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CAPITAL ASSET PRICING MODEL (CAPM)

The capital asset pricing model was developed in mid-1960s by three researchers William Sharpe, John Lintner and Jan Mossin independently. Consequently, the model is often referred to as Sharpe-Lintner-Mossin Capital Asset Pricing Model.

The capital asset pricing model or CAPM is really an extension of the portfolio theory of Markowitz. The portfolio theory is a description of how rational investors should build efficient portfolios and select the optimal portfolio. The capital asset pricing model derives the relationship between the expected return and risk of individual securities and portfolios in the capital markets if everyone behaved in the way the portfolio theory suggested.

Let us, therefore, begin by summarising the fundamental notions of portfolio theory.

FUNDAMENTAL NOTIONS OF PORTFOLIO THEORY

Return and risk are two important characteristics of every investment. Investors base their investment decision on the expected return and risk of investments. Risk is measured by the variability in returns.

Investors attempt to reduce the variability of returns through diversification of investment. This results in the creation of a portfolio. With a given set of securities, any number of portfolios may be created by altering the proportion of funds invested in each security. Among these portfolios some dominate others, or some are more efficient than the vast majority of portfolios because of lower risk or higher returns. Investors identify this efficient set of portfolios.

Diversification helps to reduce risk, but even a well diversified portfolio does not become risk free. If we construct a portfolio including all the securities in the stock market that would be the most diversified portfolio. Even such a portfolio would be subject to considerable variability. This variability is undiversifiable and is known as the market risk or **systematic risk** because it affects all the securities in the market.

The real risk of a security is the market risk which cannot be eliminated through diversification. This is indicated by the sensitivity of a security to the movements of the market and is measured by the beta coefficient of the security.

A rational investor would expect the return on a security to be commensurate with its risk. The higher the risk of a security, the higher would be the return expected from it. And since the relevant risk of a security is its market risk or systematic risk, the return is expected to be correlated with this risk only. The capital asset pricing model gives the nature of the relationship between the expected return and the systematic risk of a security.

ASSUMPTIONS OF CAPM

The capital asset pricing model is based on certain explicit assumptions regarding the behaviour of investors. The assumptions are listed below:

1. Investors make their investment decisions on the basis of risk-return assessments measured in terms of expected returns and standard deviation of returns.
2. The purchase or sale of a security can be undertaken in infinitely divisible units.
3. Purchases and sales by a single investor cannot affect prices. This means that there is perfect competition where investors in total determine prices by their actions.
4. There are no transaction costs. Given the fact that transaction costs are small, they are probably of minor importance in investment decision-making, and hence they are ignored.
5. There are no personal income taxes. Alternatively, the tax rates on dividend income and capital gains are the same, thereby making the investor indifferent to the form in which the return on the investment is received (dividends or capital gains).
6. The investor can lend or borrow any amount of funds desired at a rate of interest equal to the rate for riskless securities.
7. The investor can sell short any amount of any shares.
8. Investors share homogeneity of expectations. This implies that investors have identical expectations with regard to the decision period and decision inputs. Investors are presumed to have identical holding periods and also identical expectations regarding expected returns, variances of expected returns and covariances of all pairs of securities.

It is true that many of the above assumptions are untenable. However, they do not materially alter the real world. Moreover, the model describes the risk return relationship and the pricing of assets fairly well.

EFFICIENT FRONTIER WITH RISKLESS LENDING AND BORROWING

The portfolio theory deals with portfolios of risky assets. According to the theory, an investor faces an efficient frontier containing the set of efficient portfolios of risky assets.

Now it is assumed that there exists a riskless asset available for investment. A riskless asset is one whose return is certain such as a government security. Since the return is certain, the variability of return or risk is zero. The investor can invest a portion of his funds in the riskless asset which would be equivalent to lending at the risk free asset's rate of return, namely R_f. He would then be investing in a combination of risk free asset and risky assets.

Similarly, it may be assumed that an investor may borrow at the same risk free rate for the purpose of investing in a portfolio of risky assets. He would then be using his own funds as well as some borrowed funds for investment.

