UNIT – V

GAME THEORY

Game :- A competitive situation is called a game if it has the following properties.

(i). there are finite number of participants (players).

(ii). Each player has finite number of strategies available to him.

(iii). Every game results in an outcome.

Number of players :— If a game involves only two players, then it is called two-person game.

If the numbers of players are more than two, the game is known as n-person game.

Sum of gains and losses :— If in a game the gains of one player are exactly the losses to another player such that sum of gains and losses equals zero, then the game is said to be a zero-sum game.

otherwise it is said to be non-zero sum game.

strategy :- The list of all possible actions (moves) (courses of action) that the player will take for every pay off (outcome) is called a strategy.

Optimal strategy :- The particular strategy (or complete plan) by which a player optimises his gains or losses without knowing the competitor's strategies is called optimal strategy.

Value of the game :- The expected outcome per play when players follow their optimal strategy is called as value of the game.

Pure strategy :- It is the decision rule which is always used by the player to select particular strategy (course of action)

Thus, each player knows in advance of all strategies.

Mixed strategy :- The courses of action that are to be selected on a particular occasion with some fixed probability are called mixed stratogies.

Thus, there is a probabilistic situation in selecting the mixed strategies.

Two - Person zero - sum games :—

A game with only two players is called a Two - person zero - sum game, if one player's gain is equal to the loss of other player so that total sum is zero.

Payoff Matrix :—

A quantitative measure of satisfaction a player gets at the end of the play is called as payoff of the game.

The payoffs in terms of gains or losses, when players select their particular strategies (courses of action) can be represented in the form of a matrix. such matrix is called as payoff matrix.

Since the game is zero - sum, the gain of one player is equal to the loss of other player. Hence, one player's payoff table would contain the same amounts in payoff table of other player with the change of sign.

If player A has m strategies $A_1, A_2, \ldots, A_m$ and player B has n strategies $B_1, B_2, \ldots, B_n$.

The total number of possible outcomes is $m \times n$.

Let a_{ij} be the payoff which player A gains from player B if player A chooses strategy i and B chooses strategy j.

Then the payoff matrix will be as follows.

Player A strategies	Player B strategies B_1	B_2	$\ldots\ldots$	B_n
A_1	a_{11}	a_{12}	$\ldots\ldots$	a_{1n}
A_2	a_{21}	a_{22}	$\ldots\ldots$	a_{2n}
$\vdots$	$\vdots$	$\vdots$		$\vdots$
A_m	a_{m1}	a_{m2}	$\ldots$	a_{mn}

Assumptions of the game

(1). Each player has available to him a finite number of pure strategies (courses of action). The list may not be same for each player.

(2). Players act rationally and intelligently.

(3). The amount of gain or loss on an individual's choice of strategy is known to each player in advance.

(4). List of strategies of each player in known in advance.

(5). One player attempts to maximize gains and other attempts to minimize losses.

(6). Both players make their decisions individually prior to the play without direct communication between them.

(7). The payoff is fixed and determined in advance.

Pure strategies – Games with saddle point:—

Consider the payoff matrix of the game which represents pay off of player A.

The objective of the study is to know how these players must select their respective strategies so that they may optimise their pay off.

Such a decision-making criterion is referred to as the Minimax – Maximin principle.

Maximin principle:— For player A, the minimum value in each row represents least gain (pay off) to him.

These are written in the matrix by row minima.

He then select largest gain among the row mininum values.

This choice of player A is called Maximin principle.

The corresponding gain is called Maximin value of the game.

Minimax Principle :— Player B is assumed to be the looser. The maximum value in each column represents the maximum loss to him. These are written in the matrix by column Maxima. He will then select minimum loss among the column Maximum values.

This choice of player B is called minimax principle. The corresponding loss is the minimax value of the game.

optimal strategy:— A course of action which puts the player in the most preferred position, irrespective of the course of action his competitor(s) adopt, is called as optimal strategy.

saddle point :— If the Maximin value is equals to the minimax value, then the game is said to have a saddle (equillibrium) point.

