14

PORTFOLIO SELECTION

The objective of every rational investor is to maximise his returns and minimise the risk. Diversification is the method adopted for reducing risk. It essentially results in the construction of portfolios. The proper goal of portfolio construction would be to generate a portfolio that provides the highest return and the lowest risk. Such a portfolio would be known as the **optimal portfolio.** The process of finding the optimal portfolio is described as **portfolio selection.**

The conceptual framework and analytical tools for determining the optimal portfolio in disciplined and objective manner have been provided by Harry Markowitz in his pioneering work on portfolio analysis described in his 1952 *Journal of Finance* article[1] and subsequent book[2] in 1959. His method of portfolio selection has come to be known as the **Markowitz** model. In fact, Markowitz's work marks the beginning of what is known today as **modern portfolio theory**.

FEASIBLE SET OF PORTFOLIOS

With a limited number of securities an investor can create a very large number of portfolios by combining these securities in different proportions. These constitute the feasible set of portfolios in which the investor can possibly invest. This is also known as the **portfolio opportunity set.**

Each portfolio in the opportunity set is characterised by an expected return and a measure of risk, viz., variance or standard deviation of returns. Not every portfolio in the portfolio opportunity set is of interest to an investor. In the opportunity set some portfolios will obviously be dominated by others. A portfolio will dominate another if it has either a lower standard deviation and the same expected return as the other, or a higher expected return and the same standard deviation as the other. Portfolios that are dominated by other portfolios are known as *inefficient* portfolios. An investor would not be interested in all the portfolios in the opportunity set. He would be interested only in the efficient portfolios.

Efficient Set of Portfolios

To understand the concept of efficient portfolios, let us consider various combinations of securities and designate them as portfolios 1 to *n*. The expected returns of these portfolios may be worked out. The risk of these portfolios may be estimated by measuring the standard deviation of portfolio returns. The table below shows illustrative figures for the expected returns and standard deviations of some portfolios.

Portfolio no.	*Expected return (per cent)*	*Standard deviation (Risk)*
1	5.6	4.5
2	7.8	5.8
3	9.2	7.6
4	10.5	8.1
5	11.7	8.1
6	12.4	9.3
7	13.5	9.5
8	13.5	11.3
9	15.7	12.7
10	16.8	12.9

If we compare portfolio nos. 4 and 5, for the same standard deviation of 8.1 portfolio no. 5 gives a higher expected return of 11.7, making it more efficient than portfolio no. 4. Again, if we compare portfolio nos. 7 and 8, for the same expected return of 13.5 per cent, the standard deviation is lower for portfolio no. 7, making it more efficient than portfolio no. 8. Thus, the selection of portfolios by the investor will be guided by two criteria:

1. Given two portfolios with the same expected return, the investor would prefer the one with the lower risk.
2. Given two portfolios with the same risk, the investor would prefer the one with the higher expected return.

These criteria are based on the assumption that investors are rational and also risk-averse. As they are rational they would prefer more return to less return. As they are risk averse, they would prefer less risk to more risk.

The concept of efficient sets can be illustrated with the help of a graph. The expected return and standard deviation of portfolios can be depicted on an *XY* graph, measuring the expected return on the *Y* axis and the standard deviation on the X axis. Figure 14.1 depicts such a graph.

As each possible portfolio in the opportunity set or feasible set of portfolios has an expected return and standard deviation associated with it, each portfolio would be represented by a single point in the risk-return space enclosed within the two axes of the graph. The shaded area in the graph represents the set of all possible portfolios that can be constructed from a given set of securities. This opportunity set of portfolios takes a concave shape because it consists of portfolios containing securities that are less than perfectly correlated with each other.

Let us closely examine the diagram in Fig. 14.1. Consider portfolios *F* and *E*. Both the portfolios have the same expected return but portfolio *E* has less risk. Hence, portfolio *E* would be preferred to portfolio *F*. Now consider portfolios *C* and *E*. Both have the same risk, but portfolio *E* offers more return for the same risk. Hence, portfolio *E* would be preferred to portfolio *C*. Thus, for any point in the risk-return space, an investor would like to move as far as possible in the direction of increasing returns and also as far as possible in the direction of decreasing risk. Effectively, he would be moving towards the left in search of decreasing risk and upwards in search of increasing returns.

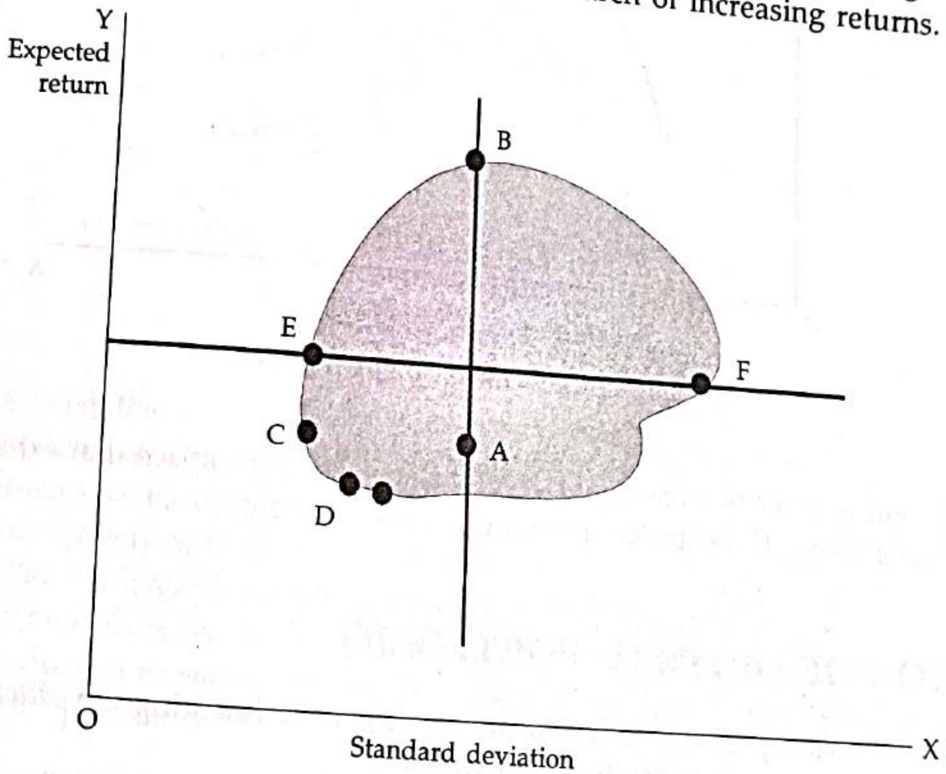

Fig. 14.1 Feasible set of portfolios.