The efficient frontier arising from a feasible set of portfolios of risky assets is concave in shape. When an investor is assumed to use riskless lending and borrowing in his investment activity the shape of the efficient frontier transforms into a straight line. Let us see how this happens.

Consider Fig. 15.1. The concave curve *ABC* represents an efficient frontier of risky portfolios. *B* is the optimal portfolio in the efficient frontier with $R_p = 15$ per cent and $\sigma_p = 8$ per cent. A risk free asset with rate of return $R_f = 7$ per cent is available for investment. The risk or standard deviation of this asset would be zero because it is a riskless asset. Hence, it would be plotted on the *Y* axis. The investor may lend a part of his money at the riskless rate, i.e. invest in the risk free asset and invest the remaining portion of his funds in a risky portfolio.

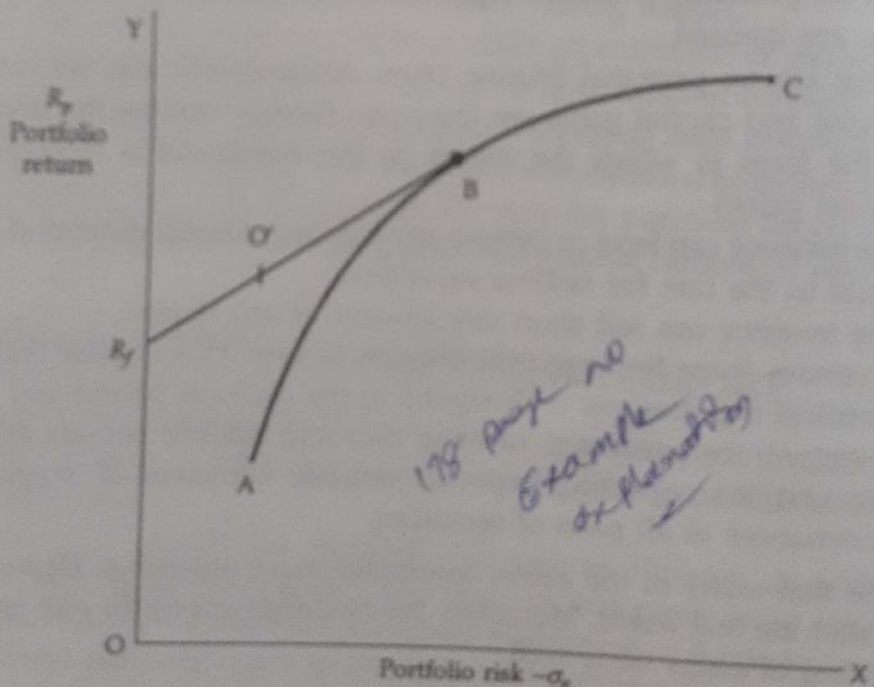

Fig. 15.1 Efficient frontier with introduction of lending.

If an investor places 40 per cent of his funds in the riskfree asset and the remaining 60 per cent in portfolio *B*, the return and risk of this combined portfolio *O'* may be calculated using the following formulas.

Return

$$R_c = \omega R_m + (1 - \omega)R_f$$

where

R_c = Expected return on the combined portfolio.
ω = Proportion of funds invested in risky portfolio.
$(1 - \omega)$ = Proportion of funds invested in riskless asset.
R_m = Expected return on risky portfolio.
R_f = Rate of return on riskless asset.

Risk

$$\sigma_c = \omega\sigma_m + (1 - \omega)\sigma_f$$

where

σ_c = Standard deviation of the combined portfolio.
ω = Proportion of funds invested in risky portfolio.
σ_m = Standard deviation of risky portfolio.
σ_f = Standard deviation of riskless asset.

