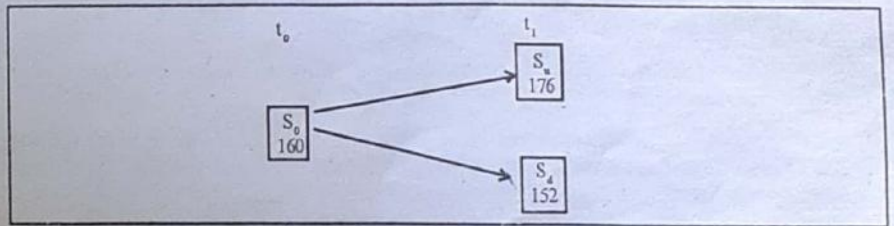


Fig. 10.3: One Step Binomial Tree

In the above example, we may calculate u factor and d factors:

$$\text{u factor} = \frac{S_u}{S_0} = \frac{176}{160} = 1.10$$

$$\text{d factor} = \frac{S_d}{S_0} = \frac{152}{160} = 0.95$$

CHAPTER 10

THE BINOMIAL OPTION PRICING MODEL (BOPM)

Learning Objectives

After studying this chapter, you will be able to:

- Understand the concept of Binomial option pricing model
- Know the assumptions and characteristics of BOPM
- Draw a binomial tree
- Calculate stock prices at each node by using forward induction method
- Calculate option value at each final node
- Determine option values at earlier nodes
- Know the strengths and limitations of BOPM
- Be a Strategic master in option valuation

Chapter Outline

Points to Note:

By definition $u \geq 1$, and $0 < d \leq 1$.

The risk neutral valuation is based on the following three factors:

- Probabilities of the stock prices moving up or down.
- The risk-free rate of interest.
- The time interval of each step.

10.2 ASSUMPTIONS OF THE BINOMIAL OPTION PRICING MODEL

There are five basic assumptions underlying these BOPM which are stated as follows:

1. Stock price movements obeys the binomial process in short periods.

This BOPM assumes that during a short interval of time, the stock can take only two values – the up move or the down move. Thus, the underlying stock price will either:

- Increase by a factor of u% (an up tick)
- Decrease by a factor of d% (a down tick)

Let the original stock price is (S_0) at the initial point of time say (t_0). Then the stock price moves to one of the two new values say S_u (up value) and S_d (down value) at the next point of time say (t_1). It is shown in the following binomial tree form:

2. Use of priori or transition probability to quantity the uncertainty about stock price movements.

The uncertainty is that we do not know which of the two states (up or down) will happen. But we can able to determine the chance of happening such upward or downward state in advance by using *priori* or *transition* probability.

The up (u) and down (d) factor are calculated using the underlying volatility (d) and the time duration of a step (t). Taking the condition that the variance of the log of the price is d^2t, we have

$$u = e^{\sigma\sqrt{t}}$$

$$d = e^{-\sigma\sqrt{t}} = \frac{1}{u}$$

Priori or transition probabilities for price movement: The probability of an up movement is assumed to be (P) and the probability of a down movement is assumed to be (1 – P).

Formula:

$$P = \frac{e^{rt} - d}{u - d}$$

Example 1

If u = 1.1, d = 0.9, r = 0.12 and t = 3 months = 0.25. Find probability of up movement and probability of down movement.

Solution:

We denote probability of up movement is P and probability of down movement is (1 – P).

P is calculated by using the following formula:

$$P = \frac{e^{rt} - d}{u - d} = \frac{e^{0.12 \times 0.25} - 0.9}{1.1 - 0.9}$$

$$= \frac{e^{0.3} - 0.9}{0.20} = 0.6523$$

The probability of down movement is 1 – P = 1 – 0.6523 = 0.3477.

Example 2

Current stock price is ₹160 at time (t_0). It will go up by 20% or down by 10% in the next point of time (t_1). Suppose the probability of the up move is 0.6. Find out the expected stock price at time t_1. Use binomial model.

Solution:

Here S_0 = ₹160 $\quad S_u = 160 \frac{120}{100} = 192 \quad S_d = 160 \times \frac{90}{100} = 142$

$P = 0.6 \quad 1 - P = 0.4$

Now we draw the binomial tree:

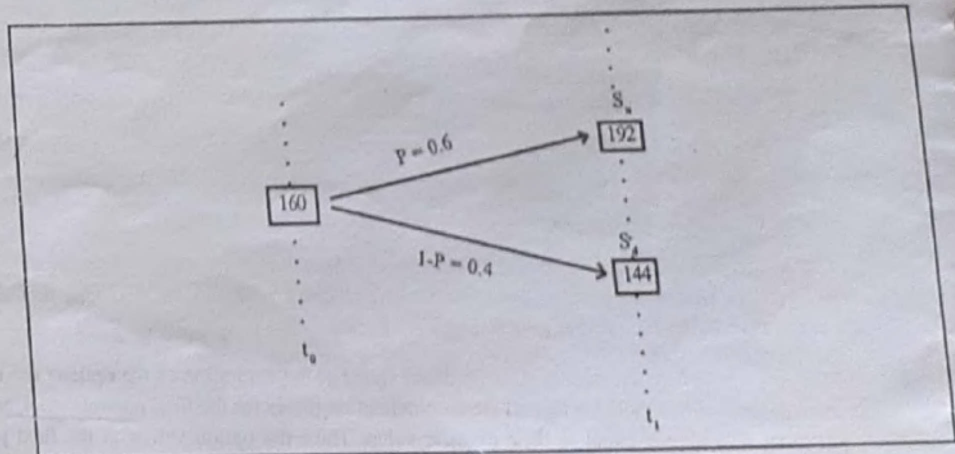


Fig. 10.4: Binomial Tree – The Stock Pricing Process

Expected stock price $= (S_u \times P) + S_d (1 - P)$

$= 192 \times 0.6 + 144 \times 0.4 = 172.80$

Example 3

A stock is currently priced at ₹ 160. In one month, the stock price may go up by 25%, or go down by 12.5%. The strike price is ₹ 180. Find pay-off of a call option. Use binomial tree.

Solution:

Computation of stock prices at t_1.

Pay-off = ₹ 200 − 180 = 20

3. **Constant Interest Rate (r% Per Period):** It is assumed that there is no interest rate uncertainty. The one period interest rate (r) is constant over the life of the option.

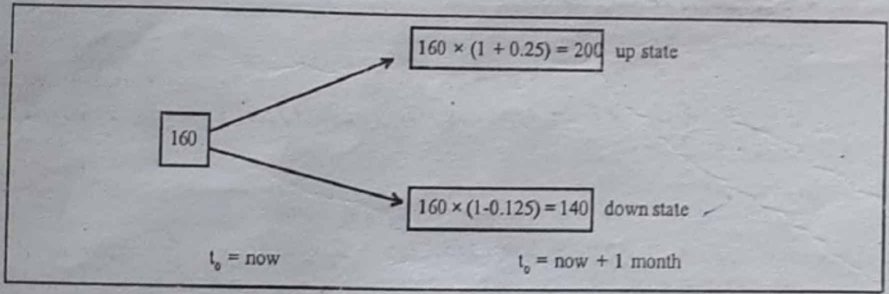


Fig. 10.5: Binomial Tree for Stock Prices

4. **Markets are Perfect:**
   - No arbitrage opportunities
   - No commision
   - No bid-ask spreads
   - No taxes
   - No margin requirements
   - No transaction cost
   - No dividends
5. **Participants Use Red Ocean Strategy:** Red ocean strategy implies to involve in full competition. Thus, the market participants act as price-takers and not the price-makers.

10.3 CHARACTERISTICS OF BINOMIAL OPTION PRICING MODEL

There are five important features of C-R-R option pricing model. These are:

1. **It is a Constant Dicrete-time Model:** This model breaks down the total time of expiration into potentially a very large number of time intervals. The length of such time intervals remain constant throughout the Binomial tree. The end of each time interval is known as 'node'.
2. **Volatility Remains Constant throughout the Model:** Volatility is the variability about the mean value of the stock price. It is measured by standard deviation (σ). The volatility represented by standard deviation remains constant throughout the binomial tree.
3. **The Probability of an Up Movement and Down Movement Remain Constant throughout the Model.**

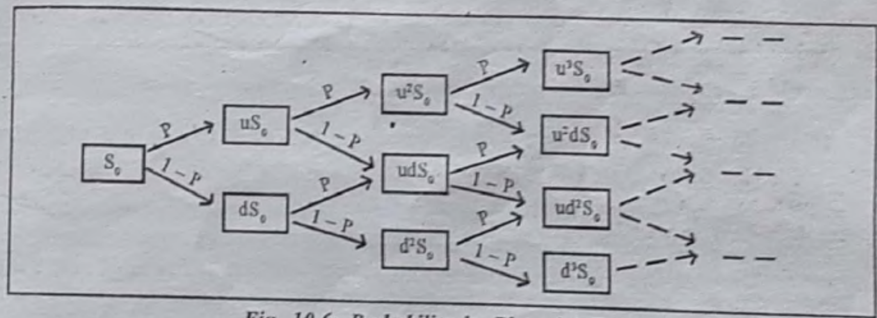


Fig. 10.6: Probability in Binomial Tree

4. **The Binomial Tree is Recombinant:** The Binomial model ensures that the tree is recombinant, i.e., if the underlying asset moves up and then down (u, d) the price will be same as if it had moved down and then up (d, u). Here the two paths merge or recombine. The following figure shows the above concept:

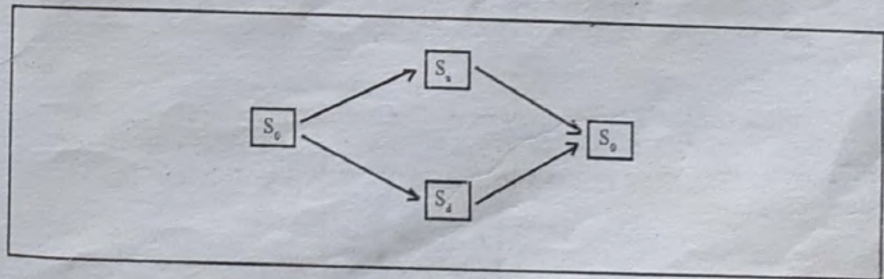
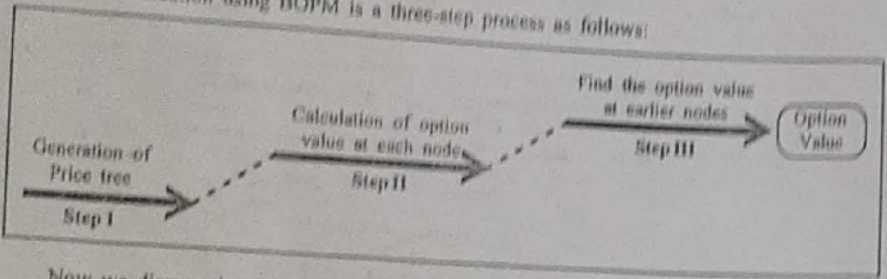


Fig. 10.7: A Combining Tree

5. **Option Price is Determined by Backward Process Calculation:** The option price at each step of the tree are calculated working back from expiration to the present. The option prices at each step are used to derive the option prices at the next step of the tree using risk neutral valuation method. The value computed at each stage is the value of the option at that point in time.

10.4 METHODOLOGY

Option valuation using BOPM is a three-step process as follows:



Now we discuss these three steps and finally derive the value of option.

10.4.1 Step I: Construct the Binomial Tree and Complete the Stock Pricing Process

A Binomial tree is to be drawn by working forward from valuation date to expiration. The procedure is known as Forward Induction (FI). At each step, it is assumed that the stock price will move up or down by a specific factor (u or d) per step of the tree. The value of $u \geq 1$ and $0 \leq d \leq 1$. Take the initial current price S. Then in the next period, it will either be $S_{up} = S \cdot u$ or $S_{down} = S \cdot d$.