Let us consider portfolios *C* and *A*. Portfolio *C* would be preferred to portfolio *A* because it offers less risk for the same level of return. In the opportunity set of portfolios represented in the diagram, portfolio *C* has the lowest risk compared to all other portfolios. Here portfolio *C* in this diagram represents the **global minimum variance portfolio**.

Comparing portfolios *A* and *B*, we find that portfolio *B* is preferable to portfolio *A* because it offers higher return for the same level of risk. In this diagram, point *B* represents the portfolio with the highest expected return among all the portfolios in the feasible set.

Thus, we find that portfolios lying in the north west boundary of the shaded area are more efficient than all the portfolios in the interior of the shaded area. This boundary of the shaded area is called the **Efficient Frontier** because it contains all the efficient portfolios in the opportunity set. The set of portfolios lying between the global minimum variance portfolio and the maximum return portfolio on the efficient frontier represents the efficient set of portfolios. The efficient frontier is shown separately in Fig. 14.2.

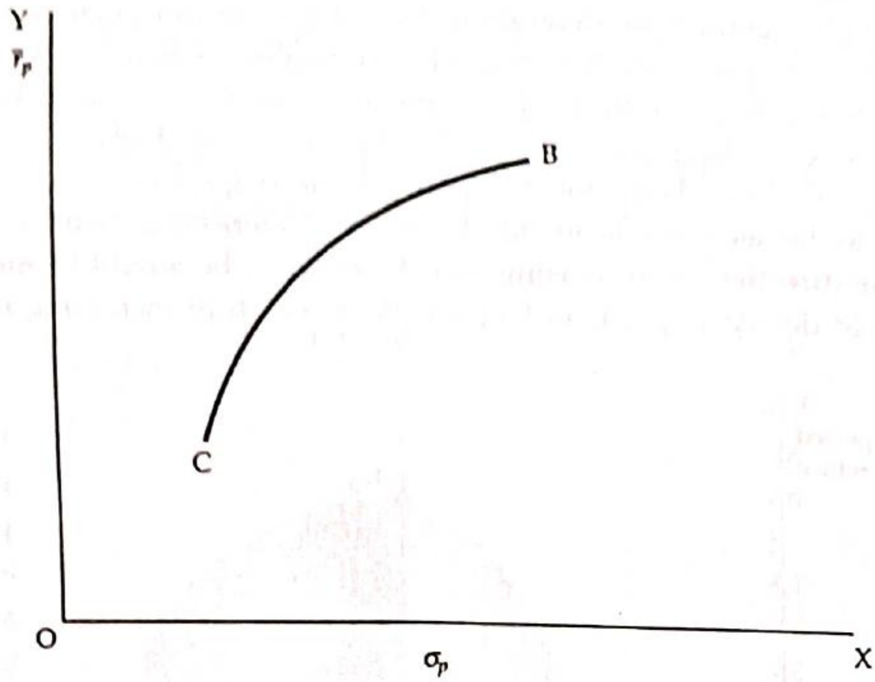

Fig. 14.2 The efficient frontier.

The efficient frontier is a concave curve in the risk-return space that extends from the minimum variance portfolio to the maximum return portfolio.

SELECTION OF OPTIMAL PORTFOLIO

The portfolio selection problem is really the process of delineating the efficient portfolios and then selecting the best portfolio from the set.

Rational investors will obviously prefer to invest in the efficient portfolios. The particular portfolio that an individual investor will select from the efficient frontier will depend on that investor's degree of aversion to risk. A highly risk averse investor will hold a portfolio on the lower left hand segment of the efficient frontier, while an investor who is not too risk averse will hold one on the upper portion of the efficient frontier.

The selection of the optimal portfolio thus depends on the investor's risk aversion, or conversely on his risk tolerance. This can be graphically represented through a series of risk return utility curves or indifference curves. The indifference curves of an investor are shown in Fig. 14.3. Each curve represents different combinations of risk and return all of which are equally satisfactory to the concerned investor. The investor is indifferent between the successive points in the curve. Each successive curve moving upwards to the left represents a higher level of satisfaction or utility. The investor's goal would be to maximise his utility by moving upto the higher utility curve. The optimal portfolio for an investor would be the one at the point of tangency between the efficient frontier and his risk-return utility or indifference curve.

This is shown in Fig. 14.3. The point O′ represents the optimal portfolio.

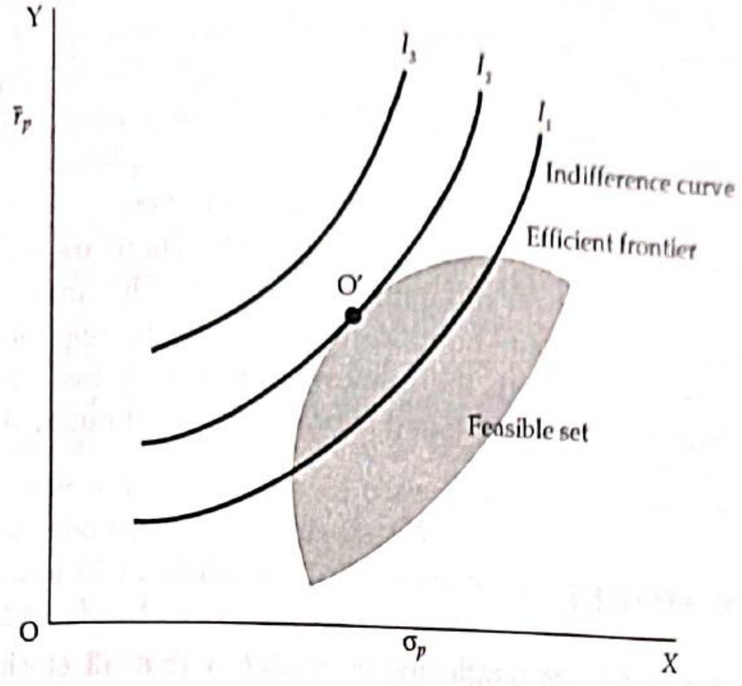

Fig. 14.3 Optimal portfolio.

Markowitz used the technique of quadratic programming to identify the efficient portfolios. Using the expected return and risk of each security under consideration and the covariance estimates for each pair of securities, he calculated risk and return for all possible portfolios. Then, for any specific value of expected portfolio return, he determined the least risk portfolio using quadratic programming. With another value of expected portfolio return, a similar procedure again gives the minimum risk portfolio. The process is repeated with different values of expected return, the resulting minimum risk portfolios constitute the set of efficient portfolios.