The second term on the right hand side of the equation, $(1 - \omega)\sigma_f$ would be zero as σ_f = zero. Hence, the formula may be reduced as

$$\sigma_c = \omega\sigma_m$$

The return and risk of the combined portfolio in our illustration is worked out below:

$$R_c = (0.60)(15) + (0.40)(7)$$
$$= 11.8 \text{ per cent}$$
$$\sigma_c = (0.60)\ (8) = 4.8 \text{ per cent}$$

Both return and risk are lower than those of the risky portfolio B.

If we change the proportion of investment in the risky portfolio to 75 per cent, the return and risk of the combined portfolio may be calculated as shown below:

$$R_c = (0.75)(15) + (0.25)(7)$$
$$= 13 \text{ per cent}$$
$$\sigma_c = (0.75)(8) = 6 \text{ per cent}$$

Here again, both return and risk are lower than those of the risky portfolio B.

Similarly, the return and risk of all possible combinations of the riskless asset and t risky portfolio *B* may be worked out. All these points will lie in the straight line from to *B* in Fig. 15.1.

Now, let us consider borrowing funds by the investor for investing in the risky portf an amount which is larger than his own funds.

If ω is the proportion of investor's funds invested in the risky portfolio, then we envisage three situations. If $\omega = 1$, the investor's funds are fully committed to the portfolio. If $\omega < 1$, only a fraction of the funds is invested in the risky portfolio an remainder is lend at the risk free rate. If $\omega > 1$, it means the investor is borrowing risk free rate and investing an amount larger than his own funds in the risky por

The return and risk of such a levered portfolio can be calculated as follows:

$$R_L = \omega R_m - (\omega - 1)R_f$$

where

R_L = Return on the levered portfolio.

ω = Proportion of investor's funds invested in the risky portfolio.

R_m = Return on the risky portfolio.

R_f = The risk free borrowing rate which would be the same as the risk free lending rate, namely the return on the riskless asset.

The first term of the equation represents the gross return earned by investing the borrowed funds as well as investor's own funds in the risky portfolio. The second term of the equation represents the cost of borrowing funds which is deducted from the gross returns to obtain the net return on the levered portfolio.

The risk of the levered portfolio can be calculated as:

$$\sigma_L = \omega \sigma_m$$

The return and risk of the investor in our illustration may be calculated assuming $\omega = 1.25$

$$R_L = (1.25)(15) - (0.25)(7)$$
$$= 17 \text{ per cent}$$
$$\sigma_L = (1.25)(8)$$
$$= 10 \text{ per cent}$$

The return and risk of the levered portfolio are larger than those of the risky portfolio. The levered portfolio would give increased returns with increased risk. The return and risk of all levered portfolios would lie in a straight line to the right of the risky portfolio B. This is depicted in Fig. 15.2.