Value of the game (V) :— The amount of payoff at a saddle point is called as Value of the game.

Remarks :— (1). A game may have more than one saddle point.

(2). If V is the value of the game,

$$\text{Maximin value} \le V \le \text{Minimax value}.$$

(3). If the Maximin and minimax values both equals to zero, then the game is said to be a fair game.

(4). If the Maximin and minimax values of the game are equal and both equal to the value of the game, then the game is said to be strictly determinable.

Method of finding saddle point:-

To determine the saddle point in the payoff matrix, we follow the following steps.

1). select the minimum (lowest) element in each row of the payoff matrix and write them under 'row minima' heading. Then select the largest among these elements and enclose it in a rectangle. □.

2). select the Maximum (largest) element in each column of the payoff Matrix and write them under 'column Maxima' heading. Then select the lowest element among these elements and enclose it in a circle. O.

3). Find out the element(s) which is same in the circle as well as rectangle.

This element represents the value of the game and is called the saddle (equlibrium) point.

Problems :-

(1). The Payoff matrix of a game is given below.

Player A	Player B B_1	B_2	B_3
A_1	-1	2	-2
A_2	6	4	-6.

Determine the optimal strategies for players A and B.
Also determine the value of the game.
Is the game (i). fair? (ii). strictly determinable?.

Sol:-

Player A	Player B B_1	B_2	B_3	Row minimum	
A_1	-1	2	$\boxed{-2}$	$\boxed{-2}$	Maximin.
A_2	6	4	-6	-6	
Column maximum	6	4	-2 Minimax		

(A_1, B_3) is the saddle point
and the value of the game is -2.
The game is strictly determinable.

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(2). Solve the following game by using Maximin, minimax principle, whose payoff matrix is given below.

Player A	Player B B_1	B_2	B_3	B_4	Row mininum	
A_1	1	7	3	4	1	
A_2	5	6	$\boxed{4}$	5	$\boxed{4}$	Maximin
A_3	7	2	0	3	0	
Column maximum	7	7	4 Minimax	5		

The optimal strategy for A is A_2
for B is B_3

and the value of the game $= 4$.

(3). A company management and the labour union are negotiating a new three-year statement.
The costs to the company are given for every pair of strategy choice.

Union strategies	Company strategies I	II	III	IV
I	20	15	12	35
II	25	14	8	10
III	40	2	10	5
IV	-5	4	11	0

What strategy will the two sides adopt?
Also determine the value of the game.

Ans:- Maximin = Minimax = 12.

∴ Company will always adopt strategy III
and union will always adopt strategy I.

The value of the game = 12.

(4). Find the range of values of p and q which will render the entry (2, 2) a saddle point for the game:

Player A	Player B B_1	B_2	B_3
A_1	2	4	5
A_2	10	7	q
A_3	4	p	6.

Sol:- Ignoring the values of p and q in the pay off matrix, We determine maximin and minimax values.

Player A	Player B B_1	B_2	B_3	Row minimum
A_1	2	4	5	2
A_2	10	$\boxed{7}$	q	$\boxed{7}$ Maximin
A_3	4	p	6	4
Column maximum	10	7	$\boxed{6}$ Minimax	

Saddle point will exist at the position (2, 2) only when $p \le 7$ and $q > 7$.

Saddle point is not unique.

5. Find the strategy selection for each player and the value of game.

	Player B				
Player A	B_1	B_2	B_3	B_4	Row Minimum
A_1	-5	3	1	10	-5
A_2	5	5	$\boxed{4}$	6	$\boxed{4}$ Maximin
A_3	4	-2	0	-5	-5
Column Maximum	5	5	4 Minimax	10	

saddle point is 4 at (A_2, B_3) position,

Value of the game = 4.

Mixed strategies and Games without Saddle point:-

In certain cases, the saddle point does not exist. To find the solution of such games, both the players must determine optimal mixed strategies.

A mixed strategy game can be solved by the following methods.

1). Algebraic Method
2). Analytical or calculus Method
3). Matrix Method
4). Graphical method
5). Linear programming Method.