LIMITATIONS OF MARKOWITZ MODEL

One of the main problems with the Markowitz model is the large number of input data required for calculations. An investor must obtain estimates of return and variance of returns for all securities as also covariances of returns for each pair of securities included in the portfolio. If there are N securities in the portfolio, he would need N return estimates, N variance estimates and $N(N-1)/2$ covariance estimates, resulting in a total of $2N + [N(N-1)/2]$ estimates. For example, analysing a set of 200 securities would require 200 return estimates, 200 variance estimates and 19,900 covariance estimates, adding upto a total of 20,300 estimates. For a set of 500 securities, the estimates required would be 1,25,750. It may be noted that the number of estimates required becomes large because covariances between each pair of securities have to be estimated.

The second difficulty with the Markowitz model is the complexity of computation required. The computations required are numerous and complex in nature. With a given set of securities infinite number of portfolios can be constructed. The expected returns and variances of returns for each possible portfolio have to be computed. The identification of efficient portfolios requires the use of quadratic programming which is a complex procedure.

Because of the difficulties associated with the Markowitz model, it has found little use in practical applications of portfolio analysis. Much simplification is needed before the theory can be used for practical applications. Simplification is needed in the amount and type of input data required to perform portfolio analysis; simplification is also needed in the computational procedure used to select optimal portfolios.

The simplification is achieved through **index models**. There are essentially two types of index models: single index model and multi-index model. The single index model is the simplest and the most widely used simplification and may be regarded as being at one extreme point of a continuum, with the Markowitz model at the other extreme point. Multi-index models may be placed at the mid region of this continuum of portfolio analysis techniques.

SINGLE INDEX MODEL

The basic notion underlying the single index model is that all stocks are affected by movements in the stock market. Casual observation of share prices reveals that when the market moves up (as measured by any of the widely used stock market indices), prices of most shares tend to increase. When the market goes down, the prices of most shares tend to decline. This suggests that one reason why security returns might be correlated and there is co-movement between securities, is because of a common response to market changes. This co-movement of stocks with a market index may be studied with the help of a simple linear regression analysis, taking the returns on an individual security as the dependent variable (R_i) and the returns on the market index (R_m) as the independent variable.

The return of an individual security is assumed to depend on the return on the market index. The return of an individual security may be expressed as:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

where

α_i = Component of security i's return that is independent of the market's performance.

R_m = Rate of return on the market index.

β_i = Constant that measures the expected change in R_i given a change in R_m.

e_i = Error term representing the random or residual return.

This equation breaks the return on a stock into two components, one part due to the market and the other part independent of the market. The beta parameter in the equation, β_i, measures how sensitive a stock's return is to the return on the market index. It indicates how extensively the return of a security will vary with changes in the market return. For example, if the β_i of a security is 2, then the return of the security is expected to increase by 20 per cent when the market return increases by 10 per cent. In this case, if the market return decreases by 10 per cent, the security return is expected to decrease by 20 per cent. For a security with β_i of 0.5, when the market return increases or decreases by 10 per cent, the security return is expected to increase or decrease by 5 per cent (that is 10 × 0.5). A beta coefficient greater than one would suggest greater responsiveness on the part of the stock in relation to the market and vice versa.

The alpha parameter α_i indicates what the return of the security would be when the market return is zero. For example, a security with an alpha of +3 per cent would earn 3 per cent return even when the market return is zero and it would earn an additional 3 per cent at all levels of market return. Conversely, a security with an alpha of −4.5 per cent would lose 4.5 per cent when the market return is zero, and would earn 4.5 per cent less at all levels of market return. The positive alpha thus represents a sort of bonus return and would be a highly desirable aspect of a security, whereas a negative alpha represents a penalty to the investor and is an undesirable aspect of a security.

The final term in the equation, e_i, is the unexpected return resulting from influences not identified by the model. It is referred to as the *random* or *residual* return. It may take on any value, but over a large number of observations it will average out to zero.

William Sharpe, who tried to simplify the data inputs and data tabulation required for the Markowitz model of portfolio analysis, suggested that a satisfactory simplification would be achieved by abandoning the covariance of each security with each other security and substituting in its place the relationship of each security with a market index as measured by the single index model suggested above. This is known as **Sharpe index model.**

In the place of $[N(N - 1)/2]$ covariances required for the Markowitz model, Sharpe model would requires only N measures of beta coefficients.

Measuring Security Return and Risk under Single Index Model

Using the single index model, expected return of an individual security may be expressed as:

$$\bar{R}_i = \alpha_i + \beta_i \bar{R}_m$$

The return of the security is a combination of two components: (a) a specific return component represented by the alpha of the security; and (b) a market related return component represented by the term $\beta_i \bar{R}_m$. The residual return disappears from the expression because its average value is zero, i.e. it has an expected value of zero.

Correspondingly, the risk of a security σ_i^2 becomes the sum of a market related component and a component that is specific to the security. Thus,

Total risk = Market related risk + Specific risk

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$$

where

σ_i^2 = Variance of individual security.

σ_m^2 = Variance of market index returns.

σ_{ei}^2 = Variance of residual returns of individual security.

β_i = Beta coefficient of individual security.

The market related component of risk is referred to as systematic risk as it affects all securities. The specific risk component is the unique risk or unsystematic risk which can be reduced through diversification. It is also called *diversifiable risk.*

The estimates of α_i, β_i and σ_{ei}^2 of a security are often obtained from regression analysis of historical data of returns of the security as well as returns of a market index. For any given or expected value of R_m, the expected return and risk of the security can be calculated. For example, if the estimated values of α_i, β_i and σ_{ei}^2 of a security are 2 per cent, 1.5 and 300 respectively and if the market index is expected to provide a return of 20 per cent, with variance of 120, the expected return and risk of the security can be calculated as shown below:

$$\bar{R}_i = \alpha_i + \beta_i \bar{R}_m$$
$$= 2 + 1.5\ (20) = 32 \text{ per cent}$$
$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$$
$$= (1.5)^2 (120) + 300$$
$$= 570$$

Measuring Portfolio Return and Risk under Single Index Model

Portfolio analysis and selection require as inputs the expected portfolio return and risk for all possible portfolios that can be constructed with a given set of securities. The return and risk of portfolios can be calculated using the single index model.

The expected return of a portfolio may be taken as portfolio alpha plus portfolio beta times expected market return. Thus,

$$\bar{R}_p = \alpha_p + \beta_p \bar{R}_m$$

The portfolio alpha is the weighted average of the specific returns (alphas) of the individual securities. Thus,

$$\alpha_p = \sum_{i=1}^{n} \omega_i \alpha_i$$

where

ω_i = Proportion of investment in an individual security.

α_i = Specific return of an individual security.