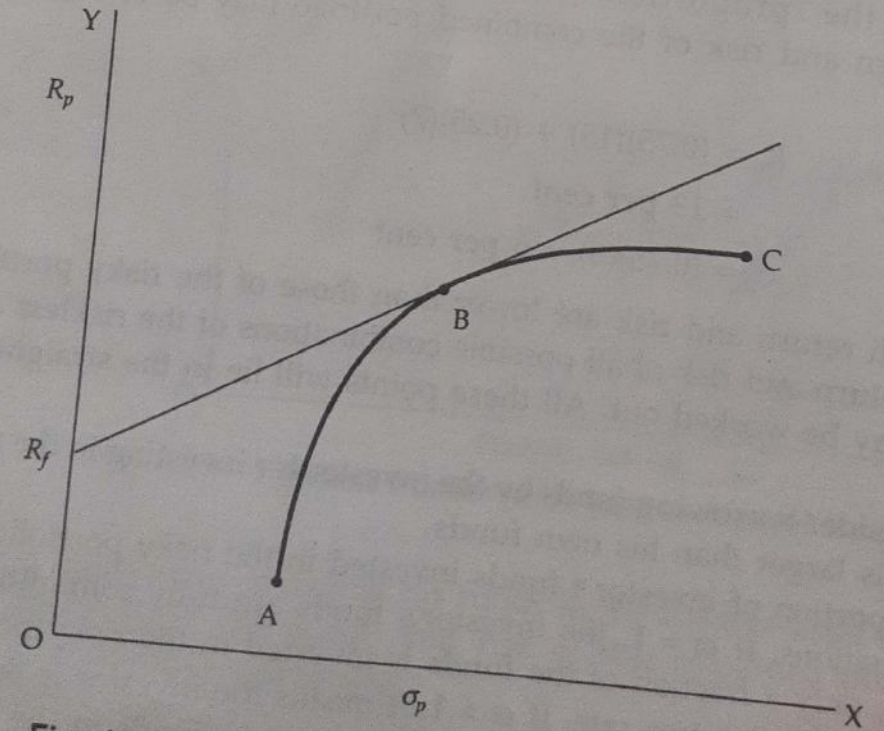

Fig. 15.2 Efficient frontier with borrowing and lending

Thus, the introduction of borrowing and lending gives us an efficient frontier that is a straight line throughout. This line sets out all the alternative combinations of the risky portfolio B with risk free borrowing and lending.

The line segment from R_f to B includes all the combinations of the risky portfolio and the risk free asset. The line segment beyond point B represents all the levered portfolios (that is combinations of the risky portfolio with borrowing). Borrowing increases both the expected return and the risk, while lending (that is, combining the risky portfolio with risk free asset) reduces the expected return and risk. Thus, the investor can use borrowing or lending to attain the desired risk level. Those investors with a high risk aversion will prefer to lend and thus, hold a combination of risky assets and the risk free asset. Others with less risk aversion will borrow and invest more in the risky portfolio.

THE CAPITAL MARKET LINE

All investors are assumed to have identical (homogeneous) expectations. Hence, all of them will face the same efficient frontier depicted in Fig. 15.2. Every investor will seek to combine the same risky portfolio B with different levels of lending or borrowing according to his desired level of risk. Because all investors hold the same risky portfolio, then it will include all risky securities in the market. This portfolio of all risky securities is referred to as the market portfolio M. Each security will be held in the proportion which the market value of the security bears to the total market value of all risky securities in the market. All investors will hold combinations of only two assets, the market portfolio and a riskless security.

All these combinations will lie along the straight line representing the efficient frontier. This line formed by the action of all investors mixing the market portfolio with the risk free asset is known as the **capital market line (CML)**. All efficient portfolios of all investors will lie along this capital market line.

The relationship between the return and risk of any efficient portfolio on the capital market line can be expressed in the form of the following equation.

$$\bar{R}_e = R_f + \left[\frac{\bar{R}_m - R_f}{\sigma_m}\right]\sigma_e$$

where the subscript e denotes an efficient portfolio.

The risk free return R_f represents the reward for waiting. It is, in other words, the price of time. The term $[(\bar{R}_m - R_f)/\sigma_m]$ represents the price of risk or risk premium, i.e. the excess return earned per unit of risk or standard deviation. It measures the additional return for an additional unit of risk. When the risk of the efficient portfolio, σ_e, is multiplied with this term, we get the risk premium available for the particular efficient portfolio under consideration.

Thus, the expected return on an efficient portfolio is:

(Expected return) = (Price of time) + (Price of risk) (Amount of risk) return of the portfolio. There is a linear relationship between the risk as measured by the standard deviation and the expected return for these efficient portfolios.

THE SECURITY MARKET LINE

The CML shows the risk-return relationship for all efficient portfolios. They would all lie along the capital market line. All portfolios other than the efficient ones will lie below the capital market line. The CML does not describe the risk-return relationship of inefficient portfolios or of individual securities. The capital asset pricing model specifies the relationship between expected return and risk for all securities and all portfolios, whether efficient or inefficient.

We have seen earlier that the total risk of a security as measured by standard deviation is composed of two components: systematic risk and unsystematic risk or diversifiable risk. As investment is diversified and more and more securities are added to a portfolio, the unsystematic risk is reduced. For a very well diversified portfolio, unsystematic risk tends to become zero and the only relevant risk is systematic risk measured by beta (β). Hence, it is argued that the correct measure of a security's risk is beta.

