

## Unit - III

### Production Analysis

- ✓ Production Functions ✓
- ✓ Law of variable functions
- ✓ Isobars, Isoquants, Isocost
- ✓ Least cost combination
- Return to scale - Economics of scale
- ✓ Cobb-douglas's production ~~\*~~ functions ✓
- Constant elasticity of substitution

#### Production Function :

Production means converting the raw materials or inputs into output of the organisation is consider as production.

Production Function as "The function which defines maximum output of the organisation with using of minimum set of inputs."

→ Michael ~~φ~~R. Baye

(or)

Production Function as "a Technical relationship which reveals the maximum output capable of being produced by each & every set of input."

$$\text{Production Function} = f [L_1, L_2, C, O, T]$$

$$Q = f [L_1, L_2, C, O, T]$$



where  $Q$  = Quantity of output product.

$L_1$  = Land

$L_2$  = Labour

$C$  = Capital

$O$  = Organisation

$T$  = Technology

NOTE:  $f$  = function, (relation b/w inputs & outputs)

In above production function, any positive change in inputs will leads to increasing the quantity of output of the organisation.

Types of production functions:

1. Production function with one variable
2. Production function with two variables
3. Production function with multi variables

1. Production function with one variable: & Law of returns  
The law of ~~return~~ <sup>return</sup> states that when atleast one factor of production is fixed (or) factor input is fixed when all other factors are varied.

The total output in the initial stage will increase at an increasing rate and after reaching certain level of production, the total output will increase at decline rate (decreases).

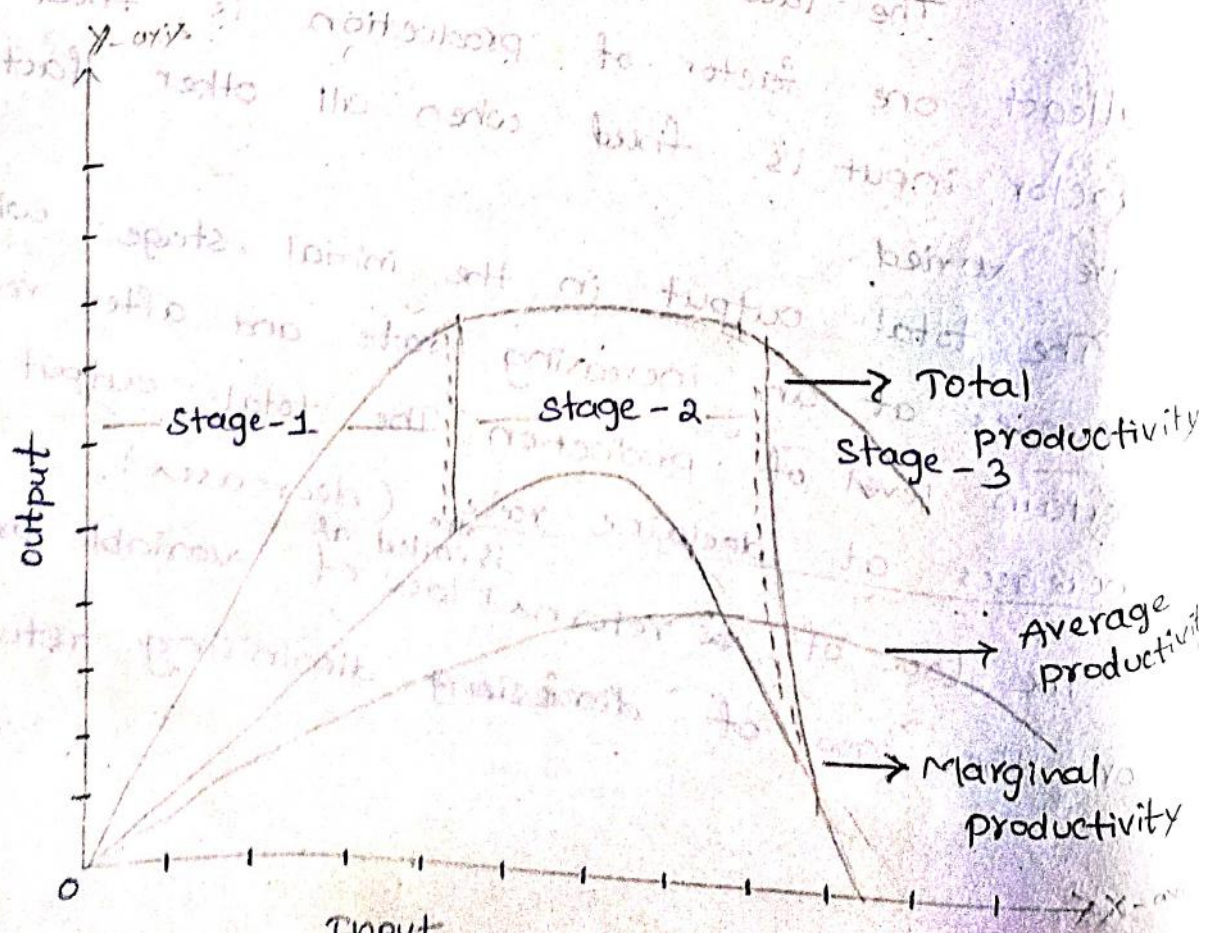
The law of ~~return~~ <sup>return</sup> is called as law of variable proportions or the law of ~~diminishing~~ <sup>diminishing</sup> diminishing returns.