The portfolio beta is the weighted average of the Beta coefficients of the individual securities. Thus,

$$\beta_p = \sum_{i=1}^{n} \omega_i \beta_i$$

where

ω_i = Proportion of investment in an individual security.

β_i = Beta coefficient of an individual security.

The expected return of the portfolio is the sum of the weighted average of the specific returns and the weighted average of the market related returns of individual securities.

The risk of a portfolio is measured as the variance of the portfolio returns. The risk of a portfolio is simply a weighted average of the market related risks of individual securities plus a weighted average of the specific risks of individual securities in the portfolio. The portfolio risk may be expressed as:

$$\sigma_p^2 = \beta_p^2\sigma_m^2 + \sum_{i=1}^{n} \omega_i^2\sigma_{ei}^2$$

The first term constitutes the variance of the market index multiplied by the square of portfolio beta and represents the market related risk (or systematic risk) of the portfolio. The second term is the weighted average of the variances of residual returns of individual securities and represents the specific risk or unsystematic risk of the portfolio.

As more and more securities are added to the portfolio, the unsystematic risk of the portfolio becomes smaller and is negligible for a moderately sized portfolio. Thus, for a large portfolio, the residual risk or unsystematic risk approaches zero and the portfolio risk becomes equal to $\beta_p^2\sigma_m^2$. Hence, the effective measure of portfolio risk is β_p.

Let us consider a hypothetical portfolio of four securities. The table below shows the basic input data such as weightage, alphas, betas and residual variances of the individual securities required for calculating portfolio return and variance.

Input Data

Security	*Weightage* (ω_i)	*Alpha* (α_i)	*Beta* (β_i)	*Residual variance* ($\sigma_{e_i}^2$)
A	0.2	2.0	1.7	370
B	0.1	3.5	0.5	240
C	0.4	1.5	0.7	410
D	0.3	0.75	1.3	285
Portfolio value	1.0	1.575	1.06	108.45

The values of portfolio alpha, portfolio beta, and portfolio residual variance can be calculated as the first step.

$$\alpha_p = \sum_{i=1}^{n} \omega_i\alpha_i$$

$$= (0.2)\ (2) + (0.1)\ (3.5) + (0.4)\ (1.5) + (0.3)\ (0.75)$$

$$= 1.575$$

$$\beta_p = \sum_{i=1}^{n} \omega_i\beta_i$$

$$= (0.2)\ (1.7) + (0.1)\ (0.5) + (0.4)\ (0.7) + (0.3)\ (1.3)$$

$$= 1.06$$

$$\text{Portfolio residual variance} = \sum_{i=1}^{n} \omega_i^2 \sigma_{ei}^2$$

$$= (0.2)^2 (370) + (0.1)^2 (240) + (0.4)^2 (410) + (0.3)^2 (285)$$

$$= 108.45$$

These values are noted in the last row of the table. Using these values, we can calculate the expected portfolio return for any value of projected market return. For a market return of 15 per cent, the expected portfolio return would be:

$$\bar{R}_p = \alpha_p + \beta_p \bar{R}_m$$

$$= 1.575 + (1.06)(15)$$

$$= 17.475$$

For calculating the portfolio variance we need the variance of the market returns. Assuming a market return variance of 320, the portfolio variance can be calculated as:

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^{n} \omega_i^2 \sigma_{ei}^2$$

$$= (1.06)^2 (320) + 108.45$$

$$= 468.002$$

The single index model provides a simplified method of representing the covariance relationships among the securities. This simplification has resulted in a substantial reduction in inputs required for portfolio analysis. In the single index model only three estimates are needed for each security in the portfolio, namely specific return α_i, measure of systematic risk β_i and variance of the residual return σ_{ei}^2. In addition to these, two estimates of the market index, namely the market return $\bar{R}_m$ and the variance of the market return σ_m^2 are also needed. Thus, for N securities, the number of estimates required would be $3N+2$. For example, for a portfolio of 100 securities, the estimates required would be 302. In contrast to this, for the Markowitz model, a portfolio with 100 securities would require 5150 estimates of input data (i.e. $2N + [N(N-1)/2]$ estimates).

Using the expected portfolio returns and portfolio variances calculated with the single index model, the set of efficient portfolios is generated by means of the same quadratic programming routine as used in the Markowitz model.

MULTI-INDEX MODEL

The single index model is in fact an oversimplification. It assumes that stocks move together only because of a common co-movement with the market. Many researchers have found that there are influences other than the market that cause stocks to move together. Multi-index models attempt to identify and incorporate these non-market or extra-market factors that cause securities to move together also into the model. These extra-market factors are a set of economic factors that account for common movement in stock prices beyond that accounted for by the market index itself. Fundamental economic variables such as inflation, real economic growth, interest rates, exchange rates etc. would have a significant impact in determining security returns and hence, their co-movement.

A multi-index model augments the single index model by incorporating these extra market factors as additional independent variables. For example, a multi-index model incorporating the market effect and three extra-market effects takes the following form:

$$R_i = \alpha_i + \beta_m R_m + \beta_1 R_1 + \beta_2 R_2 + \beta_3 R_3 + e_i$$

The model says that the return of an individual security is a function of four factors—the general market factor R_m and three extra-market factors R_1, R_2 and R_3. The beta coefficients attached to the four factors have the same meaning as in the single index model. They measure the sensitivity of the stock return to these factors. The alpha parameter α_i and the residual term e_i also have the same meaning as in the single index model.

Calculation of return and risk of individual securities as well as portfolio return and variance follows the same pattern as in the single index model. These values can then be used as inputs for portfolio analysis and selection.

A multi-index model is an alternative to the single index model. However, it is more complex and requires more data estimates for its application. Both the single index model and the multi-index model have helped to make portfolio analysis more practical.

Example 1 An investor owns a portfolio whose market model is estimated as:

$$R_p = 2.3 + 0.85\ R_m + e_p$$

If the expected return on the market index is 17.5 per cent, what is the expected return on the investor's portfolio?

Solution Assuming that $e_p = 0$

$$R_p = 2.3 + 0.85\ (17.5)$$
$$= 2.3 + 14.875$$
$$= 17.175 \text{ per cent}$$

Example 2 An investor owns a portfolio composed of five securities with the following characteristics:

Security	*Beta*	*Random error term standard deviation (per cent)*	*Proportion*
1	1.35	5	0.10
2	1.05	9	0.20
3	0.80	4	0.15
4	1.50	12	0.30
5	1.12	8	0.25

If the standard deviation of the market index is 20 per cent, what is the total risk of the portfolio?

Solution The total portfolio risk may be expressed as:

$$\sigma_p^2 = \beta_p^2\ \sigma_m^2 + \sum_{i=1}^{n} w_i^2\sigma_{ei}^2$$

where

β_p = Portfolio beta.