How to calculate up and down factors?

The up and down factors are calculated using the following two determinants:

(i) The volatility in the price of the stock i.e., σ.

(ii) **The time duration of a step (t).** The time duration of a step in measured in years (using the day count convention up the underlying asset).

Mathematical formula for u and d

The variance of the log of the price is $\sigma^2 t$. According to this condition, we have,

$$u = e^{\sigma\sqrt{t}}$$
$$d = e^{\sigma\sqrt{t}} = \frac{1}{u}$$

While we construct binomial tree, we must ensure that the tree is recombinant. The condition 'recombinant' implies that if the stock moves up and then down (u, d) the price will be same as it had moved down and then up (d, u). Accordingly the two paths merge or recombine. This property reduces the number of tree nodes and thus accelerates the computation of the option price.

The Stock Pricing Process: Direct Method

The value of the underlying asset say stock at each node can be calculated directly by using the following formula and does not require that the binomial tree be constructed first. The formula for the node value will be:

$$S_n = S_0 \times u^{N_u - N_d}$$

Where N_u = Number of up ticks

N_d = Number of down ticks

Binomial tree representation:

Let S_0 = Initial stock price

u = Up factor

d = Down factor

t_0 = Present time

t_1 = One-step time interval (say 3 months)

t_2 = Two-step time interval (say 6 months)

We may now draw one-step binomial tree and two-step binomial trees.

One-step Binomial Tree

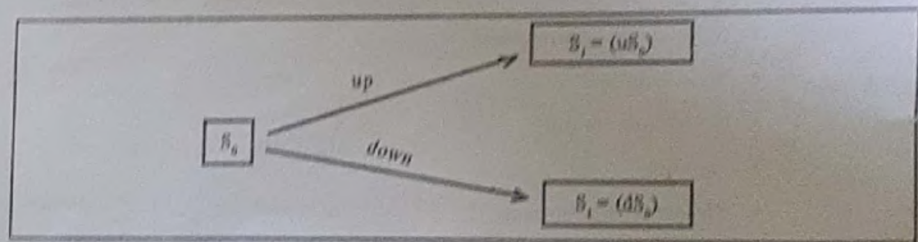


Fig. 10.8: One-step Binomial Tree

...e condition ... be same as ... is property ... price.

... by using ... formula

Two-step Binomial Trees

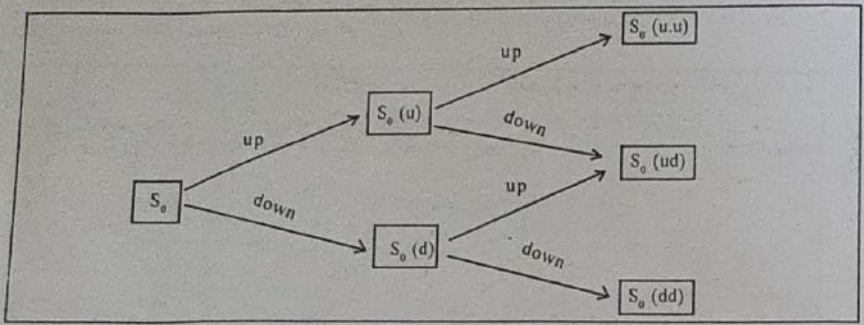


Fig. 10.9: Two-step Binomial Tree

Similarly, we can draw three steps and multi steps binomial trees.

10.4.2 Step II: Computation of Option Value at Each Final Node

At each final node of the tree (i.e., at expiration of the option), the option value is simply its ***intrinsic or exercise value***. The intrinsic value of an option is the gain to the holder of an option on immediate exercise.

It may be pointed out here that option holder cannot exercise the option before maturity in European option, hence the **intrinsic** value will be just notional and not actual. But in case of American option, the option holder can exercise his right at any time before the maturity date.

Mathematically, the value of an option will be defined as follows:

For call option: Max $[(S_n - X), 0]$

For put option: Max $[(X - S_n), 0]$

Where X is the exercise price and S_n is the spot price of stock at the nth period.

10.4.3 Step III: Computation of Option Value at Earlier Nodes

After completion of step II, then we have to compute the option value for each node using the risk neutrality assumption. The computation procedure undergoes the following three steps:

Step A

Computation of Binomial Option Values: Under the risk neutrality assumption, today's fair price of an option is equal to the expected value of its future pay-off discounted by the risk-free interest. Thus:

- Fair of value an option today = Present value of its future pay-off.
- Present value of future pay-off = Discounted value of expected pay-off.
- Discounted value is computed at risk-free interest.

Expected value is calculated using the option values from the later two nodes (option up and option down) weighted by their respective probabilities. It is already explained that the probability of an up move in the underlying is denoted by "P" and probability of a down move is denoted by "(1–P)". The expected value is then discounted at (r), the risk-free rate corresponding the life of the option.

Formulas:

1. Binomial Option Value = (Expected Option Value) e^{-rt}

2. (Expected Option Value) = {(Option up value) × P} + {(Option down value) × (1 – P)}

3. $$p = \frac{e^{rt} - d}{u - d}$$

Here P is the probability of on up tick in a risk-neutral world. In a risk-neutral world, all assets (including the stock and the option) will be priced the same riskless rate of return (r).

Step B

Continue the procedure of computing the Binomial option values by using backward induction method. The binomial option value represents fair price of an option at a particular point in time (i.e., at each node).

Step C

Evaluation of Possibility of Early Exercise: There are three styles of option – European, American and Bermudan. Depending on the style of option, evaluate the possibility of early exercise at each node subject to the following criterion:

(i) **European Option:** For European option, there is no option of early exercise. Hence, the binomial value applies at all nodes.

(ii) **American Option:** American option can be exercised at any time prior to expiry. As it is already discussed that option values at each previous node is determined by backward induction process. At each intervening node, i.e., the nodes other than the final nodes and initial node, the option value from early exercise is compared with the option value. For an American option, since the option may either be held or exercised prior to exp[illegible] value at node is: **Max {Binomial value, Exercise value}.**

(iii) **Bermudan Option:** For a Bermudan option, the binomial value at nodes (where each exercise is allowed) is: Max {Binomial value, Exercise value).

At nodes where early exercise is not allowed, only the binomial value is applied.

10.5 REPLICATING PORTFOLIO STRATEGY FOR OPTION VALUATION

A strategic (option) player always takes a decision in such a way so that he can able to hedge the risk in future and always be "In-the-Money". He has two alternatives:

1. Directly buy a call option by paying the call premium, or
2. Go for a **replicating portfolio** without holding an option.

In both the cases, he will have the same pay-off at maturity.

In the second case "Replicating Portfolio", the investor will go for investing certain portion of shares out of his own fund and invest certain amount by borrowing. At the maturity, he will sell the shares and repay the loan and have a surplus which is equal to a call pay-off.

The methodology is explained as follows:

Example 1

(i) Current stock price = ₹ 140 = ₹ 140 per share.
(ii) In one year, the stock price may:
- go up by 25%, or
- go down by 10%.

(iii) Strike price = ₹ 160 per share
(iv) Risk-free interest = 6% p.a.
(v) Option period = 1 year.

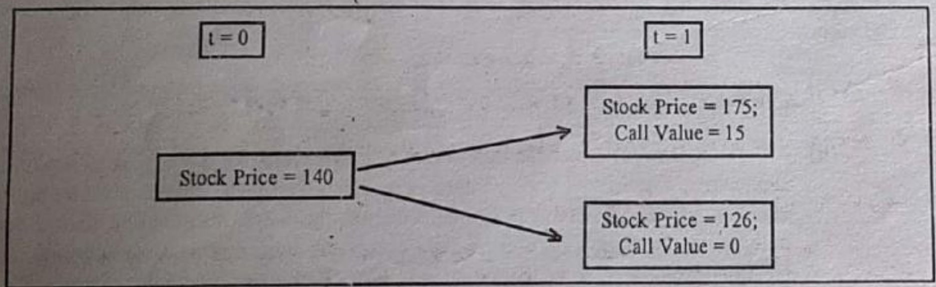


Fig. 10.10: Binomial Tree of Stock Prices and Option Values

Consider a Portfolio: Δ Shares of stock + ₹ B invested in risk-free securities.

So, the present value of this portfolio = Δ S + B = 140 Δ + B

Now, we draw the Binomial tree for portfolio value:

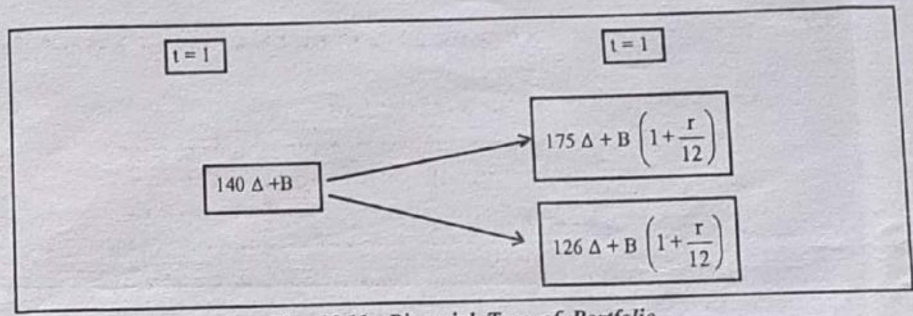


Fig. 10.11: Binomial Tree of Portfolio

Determining the Value of Δ and B

We have the following figures

Portfolio of Pay-off = Option Pay-off

$$175\Delta + B\left(1+\frac{r}{12}\right) = 15 \text{ Up state} \quad \dots\dots\dots (1)$$

$$175\Delta + B\left(1+\frac{r}{12}\right) = 0 \text{ Down state} \quad \dots\dots\dots (2)$$

Solving these equation; we get

$$175\Delta - 126\Delta = 15 - 0$$

$$\Rightarrow 49\Delta = 15 \text{ or } \Delta = \frac{15}{49} = 0.306$$

$$\text{Taking} = 175\Delta + B\left(1+\frac{r}{12}\right) = 15$$

$$\Rightarrow 175\Delta \times 0.306 + B\left(1+\frac{r}{12}\right) = 15$$

$$\Rightarrow 53.55 + B\left(1+\frac{r}{12}\right) = 15$$

$$\text{or } B\left(1+\frac{r}{12}\right) = 15 - 53.55 = -38.55$$

$$\Rightarrow B = \frac{-38.55}{\left(1-\frac{r}{12}\right)} = \frac{-38.55}{1.06} = -36.37$$

(i) 0.306 shares of stock. So, the current value of stock = 140 × 0.306 = 42.84

(ii) Partially financed by borrowing the amount B = ₹ 36.37

Point to Note:

It is observed that the value of delta changes as we move in the binomial tree. Accordingly, the replicating portfolio will change.

Now we put the results at maturity (after one year).

	Up State	Down State
(a) Stock Price	175	126
(b) 0.306 share price	53.55	38.55
(c) Repayment of loan loan	38.55	38.55
Net portfolio (b) – (c) =	15	Nil

Finding: The replicating has the same portfolio in all states as the call portfolio, the two must also have the same price.

For further understanding, consider the second example.

Example 2

Consider the following two binomial trees:

A. (i) Type of option: European call.

(ii) Current stock price = 100

(iii) Exercise price = 110

(iv) Time to expiration = 1 year

(v) In one year, the stock price may:

— go up by 25%

— go down by 20%

(vi) Risk-free interest = 5% p.a.

(vii) No Dividend

(viii) $e^{0.5} = 1.0513$

Now, we construct a binomial tree for stock price.