The Rules (Principles) of Dominance:-

The rules of dominance are used to reduce the size of the payoff matrix.

The Dominance principles are stated as follows.

(1). For player A who is assumed to be the gainer, if each element in a row R_s is less than or equal to the corresponding element in other row R_t, then R_s is said to be dominated by R_t.

Then R_s can be deleted from the payoff matrix.

The player A will never use the strategy corresponding to the row R_s.

(2). For player B who is assumed to be the loser, if each element in a column, ~~say~~ C_s is greater than or equal to the corresponding element in a column C_t, then C_s is said to be dominated by C_t.

Then C_s can be deleted from the payoff matrix.

The player B will never use the strategy corresponding to the ~~row~~ column C_s.

(3). Some strategies also be dominated if it is inferior (less attractive) to an average or more other pure strategies.

Eg:- The pay off matrix for A is

Player A \ Player B	B_1	B_2	B_3	Row minima
A_1	-5	10	20	$\boxed{-5}$ maximin.
A_2	5	-10	-10	-10
A_3	5	-20	-20	-20
column maxima	5 Minimax	10	20	

There is no saddle point for the game.

Every element of $B_3 \geq$ every element of B_2.

$\therefore$ B_3 is dominated by B_2.

So we delete B_3 from pay off Matrix.

Player A \ Player B	B_1	B_2
A_1	-5	10
A_2	5	-10
A_3	5	-20

Every element of $A_3 \leq$ every element of A_2.

$\therefore$ A_3 is dominated by A_2.

So we delet A_3 from the pay off Matrix.

Player A \ Player B	B_1	B_2
A_1	-5	10
A_2	5	-10

{For this problem $V = 0$}.

After getting the 2×2 pay off Matrix, we find the saddle point by any one of the above methods.

Algebraic Method :-

consider the game with the payoff matrix as follows.

	Player B		
Player A	B_1	B_2	Probability
A_1	a_{11}	a_{12}	P_1
A_2	a_{21}	a_{22}	P_2
Probability	q_1	q_2	

Let P_1, P_2 are the probabilities when player A select the strategies A_1, A_2 respectively.

Let q_1, q_2 are the probabilities when player B select the strategies B_1, B_2 respectively.

Let V is the value of the game.

Since player A is the gainer, A expects atleast V.

$\therefore$ We must have

$$\left.\begin{aligned} a_{11}P_1 + a_{21}P_2 &\geq V. \\ a_{12}P_1 + a_{22}P_2 &\geq V. \end{aligned}\right\} \rightarrow (1).$$

where $P_1 + P_2 = 1$

Since player B is the loser. B expects atmost V.

$\therefore$ We must have

$$\left.\begin{aligned} a_{11}q_1 + a_{12}q_2 &\leq V \\ a_{21}q_1 + a_{22}q_2 &\leq V \end{aligned}\right\} \rightarrow (2).$$

where $q_1 + q_2 = 1$

consider the inequalities in eqs (1), (2) as equalities, solve for the values P_1, P_2, q_1, q_2. and find the value of the game.

Problems:-

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(1). solve the game whose payoff matrix is given below.

	Player B				
Player A	B_1	B_2	B_3	B_4	Row minimum
A_1	3	2	4	0	0
A_2	3	4	2	4	2 - Maximin.
A_3	4	2	4	0	0
A_4	0	4	0	8	0
Column maximum	4	4	4 Minimax.	8	

The game has no saddle point.

We reduce the size of the given payoff matrix by using dominance principles.

For player A, first row is dominated by third row.

i.e every element of $A_1 \leq$ every element of A_3.

∴ We delete first row.

	Player B			
Player A	B_1	B_2	B_3	B_4
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

For player B, each element of $B_1 \geq$ each element of B_3.

⇒ first column is dominated by third column.

∴ We delelete first column.

	Player B		
Player A	B_2	B_3	B_4
A_2	4	2	4
A_3	2	4	0
A_4	4	0	8

Here, none of the strategies of players A and B is inferior to any of their other strategies.