σ_m^2 = Variance of the market index.

w_i = Proportion of investment in each security.

σ_{ei}^2 = Residual variance (random error) of individual securities.

β_p or portfolio beta has to be calculated using the formula.

$$\beta_p = \sum_{i=1}^{n} w_i\beta_i$$

$$= (0.1)\ (1.35) + (0.2)\ (1.05) + (0.15)\ (0.80) + (0.3)\ (1.5) + (0.25)\ (1.12)$$

$$= 1.195$$

Portfolio residual variance $\left(\sum_{i=1}^{n} w_i^2\sigma_{ei}^2\right)$ can be calculated as:

$$= (0.1)^2(5)^2 + (0.2)^2(9)^2 + (0.15)^2(4)^2 + (0.30)^2(12)^2 + (0.25)^2(8)^2$$

$$= 20.81$$

Portfolio total risk can now be calculated as:

$$\sigma_p^2 = \beta_p^2\sigma_m^2 + \sum_{i=1}^{n} w_i^2\sigma_{ei}^2$$

$$= (1.195)^2(20)^2 + 20.81$$

$$= 571.21 + 20.81 = 592.02$$

Example 3 Consider a portfolio composed of five securities. All the securities have a beta of 1.0 and unique or specific risk (standard deviation) of 25 per cent. The portfolio distributes weight equally among its component securities. If the standard deviation of the market index is 18 per cent, calculate the total risk of the portfolio.

Solution The input data may be arranged in the form of the following table:

Security	*Beta*	*Specific risk (Standard deviation)*	*Proportion*
1	1.0	25	0.2
2	1.0	25	0.2
3	1.0	25	0.2
4	1.0	25	0.2
5	1.0	25	0.2

Standard deviation of market index is 18 per cent.

$$\beta_p = \sum_{i=1}^{n} w_i \beta_i$$

$$= (0.2 \times 1.0) \times 5 = 1.0$$

$$\text{Portfolio residual variance} = \sum_{i=1}^{n} w_i^2 \sigma_{ei}^2$$

$$= (0.2)^2 (25)^2 \times 5 = 125$$

Portfolio total risk

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^{n} w_i^2 \sigma_{ei}^2$$

$$= (1.0)^2 (18)^2 + 125$$

$$= 324 + 125 = 449$$

Example 4 How many parameters must be estimated to analyse the risk- return profile of a 50-stock portfolio using (a) the original Markowitz model, and (b) the Sharpe single index model?

Solution In Markowitz model we require the following estimates:

N return estimates
N variance estimates
$N(N-1)/2$ covariance estimates

Total estimates = $2N + [N(N-1)/2]$
= $(2 \times 50) + [50(50-1)/2]$
= $100 + 1225 = 1325$

In the Sharpe single index model we must have

N α estimates
N β estimates
N residual variance estimates.
Market return, R_m
Variance of market return, σ_m^2

Total estimates = $3N + 2$
= $(3 \times 50) + 2 = 152$

Example 5 Consider a portfolio of four securities with the following characteristics:

Security	*Weighting*	α_i	β_i	*Residual variance* (σ_{ei}^2)
1	0.2	2.0	1.2	320
2	0.3	1.7	0.8	450
3	0.1	−0.8	1.6	270
4	0.4	1.2	1.3	180

Calculate the return and risk of the portfolio under single index model, if the return on market index is 16.4 per cent and the standard deviation of return on market index is 14 per cent.

Solution 1. Portfolio return under single index model is calculated using the formula:

$$R_p = \alpha_p + \beta_p R_m$$

For applying this formula, α_p and β_p have to be calculated as:

$$\alpha_p = \sum_{i=1}^{n} w_i \alpha_i$$

$$= (0.2)\ (2.0) + (0.3)\ (1.7) + (0.1)\ (-0.8) + (0.4)\ (1.2)$$

$$= 1.31$$

$$\beta_p = \sum_{i=1}^{n} w_i \beta_i$$

$$= (0.2)\ (1.2) + (0.3)\ (0.8) + (0.1)\ (1.6) + (0.4)\ (1.3)$$

$$= 1.16$$

$$R_p = \alpha_p + \beta_p R_m$$

$$= 1.31 + (1.16)\ (16.4)$$

$$= 1.31 + 19.024$$

$$= 20.334$$

2. Portfolio risk under single index model is calculated as:

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^{n} w_i^2 \sigma_{ei}^2$$

For applying this, portfolio residual variance needs to be calculated as:

$$\sum_{i=1}^{n} w_i^2 \sigma_{ei}^2$$

Thus,

$$(0.2)^2(320) + (0.3)^2(450) + (0.1)^2(270) + (0.4)^2(180)$$

$$= 12.8 + 40.5 + 2.7 + 28.8$$

$$= 84.8$$

Now,

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^{n} w_i^2 \sigma_{ei}^2$$

$$= (1.16)^2(14)^2 + 84.8$$

$$= 263.74 + 84.8 = 348.54$$

Hence,

$$\sigma_p = \sqrt{348.54} = 18.67$$

Example 6 The data for three stocks are given. The data are obtained from correlating returns on these stocks with the returns on the market index.

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CAPITAL ASSET PRICING MODEL (CAPM)

The capital asset pricing model was developed in mid-1960s by three researchers William Sharpe, John Lintner and Jan Mossin independently. Consequently, the model is often referred to as Sharpe-Lintner-Mossin Capital Asset Pricing Model.

The capital asset pricing model or CAPM is really an extension of the portfolio theory of Markowitz. The portfolio theory is a description of how rational investors should build efficient portfolios and select the optimal portfolio. The capital asset pricing model derives the relationship between the expected return and risk of individual securities and portfolios in the capital markets if everyone behaved in the way the portfolio theory suggested.

Let us, therefore, begin by summarising the fundamental notions of portfolio theory.

FUNDAMENTAL NOTIONS OF PORTFOLIO THEORY

Return and risk are two important characteristics of every investment. Investors base their investment decision on the expected return and risk of investments. Risk is measured by the variability in returns.

Investors attempt to reduce the variability of returns through diversification of investment. This results in the creation of a portfolio. With a given set of securities, any number of portfolios may be created by altering the proportion of funds invested in each security. Among these portfolios some dominate others, or some are more efficient than the vast majority of portfolios because of lower risk or higher returns. Investors identify this efficient set of portfolios.

Diversification helps to reduce risk, but even a well diversified portfolio does not become risk free. If we construct a portfolio including all the securities in the stock market, that would be the most diversified portfolio. Even such a portfolio would be subject to considerable variability. This variability is undiversifiable and is known as the market risk or ***systematic risk*** because it affects all the securities in the market.