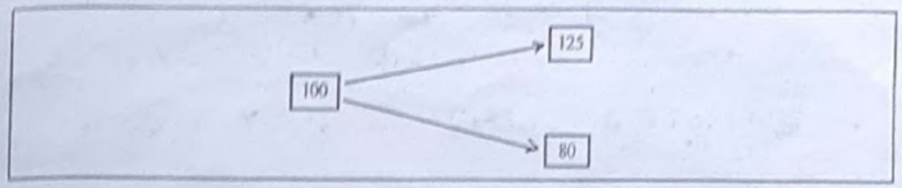


Fig. 10.12: Binomial Tree of Stock Price

When the stock price goes up, option worth is Max (0,125 – 110) = 15

When the stock goes down, option worth is Max (0.80 – 110) = 0

B. After computing the stock prices as above now, we have to construct the binomial tree of option prices.

Assume the Risk neutral probability, P = 0.6

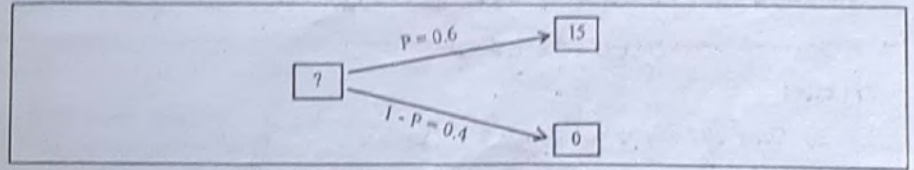


Fig. 10.13: Binomial Tree of Option Price

Expected value of option = 15 × 0.6 + 0 × 0.4 = 9

Value of option = $9\,e^{-rt} = 9\,e^{(-0.5\times1)} = 9 \times 0.9512 = 8.56$

In the above example, the stock price can be either 125 or 80 (Figure 10.12). Corresponding to these, the call option values are 15 and 0 respectively (Figure 10.13).

Observation: Change in stock prices = 125 – 80 = 45

Change is option prices = 15 – 0 = 15

When stock prices changes ₹ 45, then the corresponding change in option price is ₹ 15. If the change in stock price is 1, then the corresponding change in option price is $\frac{15}{45} = 0.3334$.

What is Option Delta?

The delta of an option is the sensitivity of an option price relative to changes in the price of the underlying asset. Thus, it is the change in the option value when the stock price changes, by 1.

Mathematically, Option Delta $C_\Delta = \frac{\text{Change in option value}}{\text{Change in stock price}}$

$$= \frac{C_u - C_d}{S_u - S_d}$$

Where C_u = Call option up price, C_d = Call option down price, S_u = Stock price up tick

S_d = Stock price down tick

In this example, $C_\Delta = \frac{15-0}{125-80} = \frac{15}{45} = 0.3333$

The option delta is usually have a decimal value. It indicates how fast the price of the option will change as the underlying stock moves.

Points to Note:

1. Option Delta values range between 0 and 1 for call options and –1 to 0 for put options.
2. Call options and put options have opposite delta.

Graphic Representation of the Behaviour of Option Delta

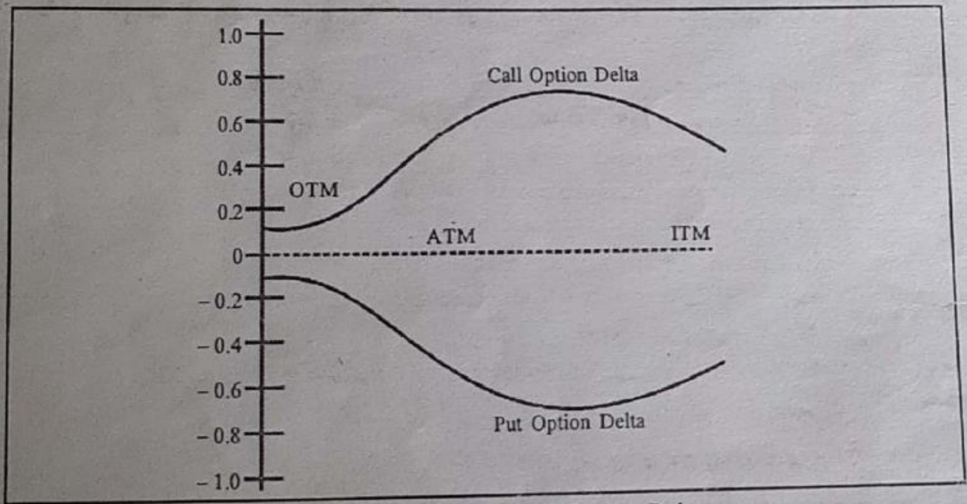


Fig. 10.14: Behaviour of Option Delta

What is Replicating Portfolio?

In the above example, we get the following values:

- Value of Call Option = ₹ 8.56
- Value of Call Delta = 0.3333 = 1/3

If the call option holder holds $\frac{1}{3}$ of the share, then he will have the same exposure. In other words, the trader only involves investing in cash for the following two forms:

- Option price, i.e., ₹ 8.56
- $\frac{1}{3}$ of underlying stock, i.e., $\frac{1}{3} \times 100 = 33.33$

Thus, the intial investment for a call option should be 33.33 – 8.56 = 24.77. This amount will be borrowed.

Here the option holder has the following composite position at present.

$\frac{1}{3}$ share @ 100 option with ₹ 8.56. (one call)

This composite position is known as **Replicating portfolio.**

The call option holder has to repay the borrowed fund of ₹ 24.77. Hence, the repayment amount will be = 24.77 e^{rt} = 24.77 × 1.0513 = 26.04

The value of replicating portfolio is shown as follows:

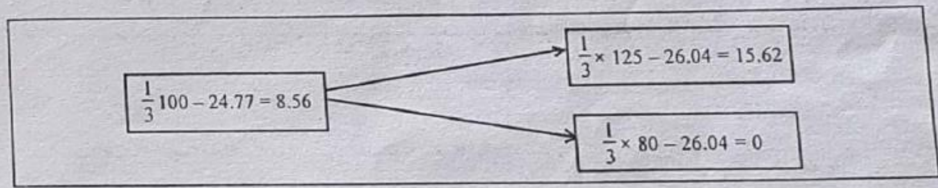


Fig. 10.15: Valuation of Option by Replicating Portfolio Strategy

What is Dynamic Portfolio?

When the Stock Price go on changing in subsequent sub-periods, we have to continuous adjustment in the replacing portfolio. This process is known as dynamic delta hedging or dynamic portfolio.

10.6 DYNAMIC APPROACH TO BOPM – STRATEGIC ILLUSTRATIONS

Illustration 1

Draw binomial trees from the following date and show the stock prices:

Parameter	European	
	Call option	Put option
Stock price	100	100
Strike price	120	80
Maturity	1 year	1 year

Sub-periods (t)	Two	Two
u factor	1.40	1.40
d factor	0.80	0.80

Solution

The stock price tree for the both option will be same as shown in the following figure.

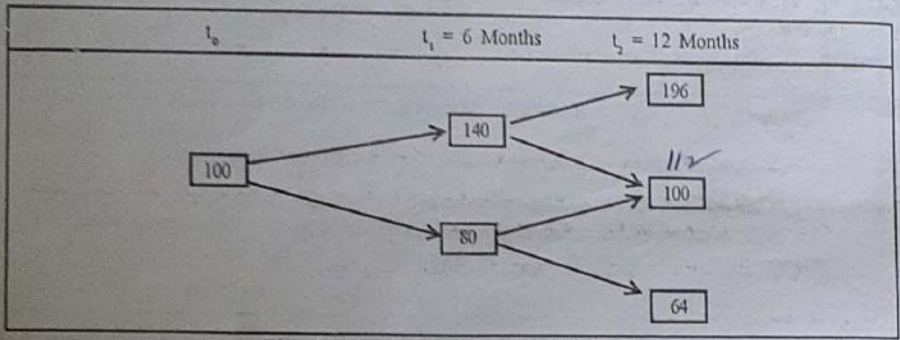


Fig. 10.16: Stock Price Binomial Tree

Illustration 2

Take the data in illustration 1, and determine the call option value taking risk neutral probability P = 0.48. Risk-free interest = 10% p.a.

(a) Computation European call option value by backward induction process:

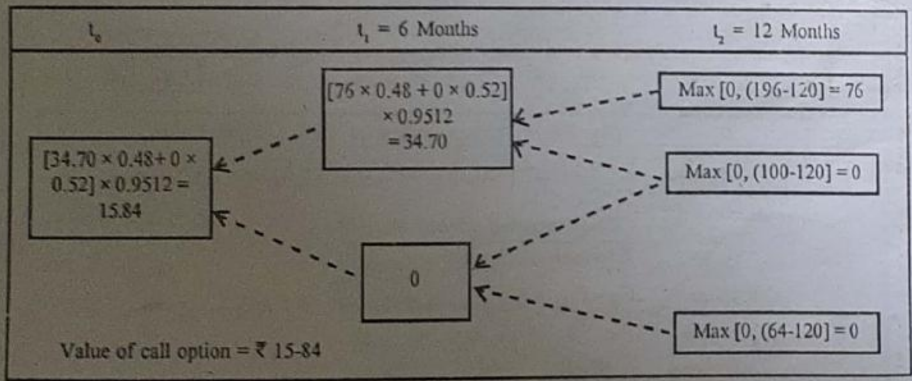


Fig. 10.17: Binomial Tree for European Call Option Values

(b) Computation of European put option value by backward induction process:

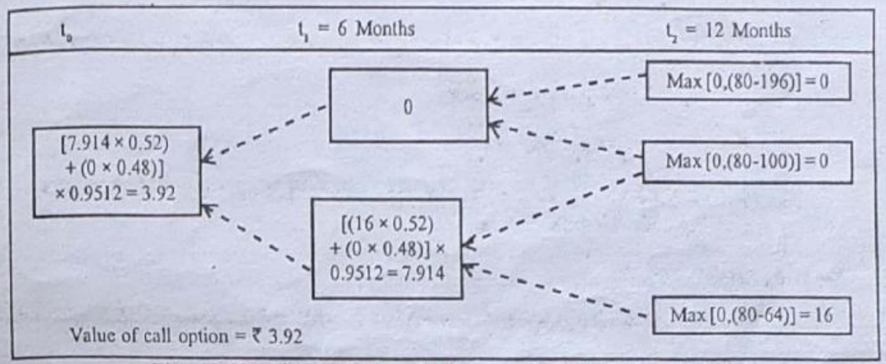


Fig. 10.18: Binomial Tree for European Put Option Values

Illustration 3

From the following data, calculate: (a) Expected stock price, (b) Value of an option, (c) Option Delta.

(i)	Type of option:	European
(ii)	Stock price at the intial point of option =	₹ 240
(iii)	Option period (Time):	one year
(iv)	Stock price movement in the next year.	
—	go up by 25%	
—	go down by 20%	
(v)	Exercise price =	₹ 260
(vi)	Risk-free interest rate =	5% p.a.
(vii)	Risk neutral probability of up move is 0.60 and down move is 0.40.	

Solution:

Binomial Tree of Stock Prices

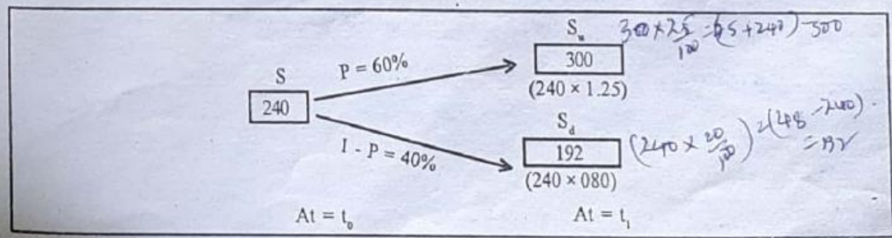


Fig. 10.19: Binomial Tree for Stock Prices

The expected stock price at t_1 = (300 × 0.60) + 192 × 0.4) = 180 + 76.80 = 256.80

Now we can find the expected return:

Investment	Return
240	16.80 (256.80 − 240)
100	$\frac{16.8}{240} \times 100 = 7$ per cent.