S.NO.	Labour	Total productivity	Marginal productivity	Average productivity
1.	0	0	0	0
2.	10	10 (10-0)	10 ( $\frac{0+10}{2}$ )	5
3.	22	22 (22-10)	12 ( $\frac{12+10}{2}$ )	11
4.	33	33 (33-22)	11	11.5
5.	40	40 (40-33)	7	9
6.	5	45 (45-40)	5	6
7.	6	48 (48-45)	3	4
8.	7	45 (45-48)	-3	-0

**NOTE:** Assumption  
 In the short run, it is assumed that Capital is fixed factor input and labour is variable input. It is also assumed that technology is given and it is not going to change.

**Graphical Representation :**





Stage - 1 → Increasing level rate of returns

Stage - 2 → constant level

Stage - 3 → Decreasing level

2. Production Function with Two Variable:

Production function with two variables consists of both Capital (C) & Labour (L) to produce a given output. There could be a more than two inputs in real life situations. But for simple analysis we restrict the number of inputs to two only.

$$Q = f(C, L)$$

where, Q = Quantity of output

C = Capital

L = Labour

Both Capital and Labour are required to produce a product

Isoquants:

Iso means equal, Quant means quantity

Isoquants means that quantities through out a given Isoquants are equal. Isoquant curves are also called as "Iso product curves".

An Isoquant curves shows various combination of two inputs such as capital and labour, which yields same level of outputs.

Different combination of inputs that gives same level of output.

As an Isoquant curve represents all such combinations

which yield same equal quantity of output, every combination is a good combination, since all these combinations equally, an isoquant curve is also called "product Indifference curve".



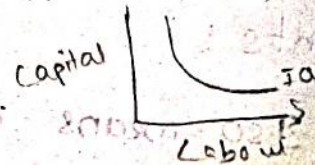
Combination	capital (1)	Labour (2)	Quantity output (unit)
A	1	20	40,000
B	2	15	40,000
C	3	11	40,000
D	4	8	40,000
E	5	6	40,000
F	6	5	40,000

The above table shows that different table factors of inputs, which yields an output of 40,000 output.