The real risk of a security is the market risk which cannot be eliminated through diversification. This is indicated by the sensitivity of a security to the movements of the market and is measured by the beta coefficient of the security.

A rational investor would expect the return on a security to be commensurate with its risk. The higher the risk of a security, the higher would be the return expected from it. And since the relevant risk of a security is its market risk or systematic risk, the return is expected to be correlated with this risk only. The capital asset pricing model gives the nature of the relationship between the expected return and the systematic risk of a security.

ASSUMPTIONS OF CAPM

The capital asset pricing model is based on certain explicit assumptions regarding the behaviour of investors. The assumptions are listed below:

1. Investors make their investment decisions on the basis of risk-return assessments measured in terms of expected returns and standard deviation of returns.
2. The purchase or sale of a security can be undertaken in infinitely divisible units.
3. Purchases and sales by a single investor cannot affect prices. This means that there is perfect competition where investors in total determine prices by their actions.
4. There are no transaction costs. Given the fact that transaction costs are small, they are probably of minor importance in investment decision-making, and hence they are ignored.
5. There are no personal income taxes. Alternatively, the tax rates on dividend income and capital gains are the same, thereby making the investor indifferent to the form in which the return on the investment is received (dividends or capital gains).
6. The investor can lend or borrow any amount of funds desired at a rate of interest equal to the rate for riskless securities.
7. The investor can sell short any amount of any shares.
8. Investors share homogeneity of expectations. This implies that investors have identical expectations with regard to the decision period and decision inputs. Investors are presumed to have identical holding periods and also identical expectations regarding expected returns, variances of expected returns and covariances of all pairs of securities.

It is true that many of the above assumptions are untenable. However, they do not materially alter the real world. Moreover, the model describes the risk return relationship and the pricing of assets fairly well.

EFFICIENT FRONTIER WITH RISKLESS LENDING AND BORROWING

The portfolio theory deals with portfolios of risky assets. According to the theory, an investor faces an efficient frontier containing the set of efficient portfolios of risky assets.

Now it is assumed that there exists a riskless asset available for investment. A riskless asset is one whose return is certain such as a government security. Since the return is certain, the variability of return or risk is zero. The investor can invest a portion of his funds in the riskless asset which would be equivalent to lending at the risk free asset's rate of return, namely R_f. He would then be investing in a combination of risk free asset and risky assets.

Similarly, it may be assumed that an investor may borrow at the same risk free rate for the purpose of investing in a portfolio of risky assets. He would then be using his own funds as well as some borrowed funds for investment.

The efficient frontier arising from a feasible set of portfolios of risky assets is concave in shape. When an investor is assumed to use riskless lending and borrowing in his investment activity the shape of the efficient frontier transforms into a straight line. Let us see how this happens.

Consider Fig. 15.1. The concave curve *ABC* represents an efficient frontier of risky portfolios. *B* is the optimal portfolio in the efficient frontier with R_p = 15 per cent and σ_p = 8 per cent. A risk free asset with rate of return R_f = 7 per cent is available for investment. The risk or standard deviation of this asset would be zero because it is a riskless asset. Hence, it would be plotted on the *Y* axis. The investor may lend a part of his money at the riskless rate, i.e. invest in the risk free asset and invest the remaining portion of his funds in a risky portfolio.

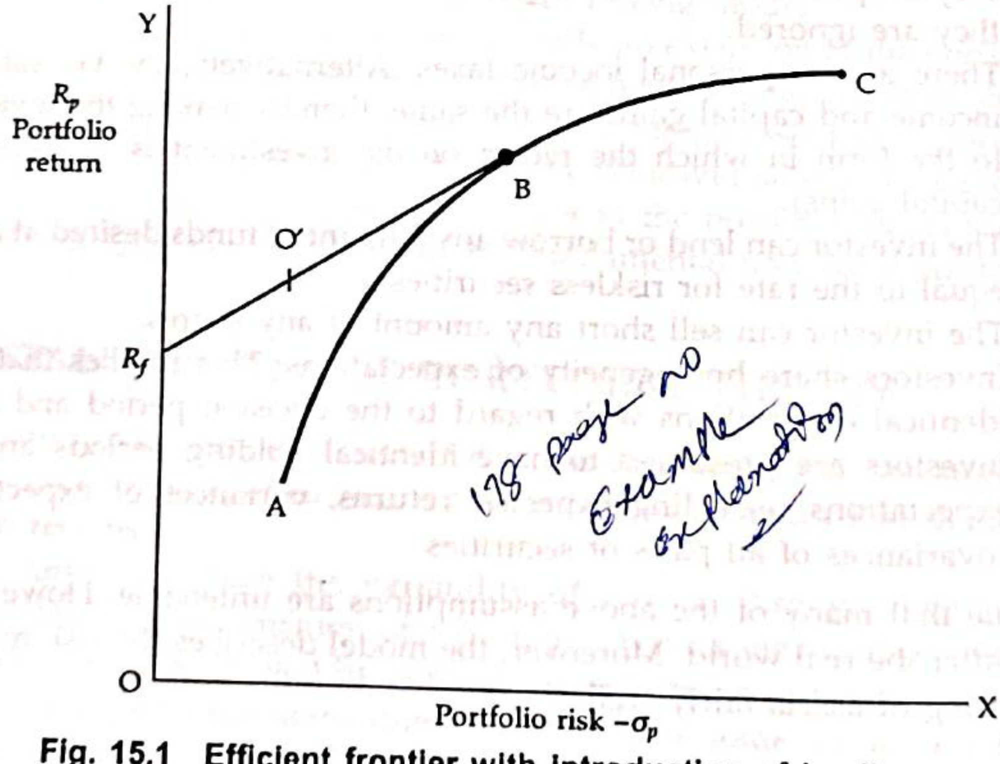

Fig. 15.1 Efficient frontier with introduction of lending.

If an investor places 40 per cent of his funds in the riskfree asset and the remaining 60 per cent in portfolio B, the return and risk of this combined portfolio O' may be calculated using the following formulas.

Return

$$R_c = \omega R_m + (1 - \omega)R_f$$

where

R_c = Expected return on the combined portfolio.

ω = Proportion of funds invested in risky portfolio.

$(1 - \omega)$ = Proportion of funds invested in riskless asset.

R_m = Expected return on risky portfolio.

R_f = Rate of return on riskless asset.

Risk

$$\sigma_c = \omega\sigma_m + (1 - \omega)\sigma_f$$

where

σ_c = Standard deviation of the combined portfolio.

ω = Proportion of funds invested in risky portfolio.

σ_m = Standard deviation of risky portfolio.