If we discount ₹ 256.80 at the rate of 7 per cent, then we get the present value of ₹ 240. This rate is known as Risk Adjusted Discount Rate.

But to get risk neutral expected stock price, first we have to find this risk neutral probability such that the stock earns only at the risk-free rate, i.e., 5 per cent (given).

In this case, up move is 25% and down move is 20%. Since the total probability is 1, then $P = \frac{25}{25+20} = \frac{3}{9} = \frac{5}{9}$ and $1 - P = 4/9$

Taking these risk neutral probabilities, we may now draw the binomial tree and calculated the stock price.

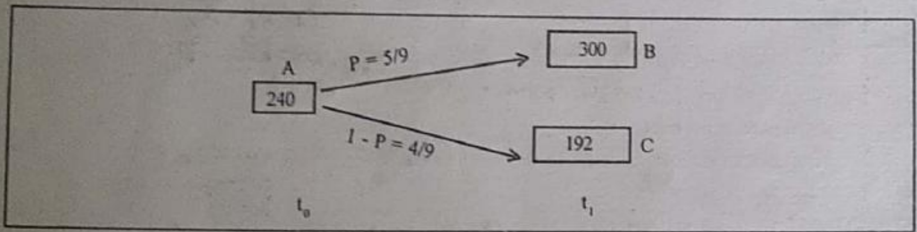


Fig. 10.20: Binomial Tree for Stock Prices

Expected Stock Price at (t_1) = $300 \times \frac{5}{9} + 192 \times \frac{4}{9}$ = 166.66 + 85.33 = 251.99 = 252

Determination of European Call Value

Now we have to calculate the option price by using the binomial free approach and backward induction process.

Step 1: Computation of option values at nodes B and C.

Node	Max [(S − X), 0]	Option Value
B	Max [300 − 260), 0]	40
C	Max [192 − 240), 0]	0

Step 2: Computation of option values at the preceeding node A by backward induction process:

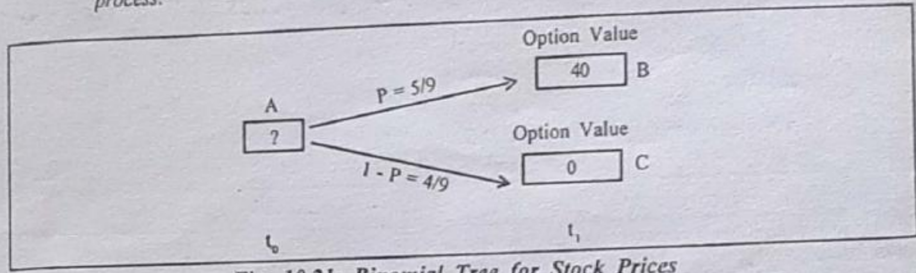


Fig. 10.21: Binomial Tree for Stock Prices

Expected value of option = $40 \times \frac{5}{9} + 0 \times \frac{4}{9} = 22.22$

Now to calculate the option value at A node A, i.e., at the initial point of time. We have to find out the present value of 22.22 by discounting at this risk-free rate 5% p.a.

Thus, the present value of the European call option $= 22.22\ e^{-rt} = 22.22\ e^{-0.5 \times 1}$

$= 22.22 \times .9512 = 21.14$ (Approx.)

Binomial tree representation:

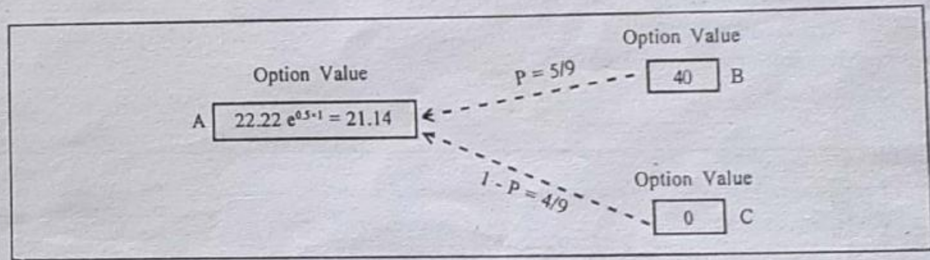


Fig. 10.21: Computation of Call Option Value

(c) Call Option Delta

$$C_\Delta = \frac{\text{Change in option price}}{\text{Change in stock price}}$$

$$= \frac{C_u - C_d}{S_u - S_d}$$

$$= \frac{40-0}{3000-192} = \frac{40}{108} = 0.3704$$

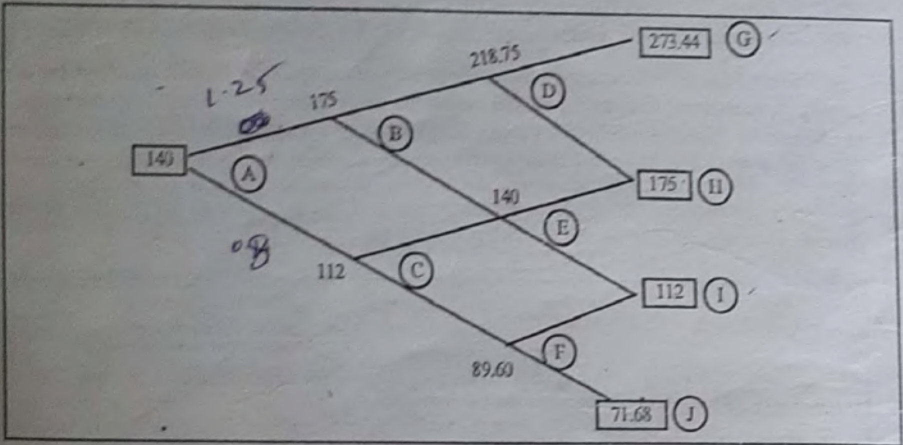
Illustration 4

Use three-step binomial model and determine the option value from the following data:

Type of option:	Call option
Style of option:	European
Current stock price:	₹ 140
Exercise price =	₹ 150
Up factor (u) =	1.25,
Down factor (d) =	0.8
Probability of up =	P = 0.6
Probability of down =	1 – P = 0.4
Risk-free interest rate =	10% p.a.

Solution:

Now, we first draw the binomial tree of stock prices for three time intervals:



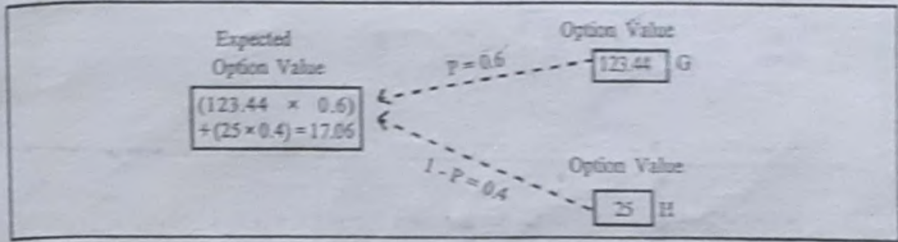
Next we calculate the option prices by backward induction process.

Step 1: Computation of option values at the final nodes G, H, I and J.

Node	Max [(S – X), 0]	Option Value
G	Max [273.44 – 150), 0]	123.44
H	Max [175 – 150), 0]	25
I	Max [112 – 150), 0]	0
J	Max [71.68 – 150), 0]	0

Step 2: Computation of option value of the proceeding nodes at D, E and F.

(a) Option Value at D.

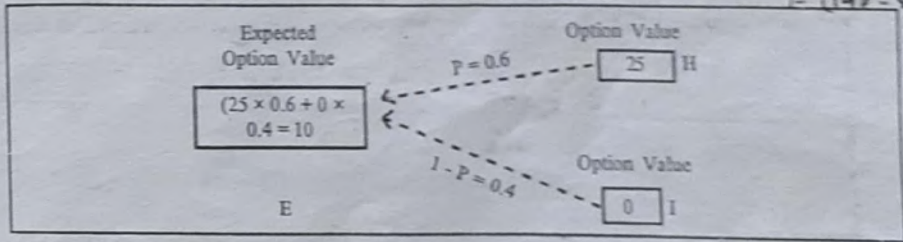


Expected value of option = 123.44 × 0.6 + 25 × 0.4 = 84.06

Present value of the expected option value at D = 84.06 e^{-rt} = 84.06$e^{-0.10\times\frac{1}{12}}$

= 84.06 × 0.9917 = 83.36

(b) Option Value at E.



Present value of the expected option value at node E.

= 10 e^{-rt} = 10 $e^{-0.10\times\frac{1}{12}}$ = 10 $e^{-0.183}$

= 10 × 0.9917 = 9.917

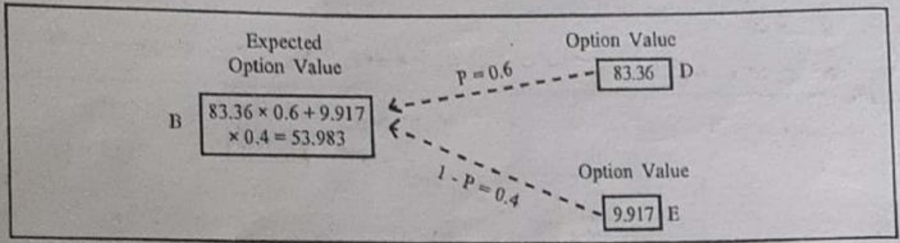
(c) Option Value at F = 0

Step 3: (a) Computation of option value at node B

Expected value of option at B

= 83.36 × 0.6 + 9.917 × 0.4 = 53.983

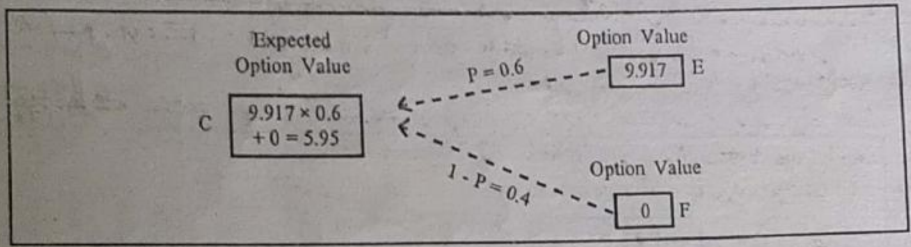
Present value of the expected option value at B = 53.983 e^{-rt} = 53.983 × 0.9917 = 53.53



(b) *Computation of option value at node C*

Expected value of option at C = 9.917 × 0.6 + 0 × 0.4 = 5.95

Present value of the expected value of option at node C = 5.95 e^{-rt} = 5.95 × 0.9917 = 5.90

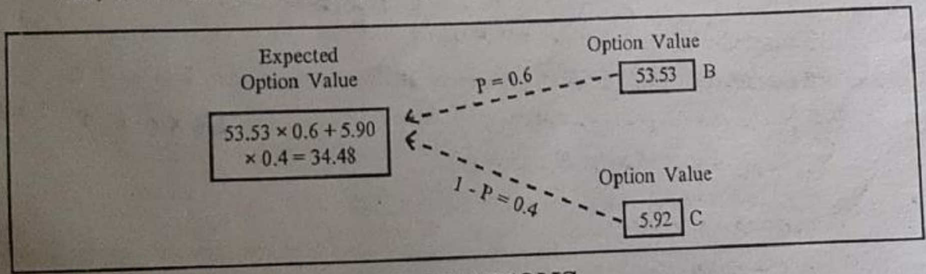


Step 4: Computation of option value at initial node A:

Expected value of option at next node = 53.53 × 0.6 + 5.90 × 0.4 = 32.12 + 2.36 = 34.48

Present value of expected value of option at node A = 34.48 e^{-rt} = 34.48 × 0.0017 = 34.19

Thus, the current value of European option is ₹ 34.19.