The average payoffs of B_3, B_4 are $\frac{2+4}{2}, \frac{4+0}{2}, \frac{0+8}{2}$

$= 3, 2, 4$

Then every element of $B_2 \geq$ average payoffs of B_3, B_4.

$\therefore$ The strategy B_2 may be deleted.

Player A	Player B	
	B_3	B_4
A_2	2	4
A_3	4	0
A_4	0	8

The averages of A_3, A_4 are 2, 4 is same as A_2.

$\therefore$ We delete strategy A_2.

Player A	Player B		
	B_3	B_4	Probabilities
A_3	4	0	p_1
A_4	0	8	p_2
Probabilities	q_1	q_2	

$$4p_1 + 0p_2 = 0p_1 + 8p_2$$

$$4p_1 = 8p_2$$

$$4p_1 = 8(1-p_1), \quad p_1 + p_2 = 1$$

$$p_2 = 1 - p_1$$

$$4p_1 = 8 - 8p_1$$

$$12p_1 = 8$$

$$p_1 = \frac{2}{3}, \quad p_2 = \frac{1}{3}.$$

$$4q_1 + 0q_2 = 0q_1 + 8q_2$$

$$4q_1 = 8(1-q_1)$$

$$q_1 = \frac{2}{3}.$$

$$q_2 = \frac{1}{3}.$$

The value of the game

$$= 4p_1 + 0p_2$$

$$= 4\left(\frac{2}{3}\right) + 0 = \frac{8}{3}.$$

The optimal strategies for A are $\left\{0, 0, \frac{2}{3}, \frac{1}{3}\right\}$.

The optimal strategies for B are $\left\{0\ 0, \frac{2}{3}, \frac{1}{3}\right\}$.

(2). Using dominance rules to reduce the size of the following payoff matrix to (2×2) size and hence find the optimal strategies and value of the game.

Player A	Player B B_1	B_2	B_3
A_1	3	-2	4
A_2	-1	4	2
A_3	2	-2	6

column Max			Row mini
3	4	6	-2
			-1
			-2

no saddle point.

Sol:- For player B
each element in $B_3 \geq$ each element in B_1,

B_3 is dominated by B_1.

So we delete B_3.

Player A	Player B B_1	B_2
A_1	3	-2
A_2	-1	4
A_3	2	-2

For player A, each element in $A_3 \leq$ each element in A_1.

$\therefore A_3$ is dominated by A_1.

$\therefore$ We delete A_3.

Player A	Player B B_1	B_2	Probabilities
A_1	3	-2	P_1
A_2	-1	4	P_2
Probability	q_1	q_2	

Here also, there is no saddle point.

$3P_1 - P_2 = -2P_1 + 4P_2$

$5P_1 = 5P_2 = 5(1-P_1)$

$\Rightarrow 10P_1 = 5$, $\boxed{P_1 = \frac{1}{2}}$ $\boxed{P_2 = \frac{1}{2}}$

$3q_1 - 2q_2 = -q_1 + 4q_2$

$4q_1 = 6q_2 \Rightarrow 4q_1 = 6(1-q_1)$

$\Rightarrow 10q_1 = 6$, $\boxed{q_1 = \frac{3}{5}}$ $\boxed{q_2 = \frac{2}{5}}$

The value of the game $= 3P_1 - P_2 = \frac{3}{2} - \frac{1}{2} = 1$.

(3). Solve the game with pay off matrix

Player A	Player B B_1	B_2
A_1	1	$-1/2$
A_2	$-1/2$	0

Sol:- $V = \frac{-1}{8}$.

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4. Reduce the following game by dominance property and find the value of the game.

Player B

Player A	I	II	III	IV	V
I	1	3	2	7	4
II	3	4	1	5	6
III	6	5	7	6	5
IV	2	0	6	3	1

Sol:- This will be reduced to

	I	II	III
III	6	5	7

The optimal strategies for A is III

B is II {5 is min. loss}.

and value of the game = 5.

(OR).

Directly the saddle point = 5.