σ_f = Standard deviation of riskless asset.

The second term on the right hand side of the equation, $(1 - \omega)\sigma_f$ would be zero as σ_f = zero. Hence, the formula may be reduced as

$$\sigma_c = \omega\sigma_m$$

The return and risk of the combined portfolio in our illustration is worked out below:

$$R_c = (0.60)(15) + (0.40)(7)$$
$$= 11.8 \text{ per cent}$$
$$\sigma_c = (0.60)\ (8) = 4.8 \text{ per cent}$$

Both return and risk are lower than those of the risky portfolio *B*.

If we change the proportion of investment in the risky portfolio to 75 per cent, the return and risk of the combined portfolio may be calculated as shown below:

$$R_c = (0.75)(15) + (0.25)(7)$$
$$= 13 \text{ per cent}$$
$$\sigma_c = (0.75)(8) = 6 \text{ per cent}$$

Here again, both return and risk are lower than those of the risky portfolio *B*.

Similarly, the return and risk of all possible combinations of the riskless asset and the risky portfolio *B* may be worked out. All these points will lie in the straight line from R_f to *B* in Fig. 15.1.

Now, let us consider borrowing funds by the investor for investing in the risky portfolio an amount which is larger than his own funds.

If ω is the proportion of investor's funds invested in the risky portfolio, then we can envisage three situations. If $\omega = 1$, the investor's funds are fully committed to the risky portfolio. If $\omega < 1$, only a fraction of the funds is invested in the risky portfolio and the remainder is lend at the risk free rate. If $\omega > 1$, it means the investor is borrowing at the risk free rate and investing an amount larger than his own funds in the risky portfolio.

The return and risk of such a levered portfolio can be calculated as follows:

$$R_L = \omega R_m - (\omega - 1)R_f$$

where

R_L = Return on the levered portfolio.

ω = Proportion of investor's funds invested in the risky portfolio.

R_m = Return on the risky portfolio.

R_f = The risk free borrowing rate which would be the same as the risk free lending rate, namely the return on the riskless asset.

The first term of the equation represents the gross return earned by investing the borrowed funds as well as investor's own funds in the risky portfolio. The second term of the equation represents the cost of borrowing funds which is deducted from the gross returns to obtain the net return on the levered portfolio.

The risk of the levered portfolio can be calculated as:

$$\sigma_L = \omega\sigma_m$$

The return and risk of the investor in our illustration may be calculated assuming $\omega = 1.25$

$$R_L = (1.25)(15) - (0.25)(7)$$
$$= 17 \text{ per cent}$$
$$\sigma_L = (1.25)(8)$$
$$= 10 \text{ per cent}$$

The return and risk of the levered portfolio are larger than those of the risky portfolio. The levered portfolio would give increased returns with increased risk. The return and risk of all levered portfolios would lie in a straight line to the right of the risky portfolio *B*. This is depicted in Fig. 15.2.

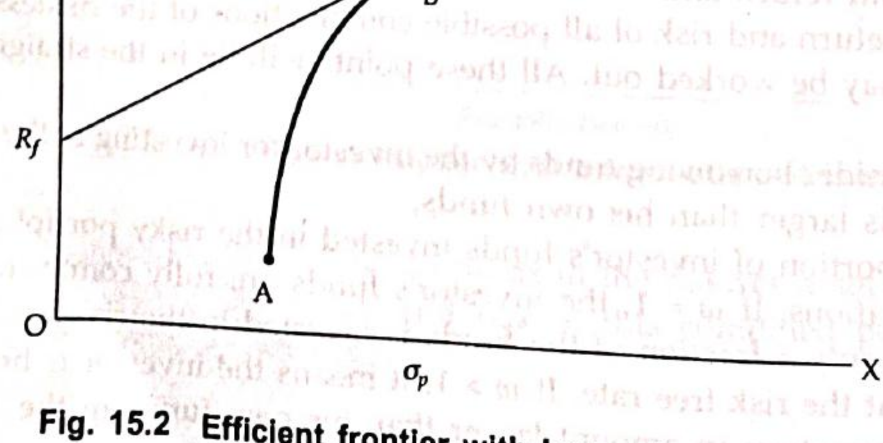

Fig. 15.2 Efficient frontier with borrowing and lending.

Thus, the introduction of borrowing and lending gives us an efficient frontier that is a straight line throughout. This line sets out all the alternative combinations of the risky portfolio B with risk free borrowing and lending.

The line segment from R_f to B includes all the combinations of the risky portfolio and the risk free asset. The line segment beyond point B represents all the levered portfolios (that is combinations of the risky portfolio with borrowing). Borrowing increases both the expected return and the risk, while lending (that is, combining the risky portfolio with risk free asset) reduces the expected return and risk. Thus, the investor can use borrowing or lending to attain the desired risk level. Those investors with a high risk aversion will prefer to lend and thus, hold a combination of risky assets and the risk free asset. Others with less risk aversion will borrow and invest more in the risky portfolio.

THE CAPITAL MARKET LINE

All investors are assumed to have identical (homogeneous) expectations. Hence, all of them will face the same efficient frontier depicted in Fig. 15.2. Every investor will seek to combine the same risky portfolio B with different levels of lending or borrowing according to his desired level of risk. Because all investors hold the same risky portfolio, then it will include all risky securities in the market. This portfolio of all risky securities is referred to as the market portfolio M. Each security will be held in the proportion which the market value of the security bears to the total market value of all risky securities in the market. All investors will hold combinations of only two assets, the market portfolio and a riskless security.

All these combinations will lie along the straight line representing the efficient frontier. This line formed by the action of all investors mixing the market portfolio with the risk free asset is known as the **capital market line (CML)**. All efficient portfolios of all investors will lie along this capital market line.

The relationship between the return and risk of any efficient portfolio on the capital market line can be expressed in the form of the following equation.

$$\bar{R}_e = R_f + \left[\frac{\bar{R}_m - R_f}{\sigma_m}\right]\sigma_e$$

where the subscript e denotes an efficient portfolio.

The risk free return R_f represents the reward for waiting. It is, in other words, the price of time. The term $[(\bar{R}_m - R_f)/\sigma_m]$ represents the price of risk or risk premium, i.e. the excess return earned per unit of risk or standard deviation. It measures the additional return for an additional unit of risk. When the risk of the efficient portfolio, σ_e, is multiplied with this term, we get the risk premium available for the particular efficient portfolio under consideration.

Thus, the expected return on an efficient portfolio is:

(Expected return) = (Price of time) + (Price of risk) (Amount of risk)

The CML provides a risk return relationship and a measure of risk for efficient portfolios. The appropriate measure of risk for an efficient portfolio is the standard deviation of return of the portfolio. There is a linear relationship between the risk as measured by the standard deviation and the expected return for these efficient portfolios.