### Features of Isoquants :

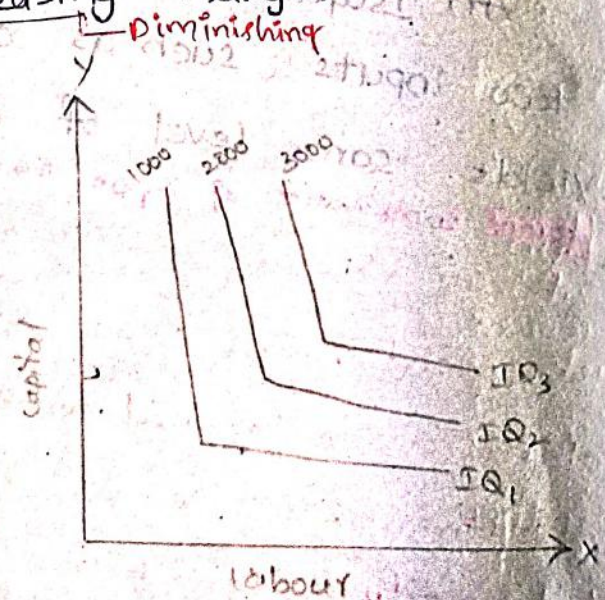
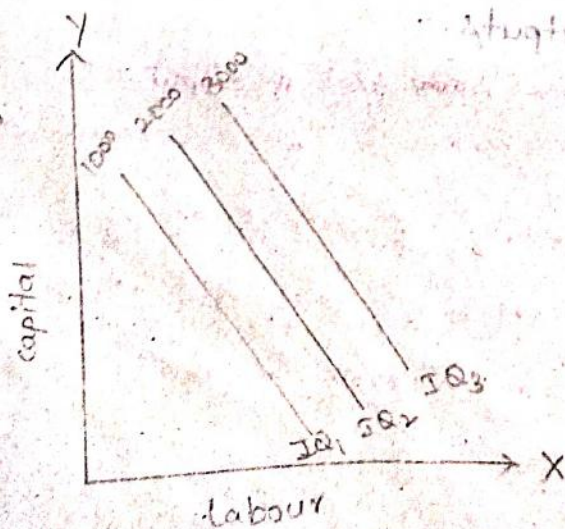
#### 1. Downward sloping :

Isoquants are downward sloping because if one input increases, the other one is decreases. There is no question of increasing in both in capital and labour to yield a given output.



#### 2. convex shape / convex origin :

Isoquants are convex to the origin. It is because the input factors are not a perfect substitutions. One input factor can be substituted by the another input factor at a decreasing marginal rate.





3. Don't touch the axes:

The Isoquants touches neither x-axis nor y-axis as both inputs are required to produce a given product

4. Donot Intersect:

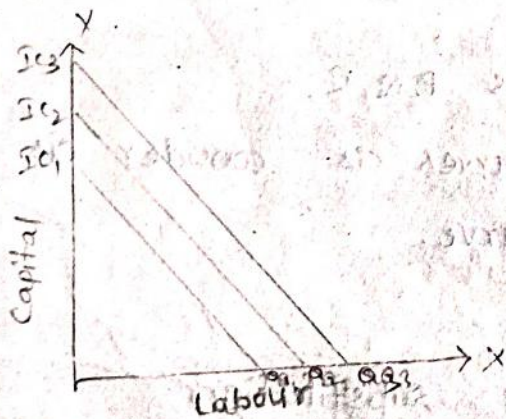
Two Isoquants donot intersect with each other because of each of these denotes a particular level of output.

Iso cost:

Iso means equal, Iso cost refers the combination of inputs that will cost the producer the same amount of money.

→ If the level of production changes total cost changes so that the Iso cost curves moves upwards as well as downwards.

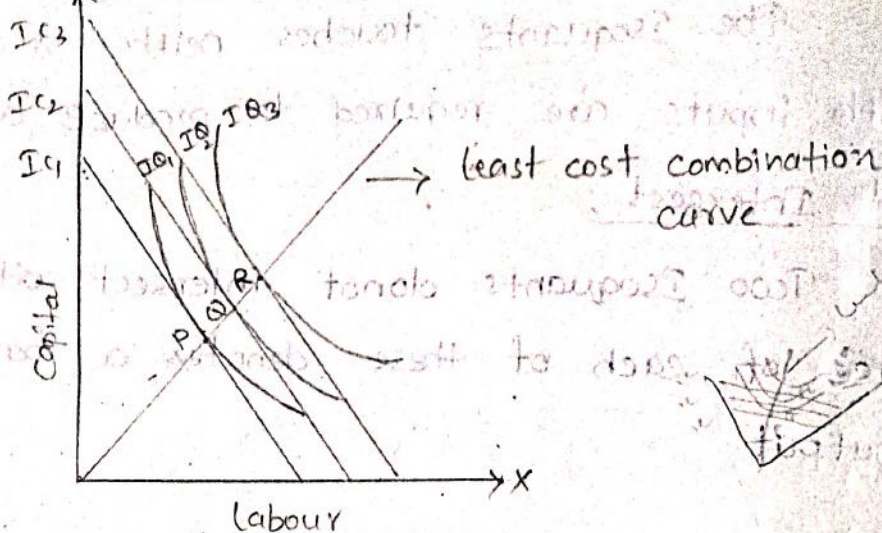
→ Iso cost curves downwards sloping with straight line.