10.7 ADVANTAGES AND LIMITATIONS

Binomial Option Pricing Model (BOPM) was originally developed by Cox, Ross and Rubinstein in 1974. It is a numerical method. This option pricing model has the following advantages and limitations:

Advantages

1. It is more accurate pricing model for American style option.
2. It facilitate the possibility of early exercise because with the binomial model it is possible to check at every point in an options life (i.e., at every step of the binomial tree).
3. This model uses simple mathematical calculations for stock prices as well as option prices.
4. It is easy to calculate the option price with a computer spreadsheet.

Limitations

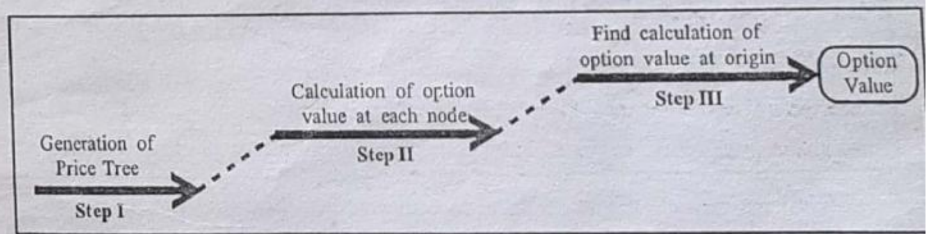
1. It is relatively slow speed. It is very difficult to calculate thousands of option prices quickly for a multi-step binominal model.
2. It is much more complex than the Black-Scholes model.

CHAPTER SUMMARY

The Binomial Option Pricing Model (BOPM) was developed by Cox, Ross and Rubinstein in 1979. This a very versatile numerical method for valuing American and European Options.

This model is a "discrete-time" model. Total time of the option is divided into a number of sub-periods. It is assumed that during a time period the stock price can move in only two ways, i.e., an up move by a multiplicative factor u or a down move by a multiplicative factor d. For this reason the model is known as Binomial Model.

Under this method, first we have to draw a binomial tree which obeys the binomial generating process. Then we proceed in the following three steps:



For computing the stock prices as well as option prices at various nodes of a Binomial Tree we have to depend on the following parameters:

1. The volatility of stock prices measured by Standard Deviation (σ)
2. The risk-free interest rate (r) continuously compounded
3. The number of sub-periods or bits.

11.1 INTRODUCTION

Professor Fisher Black, Professor Myron Scholes and Professor Robert Merton published an article *"The Pricing of Options and Corporate Liabilities"* in 1973. It was a path-breaking paper published in *"Journal of Political Economy"*. In their article, they have developed a model on *"Pricing of European call Option"*. We should offer hearty regards to these financial engineers.

Myron Scholes and Fischer Black

Fischer Black

Born: 1938 Died: 1995

1964 – Earned Ph. D. from Harvard in applied math
1971 – Joined faculty of University of Chicago Graduate School of Business
1973 – Published "The Pricing of Options and Corporate Liabilities"
19XX – Left the University of Chicago to teach at MIT
1984 – Left MIT to work for Goldman Sachs & Co.

Myron Scholes

Born: 1941

1973 – Published "The Pricing of Options and Corporate Liabilities"

Currently he is working in the derivatives trading group at Salomon Brothers.

The Black-Scholes Model is a mathematical model of a financial market containing certain derivative investment instruments. From this model, we can deduce the Black-Scholes formula for pricing of European style option.

The Black-Scholes formula is widely used by option market participants. Many empirical tests have shown that Black-Scholes price is "fairly-close" to the observed price. To acknowledge the benefits of this model, **Myron Scholes and Robert Merton** were awarded the Nobel Prize in Economics in 1997 for their noble contribution. Sadly Professor Fisher Black died in 1995. Otherwise he would undoubtedly also being one of the recipients of the Nobel Prize.

In this chapter, we will not explain the derivation of this BSOPM. However, we will discuss the assumptions and applications of this model. We have already discussed the various determinants for option pricing in the previous chapter.

11.2 ASSUMPTIONS OF THE BLACK-SCHOLES OPTION PRICING MODEL

1. No Dividends

   The B-S model assumes that the stock pays no dividend during the option's life.

2. No transaction cost or taxes.
3. Shares are infinitely divisible, i.e., all stocks are perfectly divisible.
4. Prices are lognormally distributed.
5. No brokerage or commission to buy or sell options.
6. Interest rates remain constant and known.
7. Borrowing and lending at the same risk-free rate of interest.
8. Market operates continuously, i.e., stock trading is continuous. This assumption suggests that the participants cannot consistently predict the direction of the market or an individual stock. The continuous stock trading follows the Markov Process.

Points to Note:

The BSOPM uses the European exercise terms.

Thus, the assumptions are shown in figure 11.1.

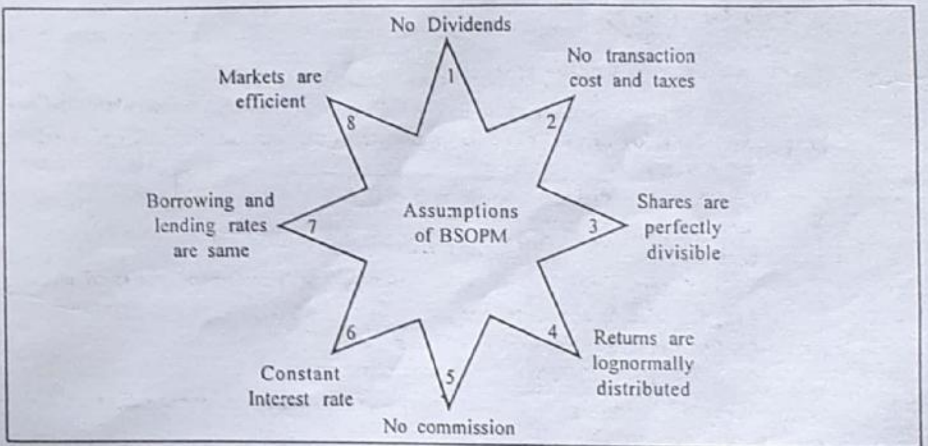


Fig. 11.1: Eight Assumptions of BSOPM

Option pricing model — Intrinsic → excess of stock price over option strike
Time value → excess of the option price over intrinsic

11.3 BLACK-SCHOLES OPTION PRICING FORMULA

(A) The price of a call option will be determined by the following formula:

$$C = SN(d_1) - Xe^{-rT} N(d_2)$$

Where

C = Price of the call option

S = Current Stock price

X = Exercise price of the Option

r = Risk-free interest rate continuously compounded

T = Current time until expiration, expressed as a per cent of a year

N() = Cumulative standard normal distribution i.e., area under the normal curve

σ = Annualised standard deviation of returns on the underlying asset, expressed as decimal

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$N(d_1)$ and $N(d_2)$ are cumulative distribution function at d_1 and d_2 respectively

e = exponential function (2.7183)

l_n = natural logarithm.

(B) The price of a put option will be determined by the following formula with the basis of put-call parity:

$$\text{Put-call parity is } P = C - S + \times e^{-rt}$$

Q. 16 //

Where

P = Price of the put option

C = Price of the call option

X = Strike price of the option

r = Risk-free interest rate

T = Time to expiration

e = Exponential function, i.e., 2.7183

Then the price of the put option is:

$$P = X e^{-rt} N(-d_2) - SN(-d_1)$$

11.4 USE OF STATISTICS AND MATHEMATICS FOR B-S OPTION PRICING FORMULA

Black-Scholes Option Pricing Model is used to calculate a theoretical call price using the six determinants of an option's price.

The original formula for calculating a European call option (non-dividend) is:

$$C = S\ N(d_1) - Xe^{-rT}N(d_2)$$

Where

S = Current Stock Price

X = Exercise Price

r = Risk-free interest

T = Time to expiration expressed as proportion of a year

But we have to calculate the values of $N(d_1)$, $N(d_2)$ and e^{-rT} for inserting in the above formula.

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$N(d_1)$ = Cumulative normal probability value of (d_1) which will be identified from the Normal Distribution Table

$N(d_2)$ = Cumulative normal probability value of (d_2) which will be identified from the Normal Distribution Table

The symbol l_n stands for natural logarithm.

So, for correct computation of all elements in the B-S formula, the basic requirements are:

1. Natural Logarithm (l_n)
2. Power of e, i.e., e^x
3. Volatility – historical and implied (σ)
4. Lognormal Distribution
5. Cumulative normal probability value

We will discuss these elements in brief. For details, one may refer any Standard Statistics or Mathematics books on these subjects.

11.4.1 Natural Logarithm (l_n)

The natural logarithm is the logarithm to the base e, where e is an irrational constant approximately equal to 2.718281828. The natural logarithm is generally written as $l_n (x)$ or $\log_e x$.

Mathematicians, Statisticians, and some Engineers generally understand either "$\log_e^{(x)}$ or $l_n (x)$ that is "log x to the base e".

Points to Note:

(i) $l_n 1 = 0$ because $e^0 = 1$.

(ii) When x is less than 1, then the natural logarithm will be negative.

(iii) When $n \geq 1$, then the natural logarithm is positive.

Example 1.

$l_n 0.01 = -4.60517$

$l_n 0.09 = 0.01005$

$l_n 1.00 = 0.0000$

$l_n 1.1 = 0.09531$

$l_n 10 = 2.39789$

Example 2. Find the natural logarithm 1.085.

Solution. From the statistical table of Natural Logarithm Values, we can identify the natural logarithm values of 1.08 and 1.09. These are as follows:

$l_n 1.08 = 0.07696$ $\quad$ $l_n 1.09 = 0.08618$

By interpolating these two values, we can get the value as:

$$l_n 1.085 = l_n 1.08 + 0.5\,[l_n(1.09) - l_n(1.08)]$$
$$= 0.07696 + 0.5\,(0.08618 - 0.07696)$$
$$= 0.07696 + 0.5 \times 0.00921$$
$$= 0.07696 + 0.00460$$
$$= 0.08157$$

11.4.2 Power of e, i.e., e^x or e^{-x}

We can calculate the e^x by referring to the statistical table "power of e".

Example 3

If $x = 0.00$, then $e^x = 1.0000$

If $x = 1.00$, then $e^x = 2.7183$

If $x = -1.00$, then $e^x = 0.36788$

Example 4. Find $e^{-0.04}$.

Solution: $e^{-0.04} = 0.96079$

Example 5. Find the values of the following

(a) $e^{0.565}$ (b) $e^{-0.565}$

Solution:

(a) $e^{0.565}$

From the statistical table, we can get the following values directly:

$e^{0.56} = 1.7507$, $e^{0.57} = 1.7683$

$$\text{Now } e^{0.565} = e^{0.56} + 0.5\,(e^{0.57} - e^{0.56})$$
$$= 1.7507 + 0.5\,(1.7683 - 1.7507)$$
$$= 1.7507 + 0.5 \times 0.0176$$
$$= 1.7507 + 0.0088$$
$$= 1.7595$$

(b) $e^{-0.565}$

From the statistical table, we can get the following values directly:

$e^{-0.56} = 0.57121$ $e^{-0.57} = 0.56553$

$$\text{Now } e^{-0.565} = e^{-0.570} + 0.5\,(e^{-0.560} - e^{-0.570})$$
$$= 0.56553 + 0.5\,(0.57121 - 0.56553)$$
$$= 0.56553 + 0.00289$$
$$= 0.56837$$

11.4.3 Volatility – Historical and Implied

Volatility is a measure of the variability in the asset's rate of return. It is estimated by using a statistical measure, i.e., Standard Deviation (σ). The degree of variability and chance of realising the asset's rate of return determine the option's pay-off.

Option prices are very sensitive to changes in volatility. Volatility cannot be directly observed and must be estimated.

Forms of Volatility: There are two types of volatility. These are:

(i) Historical volatility

(ii) Implied volatility

Historical Volatility: It is calculated from the historical stock prices. The historical stock prices may be daily or weekly stock prices. It is estimated by the standard deviation. The historical volatility will be calculated before the computation of option price by B-S formula because this historical volatility is one of the important input variable (determinant) of B-S formula.