It follows that the expected return of a security or of a portfolio should be related to the risk of that security or portfolio as measured by β. Beta is a measure of the security's sensitivity to changes in market return. Beta value greater than one indicates higher sensitivity to market changes, whereas beta value less than one indicates lower sensitivity to market changes. A β value of one indicates that the security moves at the same rate and in the same direction as the market. Thus, the β of the market may be taken as one.

The relationship between expected return and β of a security can be determined graphically. Let us consider an *XY* graph where expected returns are plotted on the *Y* axis and beta coefficients are plotted on the *X* axis. A risk free asset has an expected return equivalent to R_f and beta coefficient of zero. The market portfolio *M* has a beta coefficient of one and expected return equivalent to $\bar{R}_m$. A straight line joining these two points is known as the **security market line (SML)**. This is illustrated in Fig. 15.3.

The security market line provides the relationship between the expected return and beta of a security or portfolio. This relationship can be expressed in the form of the following equation:

$$\bar{R}_i = R_f + \beta_i(\bar{R}_m - R_f)$$

A part of the return on any security or portfolio is a reward for bearing risk and the rest is the reward for waiting, representing the time value of money. The risk free rate, R_f (which is earned by a security which has no risk) is the reward for waiting. The reward for bearing risk is the risk premium. The risk premium of a security is directly proportional to the risk as measured by β. The risk premium of a security is calculated as the product of beta and the risk premium of the market which is the excess of expected market return over the risk free return, that is, $[\bar{R}_m - R_f]$. Thus,

Expected return on a security = Risk free return + (Beta × Risk premiu[illegible]rket)