- $IC_1$  = Iso cost curve 1
- $IC_2$  = Iso cost curve 2
- $IC_3$  = Iso cost curve 3



## Least cost combination :



- 1) The manufacture has to produce at lower cost to attain higher profits.
- 2) The Iso cost & Isoquants can be used to determine the input usage that minimize the cost of production.
- 3) where the slope of Isoquants equal to Isocost, there lies the lowest point of the cost of the production.
- 4) This can be observed by super imposing of Iso cost and Isoquant curves
- 5) The points are identified as P, Q, R
- 6) The combination of all curves is consider as least cost combination curve.

## MRTS :

Marginal Rate of Technical Substitution

Combination	Capital	Labour	MRTS
A	1	20	<del>20:1</del>
B	2	15	$(20-15=5)$ 5:1
C	3	11	4:1
D	4	8	3:1
E	5	6	2:1
F	6	5	1:1



MRTS refers the rate at which one input factor is substituted with another to attain given level of output. In other words, lesser units of one input must be compensated by increasing amount of another input to produce the same level of output.

$$MRTS = \frac{\text{change in one input}}{\text{change in another input}} = \frac{\Delta K - \text{capital}}{\Delta L - \text{labour}}$$

In the above table, the ratio of MRTS between the two input factors Those are capital & labour. 5 units of decreasing in labour is compensated by increasing in 1 unit of capital.

So that,  $MRTS = 5:1$

25/11/17

★ Cobb-Douglas production Function:

Cobb & Douglas <sup>put forth a</sup> production function <sup>relating</sup> output in <sup>American</sup> manufacturing American Industries. It is derived in 1899-1922 to labour and capital inputs.

A/c to Cobb-Douglas production function

$$P = b \cdot L^a \cdot C^{1-a}$$

where,  $P$  = Cobb-Douglas production function output

$b$  = Production constant

$L$  = The index of employment labours of the production

$C$  = The index of the capital used in production

$a, 1-a$  = Are consider as elasticity changes in both capital & labour

After insert all values by American manufacturing industries. The function estimated by Cobb & Douglas

$$P = 1.01 \cdot L^{0.75} \cdot C^{0.25}$$

$$R^2 = 0.9409$$

where,  $R$  = Related coefficient values



The production function shows that one percent change in labour input at the time capital (C) constant is associated with 0.75% change in production output.

The production function shows that one percent change in capital input at the time labour (L) constant is associated with 0.25% change in production output.

The coefficient determination  $R^2$  means that 94% variations on the dependent variable (P) were accounted for by the variations means both capital (C) & labour (L)

(Independent variable).

Returns to economics of scale:

After investment of the money in any organisation. For firms gets certain returns, returns may be positive or negative.

The positive result of the organisation should be considered as profits of the firm. It explains the behaviour of the returns when inputs are changed. It is also considered as Law of return to scale.

Classification of return to scale:

1. Law of Increasing return to scale:
2. Law of constant return to scale
3. Law of Decreasing return to scale

1. Law of Increasing return to scale:



capital	Labour	Percentage increase in capital	Percentage increase in Labour	output (Assumed)	% ↑ output	
1	3	-	-	-	-	
2	6	100	100	120	140	LIRS
4	12	100	100	240	100	LCRS
8	24	100	100	360	50	LDRS
16	48	100	100	480	-	

1. Law of Increasing return to scale:  
 It states that the volume of outputs keeps on increasing with every increasing in the inputs where a given increase in the inputs leads to a more than proportional change in output.

2. Law of constant return to scale:  
 It states that the rate of increasing inputs is almost are equal to the rate of proportional output of the organisation.

3. Law of decreasing return to scale:  
 It states that the proportional increasing in the inputs doesn't lead to equivalent increase in the output. The output increase at decreasing rate.



From the above table, it is clear that with 1 unit of capital and 3 units of labour, the firm produces 50 units of output. When the inputs are doubled two units of capital and six units of labour, the output has gone up to 120 units. (From 50 units to 120 units). Thus, when inputs are increased by 100 percent, the output has increased by 140 percent. That is, output has increased by more than double. This is governed by *Law of Increasing Returns to Scale*.