Steps in Calculation:

Step 1. Compute the daily changes in the price of the stock in a market. Then calculate the natural log of the ratio of a Stock's Price S_t its previous day's price S_{t-1}

$$R_t = l_n (S_t/S_{t-1})$$

Where

R_t = Changes in the daily return

Step 2. Computation of average returns over a specified period say a month.

$$R_m = \frac{\sum_n R_t}{n}$$

Where

R_m = Mean of the daily returns

n = No. of days taken or No. of sample units

Step 3. Calculate the Standard deviation

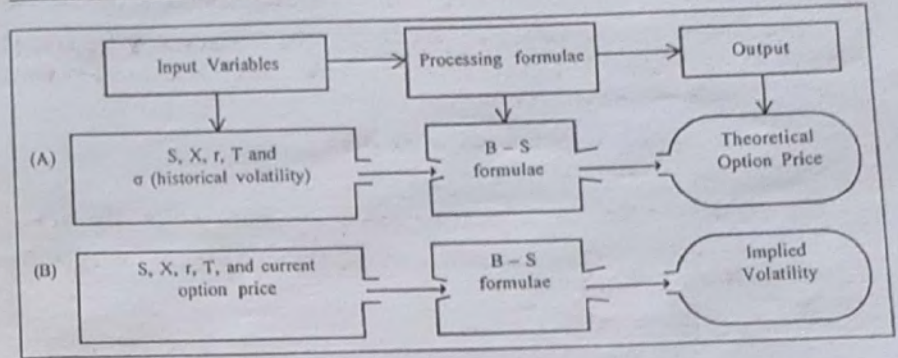
$$\sigma_{daily} = \sqrt{\frac{\sum (R_t - R_m)^2}{n-1}}$$

But in B-S formula, the annualised volatility is used. The daily volatility can be converted into equivalent annual volatility by using the following formula:

$$\sigma_{annual} = \sigma_{daily} \times \sqrt{\text{No of trading days in the year}}$$

- The normal assumption in equity market is that there are 252 trading days per year.

Implied Volatility: By using historical volatility in the Black-Scholes formula, we calculate the option value. In other words, we can find the option value when a volatility (historical) is given. But we can put a different question: What is the volatility (or standard deviation) for the observed option price to be consistent with the Black-Scholes formula? The answer is **"Implied Volatility"**. Thus, when the volatility of the stock implies the current option price, we consider it is **implied volatility**.



Points to Note:

(i) Basically implied volatility helps in determining the fair option price, while historical volatility helps in determining the theoretical value of an option.

(ii) If the historical volatility truely represents the future (implied) volatility, the actual option prices will be fair and identical to the theoretical prices.

How to Estimate Implied Volatility?

We know that option price is a function of volatility. Thus $C = f(\sigma)$ or $p = f(\sigma)$ where C = call option price, p = put option price and σ = volatility of the stock price. Consider the following two cases:

Case (a): If market price of the call option is greater than the theoretical call price determined by B-S formula, then it is assumed that implied volatility is greater than the historical volatility.

Case (b): If the market price of the call option is greater than the theoretical call price determined by B-S formula, then it is assumed that implied volatility is smaller than the historical probability

To estimate the implied volatility, we have to adopt trial and error method in the B-S formula.

11.4.4 Lognormal Distribution

In probability theory, a lognormal distribution of a random variable whose log is normally distributed is a continuous distribution in which logarithm of a variable has a normal distribution.

Definition: A random variable x is said to have the lognormal distribution with parameter $\beta \in R$ and $\sigma > O$ if logarithm of x has the normal distribution with β and σ. Equivalently if $x = e^y$ where y is normally distributed with mean β and standard deviation σ.

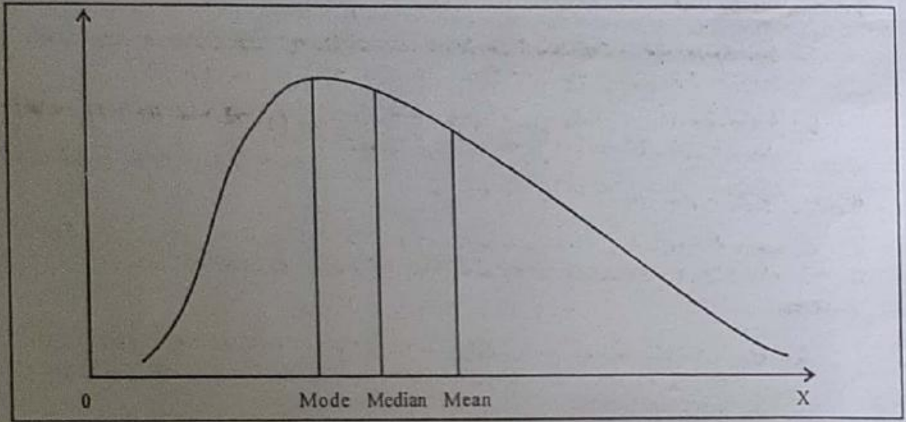
Use: The lognormal distribution is used to model continuous random quantities when the distribution is belived to be *skewed*.

In Black-Scholes option pricing model, the basic assumption that the proportional changes in the stock price in the short period follows a normal distribution. Thus, it indirectly implies that the variable (S) i.e., stock price follows a lognormal distribution.

Characteristics of a Lognormal Distribution

1. The variable of a lognormal distribution is only positive.
2. The shape of the lognormal distribution is always *skewed* with mean, median and mode.
3. If the natural logarithm of the variable is normally distributed, then only a random variable is lognormally distributed.

Shape of a Lognormal Distribution



Here Mean > Median > Mode (Positive Skewed)

11.4.5 Cumulative Normal Probability Distribution

To insert the value of $N(d_1)$ and $N(d_2)$ in Black-Scholes formula, one should know the procedure of calculating these values.

$N(d_1)$ = Cumulative normal probability value of (d_1) = The area under the normal distribution to the left of d_1.

$N(d_2)$ = Cumulative normal probability of (d_2) = The area under the normal distribution to the left d_2.

Example 6. Find the value of:

(a) N(d) = N(1.04) (b) N(d) = N(–1.04)

(c) N(d) = N(1.97) (d) N(d) = N(–1.97)

(e) N(d) = N(–0.1082) (f) N(d) = N–(0.2377)

Solution:

(a) N(d) = N(1.04) = 0.5557

(b) N(d) = N(–1.04) = 0.4443

(c) N(d) = N(1.97) = 0.9999

(d) N(d) = N – (1.97) = 0.0244

(e) N(d) = N(–0.1082)

From the table of "Area under the Normal Distribution Curve" (for negative values), we first identify the following two values:

N(–0.10) = 0.4602

N(–0.11) = 0.4562

Then by interpolation, we can calculate the value of N(–0.1082) = N(–0.10) – 0.82

{N(–0.10)} – {N(–0.11)}

= 0.4602 – 0.82 × (0.4602 – 0.4562)

= 0.4602 – 0.0033 = 0.4569 (Ans)

(f)N(d) = N(–0.2377)

From the table of "Area under the Normal Distribution Curve" (for negative values), we first identify the following two values:

N(–0.2300) = 0.4090

(N – 0.2400) = 0.4052

Then by interpolation, we can calculate the value of (N – 0.2377) = N(–0.2300) – 0.

Example 7.

(a) If N(d) = 0.9987, find d.

(b) If N(d) = 0.9772 find d.

(c) If N(d) = 0.8483 find d.

Solution:

(a) It is given that N(d) = 0.9987

From the area under the normal distribution curve (for +ve values), we find that the value of d = 3.

(b) It is given that N(d) = 0.9772

From the area under the normal distribution curve (for +ve value), we find that the value of d = 2.

(c) It is given that N(d) = 0.8483

From the area under the normal distribution curve (for +ve values), we find that the value of d = 1.

Example 8: If d = 2, find the value of N(2) + N(–2).

Solution:

N(2) = 0.9772, N(–2) = 0.0228

So, N(2) + N(–2) = 0.9772 + 0.0228 = 1.

11.5 APPLICATIONS OF BSOPM

Illustration 1

From the following information, calculate call option value and put option value:

Current market price (S)	: ₹ 100 per share
Exercise Price (X)	: ₹ 80 per share
Volality of Share Price (S)	: 30%
Risk-free interest rate (r)	: 10% p.a.
Time to expiration (T)	: 3 months

Use Black-Scholes formula.

Solution:

(a) Calculation of Call Option Value

Black-Scholes formula

$C = S.\ N(d_1) - Xe^{-rT} \times N(d_2)$

Where $d_1 = \dfrac{\ln(S/X) + \left(r + \dfrac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$

and $d_2 = d_1 - \sigma\sqrt{T}$

Here S = 100, X = 80, l_n = Natural logarithm, r = 0.10, σ = 0.30, T = 3/12 = 0.25

Step 1. Computation of values of d_1 and d_2

$$d_1 = \frac{l_n(100/80) + (0.10 + 0.030 \times 0.030/2) \times 0.25}{0.30 \times \sqrt{0.25}}$$

$$= \frac{l_n(1.25) + (0.10 + 0.045) \times 0.25}{0.30 \times 0.50}$$

$$= \frac{0.22314 + 0.03625}{0.15} = \frac{0.25939}{0.15} = 1.7293$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= 1.7293 - 0.30\sqrt{0.25}$$

$$= 1.7293 - 0.15 = 1.5793$$

Step 2. Calculation of the cumulative normal probability values of d_1, d_2. These values can be identified from the Cumulative Normal Distribution Table.

$N(d_1) = N(1.7293) = 0.9581$

$N(d_2) = N(1.5793) = 0.9429$

Step 3. Calculation of call option value by using BSOPM formula as stated above.

$$C = S \times N(d_1) - Xe^{-rT} \times N(d_2)$$

$$= 100 \times 0.9581 - 80 \times 0.9753 \times 0.9429$$

$$= 95.81 - 73.57 = 22.24$$

(b) Now we compute put option value.

Black-Scholes formula:

$P = Xe^{-rT}\ N(-d_2) - S \times N(-d_1)$

Step 1. Computation of values of d_1, and d_2

We have already got the values d_1 = 1.7293 and d_2 = 1.5793

Step 2. Computation of the values of $N(-d_1)$ and $N(-d_2)$

From Normal distribution tables, we find out the areas under the Normal distribution for these values as follows:

Direct Value	Deductive Value
$N(-d_1) = 0.0419$	$1 - N(d_1) = 1 - 0.9581 = 0.0419$
$N(-d_2) = 0.0571$	$1 - N(d_2) = 1 - 0.0429 = 0.0571$

Step 3. Calculation of put option value by using BSOPM formula as stated above.

$$P = Xe^{-rT} N(-d_2) - SN(-d_1)$$
$$= 80\ e^{-0.10 \times 0.25} \times 0.0571 - 100 \times 0.0419$$
$$= 4.4534 - 4.190 = 0.2634 \text{ or say } 0.26$$

Prove by put-call parity:

$C - P = S - X\ e^{-rT}$

LHS = 22.24 – 0.26 = 21.98 say ₹ 22 .