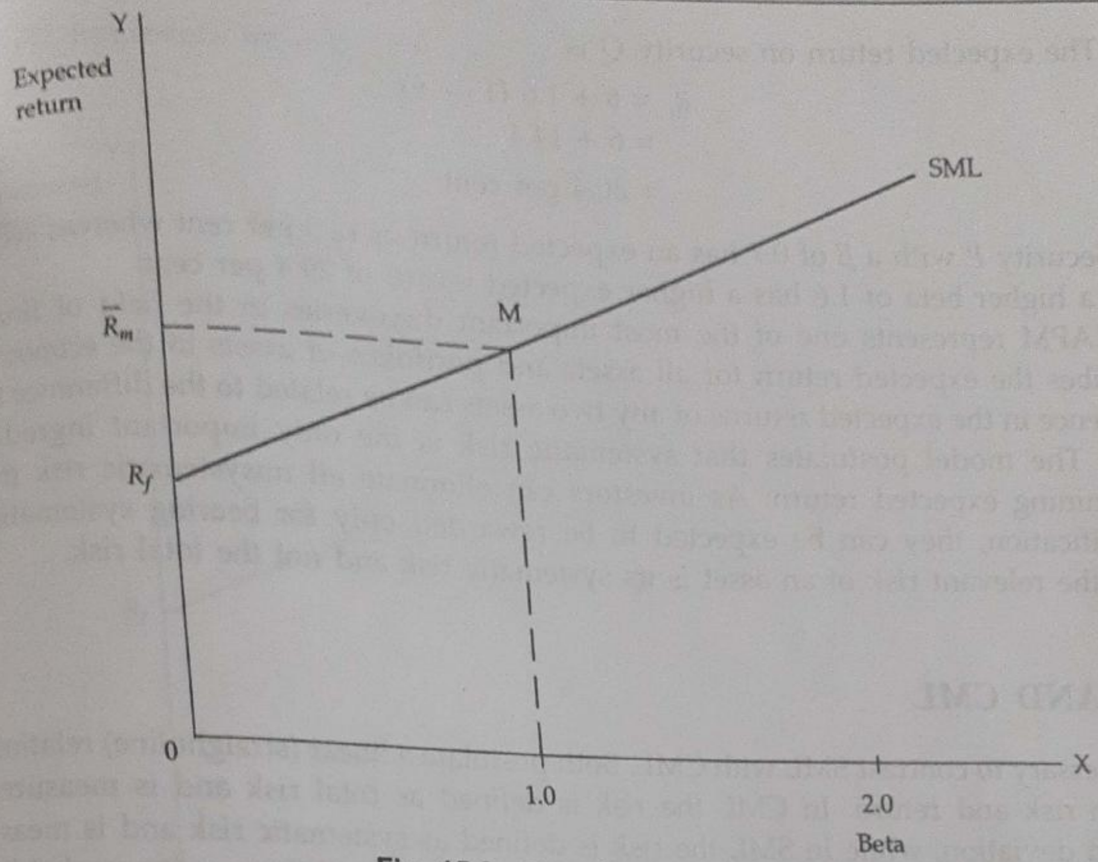

Fig. 15.3 Security market line.

CAPM

The relationship between risk and return established by the security market line is known as the **capital asset pricing model**. It is basically a simple linear relationship. The higher the value of beta, higher would be the risk of the security and therefore, larger would be the return expected by the investors. In other words, all securities are expected to yield returns commensurate with their riskiness as measured by β. This relationship is valid not only for individual securities, but is also valid for all portfolios whether efficient or inefficient.

The expected return on any security or portfolio can be determined from the CAPM formula if we know the beta of that security or portfolio. To illustrate the application of the CAPM, let us consider a simple example. There are two securities *P* and *Q* having values of beta as 0.7 and 1.6 respectively. The risk free rate is assumed to be 6 per cent and the market return is expected to be 15 per cent, thus providing a market risk premium of 9 per cent (i.e. $\overline{R}_m - R_f$).

The expected return on security *P* may be worked out as shown below:

$$\begin{aligned}\overline{R}_i &= R_f + \beta_i[\overline{R}_m - R_f] \\ &= 6 + 0.7\ (15 - 6) \\ &= 6 + 6.3 = 12.3 \text{ per cent}\end{aligned}$$

The expected return on security Q is

$$\bar{R}_i = 6 + 1.6\ (15 - 6)$$
$$= 6 + 14.4$$
$$= 20.4 \text{ per cent}$$

Security P with a β of 0.7 has an expected return of 12.3 per cent whereas security Q with a higher beta of 1.6 has a higher expected return of 20.4 per cent.

CAPM represents one of the most important discoveries in the field of finance. It describes the expected return for all assets and portfolios of assets in the economy. The difference in the expected returns of any two assets can be related to the difference in their betas. The model postulates that systematic risk is the only important ingredient in determining expected return. As investors can eliminate all unsystematic risk through diversification, they can be expected to be rewarded only for bearing systematic risk. Thus, the relevant risk of an asset is its systematic risk and not the total risk.

SML AND CML

It is necessary to contrast SML with CML. Both postulate a linear (straight line) relationship between risk and return. In CML the risk is defined as total risk and is measured by standard deviation, while in SML the risk is defined as systematic risk and is measured by β. Capital market line is valid only for efficient portfolios while security market line is valid for all portfolios and all individual securities as well. CML is the basis of the capital market theory while SML is the basis of the capital asset pricing model.

PRICING OF SECURITIES WITH CAPM

The capital asset pricing model can also be used for evaluating the pricing of securities. The CAPM provides a framework for assessing whether a security is underpriced, overpriced or correctly priced. According to CAPM, each security is expected to provide a return commensurate with its level of risk. A security may be offering more returns than the expected return, making it more attractive. On the contrary, another security may be offering less return than the expected return, making it less attractive.

The expected return on a security can be calculated using the CAPM formula. Let us designate it as the theoretical return. The real rate of return estimated to be realised from investing in a security can be calculated by the following formula:

$$R_i = \frac{(P_1 - P_0) + D_1}{P_0}$$

where

P_0 = Current market price.
P_1 = Estimated market price after one year.
D_1 = Anticipated dividend for the year.

This may be designated as the estimated return.