When the inputs are further doubled that is to 4 units of capital and 12 units of labour, the output has gone up to 240 units. (from 120 units to 240 units). Thus, when inputs are increased by 100 per cent, the output has increased by 100 per cent. That is, output also has doubled. This is governed by *Law of Constant Returns to Scale*.

When the inputs are further doubled, that is, to 8 units of capital and 24 units of labour, the output has gone up to 360 units. (from 240 units to 360 units). Thus, when input are increased by 100 per cent, the has increased only by 50 per cent. This is governed by *Law of Decreasing Returns to Scale*.



## Economies of scale:

The Economies of scale result because of increase in the scale of production. Alfred Marshall, divides the EOS into two groups.

Internal Economics of the organisation effects on output of the organisation.

Acc to Alfred Marshall, economies of scale mainly divided into two types.

1. Internal Economies of scale
2. External Economies of scale

### 1. Internal Economies of scale?

Internal Economics denotes, the economics in the production cost and their effects which are occurred to the firms alone when it expands its output of the production.

MCF - MTR - RL

#### 1) Managerial Economics:

Every firm must be need qualified managerial persons to handling & monitoring the all the activities of firm functions like finance, marketing, production, operation and R & D departments.

All the departments work together in order to increase the productivity of organisation. Specialisation in the departments will always emphasise on reduction of wastages.

#### 2) Commercial Economics:

Commercial Economics means the transaction of buying and selling of raw materials, such as <sup>Spare</sup> Square parts and soon. The procurement of raw materials cost will be directly effective on production cost and this will be leads to increase the profits of the organisation.



### 3) Financial Economics:

Financial Economics denotes there could be a cheaper credit facilities from the financial institution to meet the capital expenditure or working capital requirements. A large scale firms has larger asights to give security to the financial institutions (Banks), which can be consider increase the loan capacity and reduce the rate of intrest.

### 4) Technical Economics:

Increase the sales and production follows when there is sophisticated Technology available and the firm is in a position to hire qualified technical manpower (Employees) to make use of it.

### 5) Marketing Techno Economics:

As a firms grows larger and larger, it can be offered to maintain fullfledged marketing system or department. The marketing department handle issues related to design of the product, promotional related activities and gathering information from the customers by customer service

### 6) Risk bearing Economics:

As there is a growth in the size of the firm. There is increase in risk also. Most of the business organisations profits will be depends on how much risk they are ready to face and also depends on their risk oriented decision

### 1) Economics of R&D:

Large scale organisations are maintained Research & development departments in their organisations. It is requires high in disturbance for the purpose of to introduce new innovative & invention products. These products are (used for) help for sustain in the competitive environment.

### 2) Economics of Larger Dimension:



## 2. External Economics of scale:

### 1) Economics of concentration:

Economics of concentration refers, <sup>if</sup> all firms are located in one place, it is likely that there is a better in first infrastructure in terms of road construction, Transportation, Electricity, Banking & communication facilities and so on.

These economics will helpful to the organisation (individually) reduction of infrastructure.

### 2) Economics of welfare:

Economics of welfare refers, there could be a common facilities such as canteen, Industrial housing, Hospitals, school & colleges and so on, which can be used in common by the employees in the industry.

### 3) Economics of R&D:

Most of the firms maintained R&D department to face the competitions, but <sup>some</sup> small scale & medium scale industries are unable to maintain R&D departments. So that there are depends on large scale R&D departments.

## Constant Elasticity of Substitution:

Constant elasticity of substitution implies that any change in input factor results constant change in outputs. If a constant elasticity of substitution is constant and may not necessary equal to one (or) unity.

The constant elasticity of substitution product function

$$Q = \frac{A [\alpha K - \phi + (1 - \alpha) L - \phi] - 1}{\phi}$$



where,  $\alpha$  = constant elasticity of production function  
 $A$  = Efficiency parameter.  
 $\alpha$  = Distribution parameter  
 $\phi$  = substitution parameter, denotes elasticity of substitution.

NOTE: The homogeneity of production function can be determined by the values of substitution parameter  $\phi$ . If it is equal to 1, there should be considered as a linear homogeneous. That means proportional change in input factor result in same proportions of output.