RHS = 100 – 80 × 0.975 = 100 – 78 = ₹ 29 (Proved)

Illustration 2

Use Black-Scholes formula, calculate call option value and put option value from the following information:

Current Market Price (S)	=	₹ 100 per share
Exercise Price (X)	=	₹ 100 per share
Volatility of Share Price (σ)	=	0.30
Risk-free interest rate (r)	=	10% Per annum
Time to expiration (T)	=	3 months = 0.25 years

Solution:

(a) Calculation of call option value

Black-Scholes formula:

$C = S.N(d_1) - X\ e^{-rT}\ N(d_2)$

Here S = 100; X = 100 r = 0.10, σ = 0.30, T = 3/12 = 0.25

Step 1. Computation of values d_1 and d_2

$$d_1 = \frac{l_n(S/X) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{l_n(100/100) + (0.10 + 0.030 \times 0.030/2) \times 0.25}{0.30 \times \sqrt{0.25}}$$

$$= \frac{l_n(1) + 0.3625}{0.15} = \frac{0.0 + 0.03625}{0.15}$$

$= 0.03625 / 0.15 = 0.2417$

$d_2 = d_1 - \sigma\sqrt{T} = 0.2417 - 0.30 \times 0.30\sqrt{0.25}$

$= 0.2417 - 0.15 = 0.0917$

Step 2. Calculation of the cumulative normal probability values of d_1 and d_2. These values can be identified from the Cumulative Normal Distribution Table.

$N(d_1) = N(0.2417) = 0.5955$

$N(d_2) = N(0.0917) = 0.5365$

Step 3. Calculation of call value.

$C = N(d_1) - Xe^{-rT} N(d_2)$

$= 100 \times 0.5955 - 100\ e^{-0.1 \times 0.25} \times 0.5365$

$= 59.55 - 100 \times 0.9753 \times 0.5365$

$= 59.55 - 52.32 = 7.23$

(b) Computation of put option value by Black-Scholes Formula:

Step 1. Computation of d_1 and d_2

We have $d_1 = 0.2417$, $d_2 = 0.0917$

Step 2. Computation of the values of $N(-d_1)$ and $N(-d_2)$

From the Normal distribution table, we can obtain the areas under the Normal distributio[n] for these values as follows:

Direct value	Deductive value
$N(-d_1) = N(-0.2417) = 0.4045$	$1 - N(d_1) = 1 - 0.5955 = 0.4045$
$N(-d_2) = N(-0.0917) = 0.4635$	$1 - N(d_2) = 1 - 0.5365 = 0.4635$

Step 3. Calculation of put value

$P = X\ e^{-rT}\ N(-d_2) - S\ N(-d_1)$

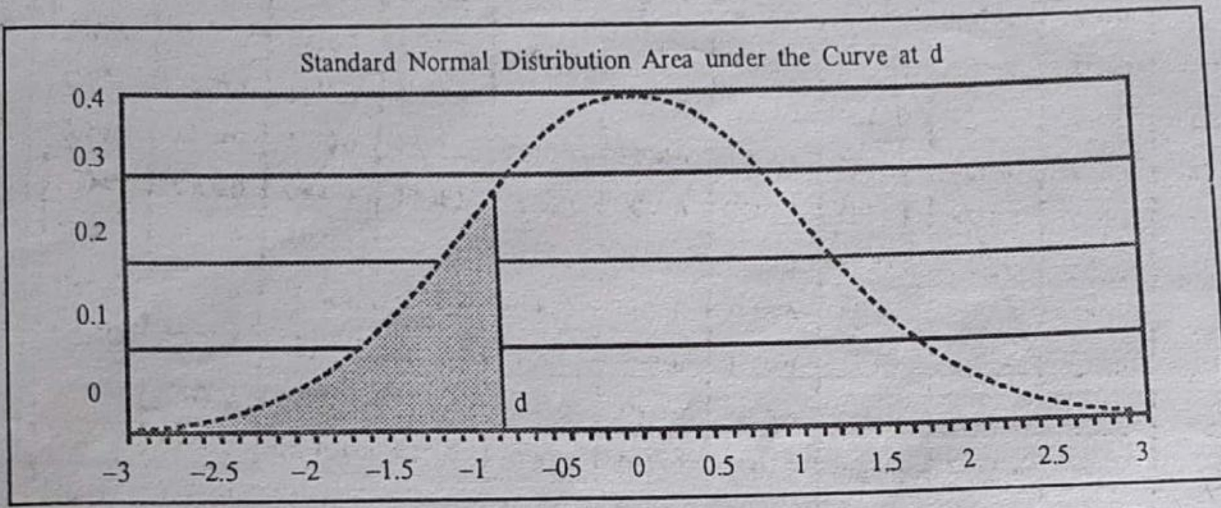
$= (100\ e^{-0.1 \times 0.25} \times 0.4635) - (100 \times 0.4045)$

$= (100 \times 0.9749 \times 0.4635) - 40.45$

$= 45.19 - 40.45 = 4.74$

APPENDIX

Table A 11.1: Cumulative Normal Distribution

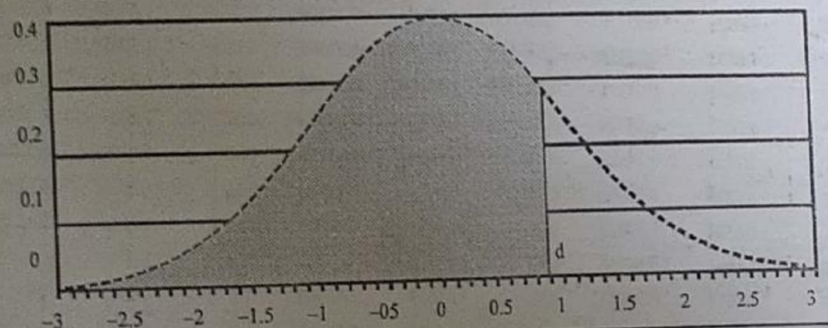


Area Under the Normal Distribution Curve (for Negative Values)

d	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
−0.1	0.4802	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.2	01151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0661
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367

-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.6	0.0047	00045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	00014
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

Standard Normal Distribution Area under the Curve at d



Area Under the Normal Distribution Curve (for Positive Values)

d	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.0	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6617
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.5621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9573	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	09984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990

Table A 11.2: Natural Logarithm. Base (e)

The Natural Logarithm Table (Equal to or Less than 1.0)

n	$\log_e n$	n	$\log_e n$	n	$\log_e n$	n	$\log_e n$
0.01	-4.60517	0.26	-1.34707	0.51	-0.67334	0.76	-0.27443
0.02	-3.91202	0.27	-1.30933	0.52	-0.65392	0.77	-0.26236
0.03	-3.50655	0.28	-1.27296	0.53	-0.63488	0.78	-0.24846
0.04	-3.21887	0.29	-.23788	0.54	-0.61618	0.79	-0.23572
0.05	-2.99573	0.30	-1.20397	0.55	-0.59783	0.80	-0.22314

0.06	–2.81341	0.31	–1.17118	0.56	–0.57982	0.81	–0.21072
0.07	–2.65926	0.32	–1.13943	0.57	–0.56212	0.82	–0.19845
0.08	–2.52573	0.33	–1.10866	0.58	–0.54472	0.83	–0.18633
0.09	–2.40894	0.34	–1.07881	0.59	–0.52763	0.84	–0.17435
0.10	–2.30258	0.35	–1.04982	0.60	–0.51082	0.85	–0.16252
0.11	–2.20727	0.36	–1.02165	0.61	–0.49430	0.86	–0.15082
0.12	–2.12026	0.37	–0.99425	0.62	–0.47803	0.87	–0.13926
0.13	–2.04022	0.38	–0.96758	0.63	–0.46203	0.88	–0.12783
0.14	–1.96611	0.39	–0.94161	0.64	–0.44629	0.89	–0.11653
0.15	–1.89712	0.40	–0.91629	0.65	–0.43708	0.90	–0.10536
0.16	–1.83258	0.41	–0.89160	0.66	–0.41551	0.91	–0.09431
0.17	–1.77196	0.42	–0.86750	0.67	–0.40047	0.92	–0.08338
0.18	–1.71480	0.43	–0.81419	0.68	–0.38566	0.93	–0.07257
0.19	–1.66073	0.44	–0.82098	0.69	–0.37106	0.94	–0.06187
0.20	–1.60944	0.45	–0.79851	0.70	–0.35567	0.95	–0.05129
0.21	–1.56065	0.46	–0.77623	0.71	–0.34249	0.96	–0.04082
0.22	–1.51412	0.47	–0.75502	0.72	–0.32850	0.97	–0.03046
0.23	–1.46968	0.48	–0.73397	0.73	–0.31471	0.98	–0.02020
0.24	–1.42711	0.49	–0.71335	0.74	–0.30110	0.99	–0.01005
0.25	–1.38629	0.50	–0.69214	0.75	–0.28768	1.00	–0.00000

The Natural Logarithm Table (Equal to or More than 1.0)

n	$\log_e n$	n	$\log_e n$	n	$\log_e n$	n	$\log_e n$
1.0	0.00000	3.0	1.09861	5.0	1.60944	25.0	3.21887
1.1	0.09531	3.1	1.13140	6.0	1.79176	26.0	3.25809
1.2	0.18232	3.2	1.16315	7.0	1.94591	27.0	3.29583
1.3	0.26236	3.3	1.19392	8.0	2.07944	28.0	3.33220
1.4	0.33647	3.4	1.22377	9.0	2.19722	29.0	3.36729
1.5	0.40546	3.5	1.25276	10.0	2.30258	30.0	3.40119
1.6	0.47000	3.6	1.28093	11.0	2.39789	40.0	3.68888
1.7	0.53063	3.7	1.30833	12.0	2.48491	50.0	3.91202
1.8	0.58779	3.8	1.33500	13.0	2.56495	60.0	4.09434
1.9	0.64185	3.9	1.36097	14.0	2.63905	70.0	4.24849
2.0	0.69314	4.0	1.38629	15.0	2.70805	80.0	4.38202
2.1	0.74193	4.1	1.41099	16.0	2.77259	90.0	4.49981
2.2	0.78845	4.2	1.43508	17.0	2.83321	100.0	4.60517

2.3	0.83291	4.3	1.45861	18.0	2.89037	200.0	5.29832
2.4	0.87547	4.4	1.48160	9.0	2.94444	300.0	5.70378
2.5	0.91629	4.5	1.50408	20.0	2.99573	400.0	5.99146
2.6	0.95551	4.6	1.52605	21.0	3.04452	500.0	6.21461
2.7	0.99325	4.7	1.54756	22.0	3.09104	600.0	6.39693
2.8	1.02962	4.8	1.56861	23.0	3.13549	700.0	6.55108
2.9	1.06471	4.9	1.58923	24.0	3.17805	800.0	6.68461

Table A 11.3: Power of e

The Powers of e for Different Positive and Negative Values

x	e_x	e^{-x}	x	e_x	e^{-x}	x	e_x	e^{-x}
0.00	1.0000	1.00000	0.26	1.2969	0.77105	0.52	1.6820	0.59452
0.01	1.0101	0.99005	0.27	1.3100	0.76335	0.53	1.6989	0.58860
0.02	1.0202	0.98020	0.28	1.3231	0.75578	0.540	1.7160	0.58275
0.03	1.0305	0.97045	0.29	1.3364	0.74826	0.55	1.7333	0.57695
0.04	1.0408	0.96079	0.30	1.3499	0.74082	0.56	1.7507	0.57121
0.05	1.0513	0.95132	0.31	1.3634	0.73345	0.57	1.7683	0.56553
0.06	1.0618	0.94176	0.32	1.3771	0.72615	0.58	1.7860	0.55990
0.07	1.0725	0.93239	0.33	1.3910	0.71892	0.59	1.8040	0.55433
0.08	1.0833	0.92312	0.34	1.4049	0.71177	0.60	1.8221	0.54881
0.09	1.0942	0.91393	0.35	1.4191	0.70469	0.61	1.8404	0.54335
0.10	1.1052	0.90484	0.36	1.4333	0.69768	0.62	1.8589	0.53794
0.11	1.1163	0.89583	0.37	1.4477	0.69073	0.63	1.8776	0.53259
0.12	1.1275	0.88692	0.38	1.4623	0.68386	0.64	1.8965	0.52729
0.13	1.1388	0.87809	0.39	1.4770	0.67706	0.65	1.9155	0.52205
0.14	1.1503	0.86936	0.40	1.4918	0.67032	0.66	1.9348	0.51685
0.15	1.1618	0.96071	0.41	1.5068	0.66365	0.67	1.9542	0.51171
0.16	1.1735	0.85214	0.42	1.5220	0.65705	0.68	1.9739	0.50662
0.17	1.1853	0.84366	0.43	1.5373	0.65051	0.69	1.9937	0.50158
0.18	1.1972	0.83527	0.44	1.5527	0.64404	0.70	2.0138	0.49659
0.19	1.2092	0.82696	0.45	1.5683	0.63763	0.71	2.0340	0.49164
0.20	1.2214	0.81873	0.46	1.5841	0.63128	0.72	2.0544	0.48675
0.21	1.2337	0.81058	0.47	1.6000	0.62500	0.73	2.0751	0.48191
0.22	1.2461	0.80252	0.48	1.6161	0.61878	0.74	2.0959	0.47711
0.23	1.2586	0.79432	0.49	1.6323	0.61263	0.75	2.1170	0.47237
0.24	1.2712	0.78663	0.50	1.6487	0.60653	0.76	2.1383	0.46767
0.25	1.2840	0.77880	0.51	1.6653	0.60050	0.77	2.1598	0.46301