5. For the following payoff matrix of the game, Determine the optimal strategies and value of the game.

Player B

Player A	B_1	B_2	B_3
A_1	30	40	-80
A_2	0	15	-20
A_3	90	20	50

Sol:- The Reduced pay off matrix is

	B_2	B_3
A_1	40	-80
A_3	20	50

$p_1 = \frac{1}{5}$ $p_2 = \frac{4}{5}$

$q_1 = \frac{13}{15}$ $q_2 = \frac{2}{15}$.

and $V = 24$.

The optimal strategies for A are $\left\{\frac{1}{5}, 0, \frac{4}{5}\right\}$.

" " B are $\left\{0, \frac{13}{15}, \frac{2}{15}\right\}$.

6. Solve the game whose payoff matrix is given by

Player B

$$\begin{array}{c|ccc|c} & I & II & III & \\ \hline \text{Player A} \; I & \boxed{-2} & 15 & \boxed{-2} & \boxed{-2} \\ II & -5 & -6 & -4 & -6 \\ III & -5 & 20 & -8 & -8 \\ \hline & -2 & 20 & -2 & \end{array}$$

Sol:- The saddle point is -2.

The value of the game $V = -2$.

The optimum strategy for A is I.

" B is I and III.

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7. Solve the game whose pay off Matrix is

$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \\ \hline A_1 & \boxed{6} & 8 & \boxed{6} & \boxed{6} \\ A_2 & 4 & 12 & 12 & 4 \\ \hline & 6 & 12 & 6 & \end{array}$$

Value of the game $V = 6$.

The optimum strategy for A is A_1.

" for B is B_1 and B_3.

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8. Find the value & optimal strategies for the two players game whose payoff matrix is given below.

$$\begin{array}{c|ccc|c} A \backslash B & I & II & III & \\ \hline I & 1 & -1 & -1 & -1 \\ II & -1 & -1 & 3 & -1 \\ III & -1 & 2 & -1 & -1 \\ \hline & 1 & 2 & 3 & \end{array}$$

Sol.- There is no saddle point.

So we reduce the size of the matrix by dominance rules.

There is no possibility to reduce the size by dominance rules.

So we take probabilities.

Player B

$$\begin{array}{c|ccc|c} \text{Player A} & I & II & III & \text{Probabilities} \\ \hline I & 1 & -1 & -1 & p_1 \\ II & -1 & -1 & 3 & p_2 \\ III & -1 & 2 & -1 & p_3 \\ \hline \text{Probabilities} & q_1 & q_2 & q_3 & \end{array}$$

For player A :-

$P_1 - P_2 - P_3 \geq V$

$-P_1 - P_2 + 2P_3 \geq V$

$-P_1 + 3P_2 - P_3 \geq V$

and $P_1 + P_2 + P_3 = 1$

For player B

$q_1 - q_2 - q_3 \leq V.$

$-q_1 - q_2 + 3q_3 \leq V.$

$-q_1 + 2q_2 - q_3 \leq 1$

and $q_1 + q_2 + q_3 = 1.$

$P_1 - P_2 - P_3 = -P_1 - P_2 + 2P_3.$

$2P_1 - 3P_3 = 0.$

$\cancel{-P_1} - P_2 + 2P_3 = \cancel{-P_1} + 3P_2 - P_3.$

$-4P_2 + 3P_3 = 0.$

9. Solve the game with payoff Matrix

$$
\begin{array}{c|c}
 & \text{Player B} \\
 & \begin{array}{ccc} \text{I} & \text{II} & \text{III} \end{array} \\
\begin{array}{c} \text{Player A I} \\ \text{II} \\ \text{III} \end{array} & \begin{bmatrix} 1 & -1 & -2 \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix} \\
 & \begin{array}{ccc} 2 & +1 & +1 \end{array}
\end{array}
\quad
\begin{array}{c} -2 \\ \boxed{-1} \\ \boxed{-1} \end{array}
$$

Sol:-

There is no saddle point.