	200.0	5.29832
	300.0	5.70378
	400.0	5.99146
	500.0	6.21461
	600.0	6.39693
	700.0	6.55108
	800.0	6.68461

e Values

e_x	e^{-x}
1.6820	0.59452
1.6989	0.58860
1.7160	0.58275
1.7333	0.57695
1.7507	0.57121
1.7683	0.56553
1.7860	0.55990
1.8040	0.55433
1.8221	0.54881
1.8404	0.54335
1.8589	0.53794
1.8776	0.53259
1.8965	0.52729
1.9155	0.52205
1.9348	0.51685
1.9542	0.51171
1.9739	0.50662
1.9937	0.50158
0138	0.49659
0340	0.49164
0544	0.48675
0751	0.48191
959	0.47711
170	0.47237
383	0.46767
598	0.46301

Table A 11.3: Power of e (Continued)
The Powers of e for Different Positive and Negative Values (Continued)

x	e_x	e^{-x}	x	e_x	e^{-x}
0.78	2.1815	0.45841	0.98	2.6645	0.37531
0.79	2.2034	0.45384	0.99	2.6912	0.37158
0.80	2.2255	0.44933	1.00	2.7183	0.36788
0.81	2.2479	0.44486	1.20	3.3201	0.30119
0.82	2.2705	0.44043	1.30	3.6693	0.27253
0.83	2.2933	0.43605	1.40	4.0552	0.24660
0.84	2.3164	0.43171	1.50	4.4817	0.22313
0.85	2.3396	0.42741	1.60	4.9530	0.20190
0.86	2.3632	0.42316	1.70	5.4739	0.18268
0.87	2.3869	0.41895	1.80	6.0496	0.16530
0.88	2.4109	0.41478	1.90	6.6859	0.14957
0.89	2.4351	0.41066	2.00	7.3891	0.13534
0.90	2.4596	0.40657	3.00	20.086	0.04979
0.91	2.4843	0.40252	4.00	54.598	0.01832
0.92	2.5093	0.39852	5.00	148.41	0.00674
0.93	2.5345	0.39455	6.00	403.43	0.00248
0.94	2.5600	0.39063	7.00	1096.6	0.00091
0.95	2.5857	0.38674	8.00	2981.0	0.00034
0.96	2.6117	0.38298	9.00	8103.1	0.00012
0.97	2.6379	0.37908	10.00	22026.5	0.00005

C12 HAPTER

HEDGING STRATEGIES W OPTIONS

Learning Objectives

After studying this chapter, you will be able to:

- Understand the basic fundamentals of hedging with options
- Learn, use and trade with the various hedge parameters (option Gree Gamma, Theta, Vega and Rho.

Chapter Outline

C 12 HAPTER

Hedging Strategies with Options

Learning Objectives

After studying this chapter, you will be able to:

- Understand the basic fundamentals of hedging with options
- Learn, use and trade with the various hedge parameters (option Greeks) such as Delta, Gamma, Theta, Vega and Rho.

Chapter Outline

12.1 INTRODUCTION

The option price for European call non-dividend paying stock is estimated by the following formula derived from Black-Scholes Option Pricing Model:

$$C = SN(d_1) - Xe^{-rT} N(d_2)$$

Where,

$$d_1 = \frac{l_n(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \text{ and}$$

$$d_2 = \frac{l_n(S/X) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

In the above formula, we have used five parameters (determinants or variables) namely current market price of the Stock (S), Strike Price (X), Volatility (σ), Time to expiration (T), and Risk-free rate of interest (r).

We may formulate functional relationship between option price and its determinants in the following expression:

$C = f(S, X, \sigma, T, r)$, and

$P = f(S, X, \sigma, T, r)$

Where,

C = Call Option Price

P = Put Option Price

S = Current Market Price of Stock

X = Strike Price

σ = Volatility

T = Time to expiry

r = Risk-free interest rate per annum

During the period of option contract, these variables are likely to change. When there is a change in one variable, the value of the option will react to change. In the previous chapter, we have discussed the direction (increase/decrease) of change in the option price due to increase in one variable assuming the other four variables remain as such. The result of such directional change in the option price is reproduced below:

Variable (Determinant)	Notation	Directional Change	
		Increase	Decrease
Current Stock Price	S	Increase	Decrease
Strike Price	X	Decrease	Increase
Time to Expiration	T	Increase	Increase
Volatility of Stock Price	σ	Increase	Increase
Risk-free interest rate	r	Increase	Decrease

The objective of trading with option derivative is to maximise profit or minimise risk of loss. In derivative market, the portfolio managers, individual investors, and corporations are very much inclined to forecast the following two aspects:

(i) The direction of change in the option price due to the influence of change in the value of one determinant.

(ii) The degree or magnitude of change in the option price due to the influence of change in the value of one determinant.

The direction and magnitude of change in the Option Price due to the influence of change in the values of determinants (except the Exercise Price) may be favourable (in-the-money) or unfavourable (out-of-the-money). When the option trader will likely to be out-of-the-money, then he will be at risk. To protect from such adverse situation, he must learn, practice, use and trade with certain risk management tools. There are various types of risks associated with options such as risk of stock price moving up and down, implied volatility moving up and down, the risk-free interest moving up and down or how much money is made or lost as time passes.

Through mathematical formulas, certain numbers are generated to estimate these risks. Collectively, these numbers are known as **"Greeks"**, because these numbers are represented by Greek letters like Δ (Delta), θ (Theta), γ (Gamma), ν (Vega) and ρ (Rho) etc. The Greeks describe how the option price changes when the parameters change. A brief concept on Option Greeks is depicted below:

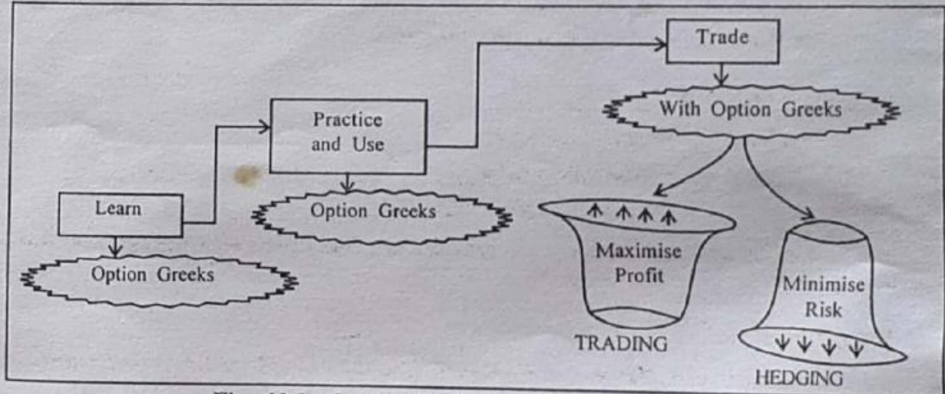


Fig. 12.1: Option Greek, Trading and Hedging

These Greek values are the partial derivatives. Delta, Theta, Vega and Rho are the first order derivatives while Gamma is the second order derivative.

Greeks can help the option traders to better understand the potential risk and reward of an option position. Each Greek measures the risk due to change in the value of one determinant in the following ways:

Delta (Δ): It measures the change in the option price due to change in stock price. It is the sensitivity of an option's theoretical value to a change in the price of the underlying stock.

Theta (θ): It measures the change in the option price due to time passing. Thus, theta is the sensitivity of an option's theoretical value to a change in the amount of time to expiration.

Gamma (γ): It measures the change in the option delta due to change in the stock price. Thus, Gamma is the sensitivity of an option's delta to a change in the price of the underlying asset.

Vega (v): It measures the change in the option price due to change in the volatility. Thus, Vega is the sensitivity of an option's theoretical value to a change in volatility.

Rho (ρ): It measures the change in the option price due to the change in the interest rates. Thus, Rho is the sensitivity of an option's theoretical value to a change in the rate of interest.

In this Chapter, we will discuss the Greeks and their applications.

12.2 OPTION DELTA (Δ)

Meaning: Option delta is the sensitivity of an option's theoretical price to changes in the value of the underlying asset.

Formula:

(i) $$\text{Option Delta } (\Delta) = \frac{\text{Change in the Option Price}}{\text{Change in the Underlying Price}}$$

Thus, Change in the Option Price = Change in the Asset Price × Delta.

(ii) Delta in the first partial derivative (in calculus) of the value of an option with respect to the price of the underlying asset. Mathematically,

$$\text{Delta} = \Delta = \frac{\partial C}{\partial S}$$

Where, ∂ = Partial derivative

C = Call option price

S = Price of the underlying asset

Thus, delta measures the rate of change of option price with respect to changes in the underlying asset.

Magnitude of Delta

The delta of an option ranges in value from 0 to 1 for calls and 0 to –1 for puts.

Notes: Delta value is always positive for calls and negative for puts.

The delta of an option reflects the increase or decrease in the price of an option in response to 1 point movement (up or down) of the underlying asset price.

Example 1: 100 long calls has a value of ₹ 3 per call. The stock price is ₹ 52 per share with delta 0.45. Estimate the call option price if the stock price moves up to ₹ 53 or moves down to ₹ 51.

Solution:

Stock Price (S)	Call Delta (Δ)	Call Price (C)
52	0.45	3
53	0.45	3.45
51	0.45	2.55

Example 2: 100 long puts on a share has a value of ₹ 3 per put. The stock price is ₹ 52 per share with delta (–)0.45. Estimate the put option price if the stock price moves up to ₹ 53 or moves down to ₹ 51.

Solution:

Stock Price (S)	Put Delta (Δ)	Put Price (P)
52	–0.45	3
53	–0.45	3 – 0.45 = 2.55
51	–0.45	3 + 0.45 = 3.55

Example 3: The change in underlying's price is ₹ 20 and call delta is (+)0.24. Estimate the change in option price.

Solution: Change in Option Price = Change in Asset Price × Option Delta

= 20 × 0.24 = ₹ 4.80

Example 4: The decrease in the underlying assets price is ₹ 10 and put delta is –0.057. Estimate the increase in the put price.

Solution: Change in the Option Price = Change in the Underlying Asset × Option Delta

= 10 × 0.057 = 0.57 (increase)

12.2.1 Moneyness and Behaviour of Option Delta

An option trader should know the relationship between the value of option delta and moneyness of an option. In brief, relationship is summarised as follows:

Call Option	
Value of Delta	Moneyness Position
1	Deep in-the-money
0.5	At-the-money
Δ → 0 (Approaches to Zero)	Out-of-the-money

Put Option	
Value of Delta	Moneyness Position
–1	Deep in-the-money
–0.5	At-the-Money
Δ → 0 (Approaches to Zero)	Out-of-the-money