For player A, I is dominated by III

So we delete I

$$
\begin{array}{cc}
 & \text{Player B} \\
 & \begin{array}{ccc} \text{I} & \text{II} & \text{III} \end{array} \\
\begin{array}{c} \text{Player A} \\ \text{II} \\ \text{III} \end{array} & \begin{array}{c} \\ \begin{bmatrix} -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix} \end{array}
\end{array}
$$

For player B, III is dominated by II.

So we delete III

$$
\begin{array}{ccc}
 & \text{Player B} & \\
 & \begin{array}{cc} \text{I} & \text{II} \end{array} & \\
\begin{array}{c} \text{Player A} \\ \text{II} \\ \text{III} \end{array} & \begin{array}{c} \\ \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \end{array} & \begin{array}{c} \\ p_1 \\ p_2 \end{array} \\
 & \begin{array}{cc} q_1 & q_2 \end{array} &
\end{array}
$$

$$-p_1 + 2p_2 = p_1 - p_2$$

$$-2p_1 + 3p_2 = 0$$

$$-2p_1 + 3(1-p_1) = 0$$

$$-2p_1 + 3 - 3p_1 = 0.$$

$$-5p_1 = -3$$

$$p_1 = \frac{3}{5} \quad p_2 = \frac{2}{5}$$

$$-q_1 + q_2 = 2q_1 - q_2.$$

$$-3q_1 = -2q_2.$$

$$3q_1 = 2(1-q_1)$$

$$3q_1 + 2q_1 = 2.$$

$$5q_1 = 2$$

$$q_1 = \frac{2}{5} \quad q_2 = \frac{3}{5}.$$

Value of the game $V = -p_1 + 2p_2$

$$= \frac{-3}{5} + \frac{4}{5} = \frac{1}{5}.$$

The optimum strategies for A are $\left\{0, \frac{3}{5}, \frac{2}{5}\right\}$.

" B are $\left\{\frac{2}{5}, \frac{3}{5}, 0\right\}$.

(10). Solve the following game by using the principle of dominance.

				Player B		
Player A	I	II	III	IV	V	VI
1	4	2	0	2	1	1
2	4	3	1	3	2	2
3	4	3	7	-5	1	2
4	4	3	4	-1	2	2
5	4	3	3	-2	2	2

Sol:-

	III	IV
2	1	3
3	7	-5

$A : \left\{0, \frac{6}{7}, \frac{1}{7}, 0, 0\right\}$.

$B : \left\{0, 0, \frac{4}{7}, \frac{3}{7}, 0, 0\right\}$.

and $V = \frac{13}{7}$.

(11). Use dominance rules to reduce the size of the payoff matrix and hence find the optimal strategies and value of the game.

		Player B	
Player A	B_1	B_2	B_3
A_1	3	-2	4
A_2	-1	4	2
A_3	2	2	6

Sol:- For player B, B_3 is dominated by B_1.

	Player B	
Player A	B_1	B_2
A_1	3	-2
A_2	-1	4
A_3	2	2

$\frac{3-1}{2}, \frac{-2+4}{2}$

$= 1, 1$

(11). By dominance theory, solve the following game.

Player B

Player A	B_1	B_2	B_3	B_4
A_1	8	10	9	14
A_2	10	11	8	12
A_3	13	12	14	13

Ans.:— $V = 12$.

Graphical Method :—

This method is useful for the game where the payoff matrix is of the size $2 \times n$ or $m \times 2$.

consider the following $2 \times n$ payoff matrix of a game without saddle point.

Player A	Player B B_1	B_2	B_3	.... B_n	Probability
A_1	a_{11}	a_{12}	a_{13}	.... a_{1n}	P_1
A_2	a_{21}	a_{22}	a_{23}	.... a_{2n}	P_2
Probability	q_1	q_2	q_3	.... q_n	

Player A has two strategies A_1, A_2 with selection probabilities P_1, P_2 respectively, such that $P_1 + P_2 = 1$. and $P_1 . P_2 \geq 0$

The pure strategies available to player B and expected payoff for player A would be as follows.

B's Pure strategies	A's expected payoff
B_1	$a_{11} P_1 + a_{21} P_2$
B_2	$a_{12} P_1 + a_{22} P_2$
B_3	$a_{13} P_1 + a_{23} P_2$
$\vdots$	$\vdots$
B_n	$a_{1n} P_1 + a_{2n} P_2$

Plot the straight lines on the graph representing player A's expected payoff values.

Player A should select the value of probability P_1 and P_2 so as to Maximize the minimum expected payoff.

i.e maximin value.

Hence, we select (maximum) highest point on the lower boundary of these lines.

By solving the corresponding lines which passes through this point, we can find the value of the game.

For $m \times 2$ games also, we follow the same procedure and select Minimum (lowest) point on the upper boundary of the lines and find the value of the game.

Problems

(1). Use graphical method to solve the following game and hence find value of the game.

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	2	2	3	-2
	A_2	4	3	2	6

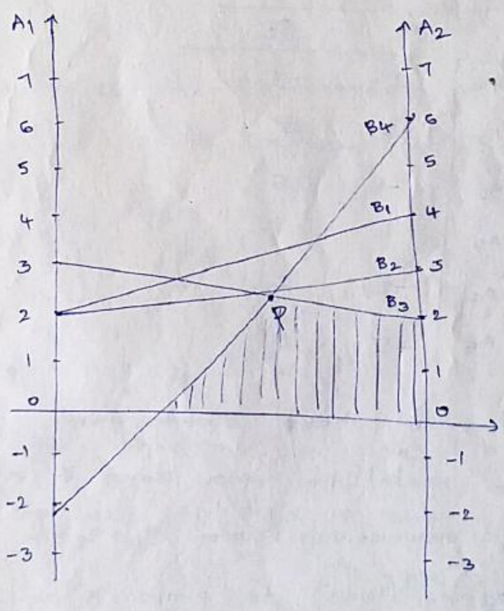
Sol:- There is no saddle point for the game.

Let P_1, P_2 be the probabilities of player A in selecting A_1, A_2 strategies respectively. and $P_1 + P_2 = 1$

The expected payoff (gain) to player A will be as follows.

B's pure strategies	A's expected Payoff
B_1	$2P_1 + 4P_2$
B_2	$2P_1 + 3P_2$
B_3	$3P_1 + 2P_2$
B_4	$-2P_1 + 6P_2$

We plot the straight lines of A's expected payoff values on the graph.



P is the maximin point.

The Maximin point is the intersection of B_3 and B_4.

∴ The payoff matrix will be reduced to (2×2) matrix as given below.

Player A	Player B: B_3	B_4	
A_1	3	-2	P_1
A_2	2	6	P_2
	q_1	q_2	

$$3p_1 + 2p_2 = -2p_1 + 6p_2 \qquad 3q_1 - 2q_2 = 2q_1 + 6q_2$$

$$5p_1 = 4p_2 \qquad q_1 = 8q_2$$

$$5p_1 = 4(1-p_1) \qquad q_1 = 8(1-q_1)$$

$$9p_1 = 4 \qquad 9q_1 = 8.$$

$$p_1 = \frac{4}{9},\ p_2 = \frac{5}{9} \qquad q_1 = \frac{8}{9},\ q_2 = \frac{1}{9}.$$

$\therefore$ Value of the game $V = 3p_1 + 2p_2$

$$= 3\left(\frac{4}{9}\right) + 2\left(\frac{5}{9}\right)$$

$$= \frac{12+10}{9} = \frac{22}{9}.$$

The optimum strategies for A are $\left\{\frac{4}{9}, \frac{5}{9}\right\}$.

" for B are $\left\{0, 0, \frac{8}{9}, \frac{1}{9}\right\}$.

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(2). obtain the optimal strategies for both persons and find value of the game for the two-person zero-sum game whose payoff matrix is as follows.

Player A \ Player B	B_1	B_2
A_1	1	-3
A_2	3	5
A_3	-1	6
A_4	4	1
A_5	2	2
A_6	-5	0

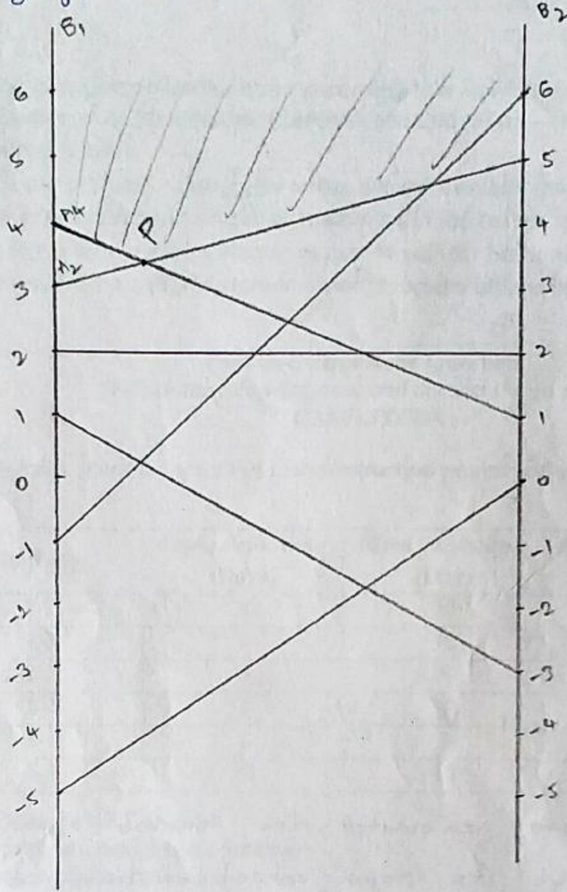
Sol:- The game does not have saddle point.

Let q_1, q_2 are probabilities for player B in selecting the strategies B_1, B_2 respectively. and $q_1 + q_2 = 1$.

The expected pay off (loss) to player B will be as follows.

A's pure strategies	B's expected payoff.
A_1	$q_1 - 3q_2$
A_2	$3q_1 - 5q_2$
A_3	$-q_1 + 6q_2$
A_4	$4q_1 + q_2$
A_5	$2q_1 + 2q_2$
A_6	$-5q_1 + 0q_2$.

We plot the straight lines of B's expected payoff values on the graph as follows.



P = Minimax point.

The minimax point is the intersection of A_2 and A_4.

∴ The payoff matrix will be reduced to (2×2) matrix as follows.

player A	Player B B_1	B_2	
A_2	3	5	p_1
A_4	4	1	p_2
	q_1	q_2	

By solving, we get, Value of the game $V = \frac{17}{5}$.

The optimum strategies for player A are $\{0, \frac{3}{5}, 0, \frac{2}{5}, 0, 0\}$.

" B are $\{\frac{4}{5}, \frac{1}{5}\}$.

(3).

Player A	Player B B_1	B_2	B_3
A_1	1	3	11
A_2	8	5	2.

Sol:- $A : \left(\frac{3}{11}, \frac{8}{11}\right)$ $B : \left(0, \frac{2}{11}, \frac{9}{11}\right)$ and $v = \frac{49}{11}$.

(4).

Player A	Player B B_1	B_2
A_1	-6	7
A_2	4	-5
A_3	-1	-2
A_4	-2	5
A_5	7	-6

Sol:- $A : \left\{0, 0, 0, \frac{13}{20}, \frac{7}{20}\right\}$.

$B : \left\{\frac{11}{20}, \frac{9}{20}\right\}$. and $V = \frac{23}{20}$.

(5). A soft drink company calculated the market share of two products against its major competitor having three products and found out the impact of additional advertisement in any one of its products against the other.

Company A	Company B B_1	B_2	B_3
A_1	6	7	15
A_2	20	12	10

What is the best strategy as well as the competitor? What is the payoff obtained by the company and the competitor in the long run?
Use graphical method to obtain the solution.

Sol:- $A : \left(\frac{2}{3}, \frac{1}{3}, 0\right)$ $B : \left(\frac{7}{12}, \frac{5}{12}\right)$ and $V = \frac{1}{3} \cdot \{11\}$.