THE SECURITY MARKET LINE

The CML shows the risk-return relationship for all efficient portfolios. They would all lie along the capital market line. All portfolios other than the efficient ones will lie below the capital market line. The CML does not describe the risk-return relationship of inefficient portfolios or of individual securities. The capital asset pricing model specifies the relationship between expected return and risk for all securities and all portfolios, whether efficient or inefficient.

We have seen earlier that the total risk of a security as measured by standard deviation is composed of two components: systematic risk and unsystematic risk or diversifiable risk. As investment is diversified and more and more securities are added to a portfolio, the unsystematic risk is reduced. For a very well diversified portfolio, unsystematic risk tends to become zero and the only relevant risk is systematic risk measured by beta (β). Hence, it is argued that the correct measure of a security's risk is beta.

It follows that the expected return of a security or of a portfolio should be related to the risk of that security or portfolio as measured by β. Beta is a measure of the security's sensitivity to changes in market return. Beta value greater than one indicates higher sensitivity to market changes, whereas beta value less than one indicates lower sensitivity to market changes. A β value of one indicates that the security moves at the same rate and in the same direction as the market. Thus, the β of the market may be taken as one.

The relationship between expected return and β of a security can be determined graphically. Let us consider an *XY* graph where expected returns are plotted on the *Y* axis and beta coefficients are plotted on the *X* axis. A risk free asset has an expected return equivalent to R_f and beta coefficient of zero. The market portfolio *M* has a beta coefficient of one and expected return equivalent to $\bar{R}_m$. A straight line joining these two points is known as the **security market line (SML)**. This is illustrated in Fig. 15.3.

The security market line provides the relationship between the expected return and beta of a security or portfolio. This relationship can be expressed in the form of the following equation:

$$\bar{R}_i = R_f + \beta_i(\bar{R}_m - R_f)$$

A part of the return on any security or portfolio is a reward for bearing risk and the rest is the reward for waiting, representing the time value of money. The risk free rate, R_f (which is earned by a security which has no risk) is the reward for waiting. The reward for bearing risk is the risk premium. The risk premium of a security is directly proportional to the risk as measured by β. The risk premium of a security is calculated as the product of beta and the risk premium of the market which is the excess of expected market return over the risk free return, that is, $[\bar{R}_m - R_f]$. Thus,

Expected return on a security = Risk free return + (Beta × Risk premium of market)

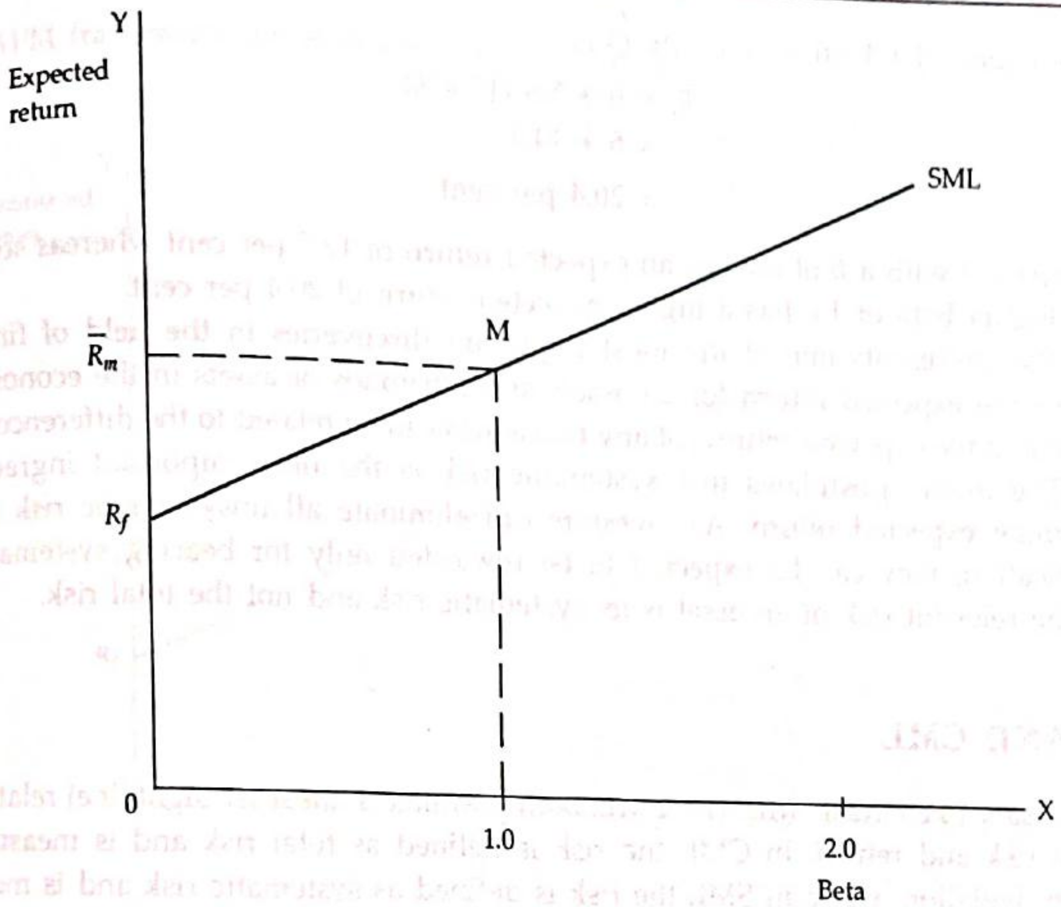

Fig. 15.3 Security market line.

CAPM

The relationship between risk and return established by the security market line is known as the **capital asset pricing model**. It is basically a simple linear relationship. The higher the value of beta, higher would be the risk of the security and therefore, larger would be the return expected by the investors. In other words, all securities are expected to yield returns commensurate with their riskiness as measured by β. This relationship is valid not only for individual securities, but is also valid for all portfolios whether efficient or inefficient.

The expected return on any security or portfolio can be determined from the CAPM formula if we know the beta of that security or portfolio. To illustrate the application of the CAPM, let us consider a simple example. There are two securities P and Q having values of beta as 0.7 and 1.6 respectively. The risk free rate is assumed to be 6 per cent and the market return is expected to be 15 per cent, thus providing a market risk premium of 9 per cent (i.e. $\bar{R}_m - R_f$).

The expected return on security P may be worked out as shown below:

$$\bar{R}_i = R_f + \beta_i[\bar{R}_m - R_f]$$
$$= 6 + 0.7\ (15 - 6)$$
$$= 6 + 6.3 = 12.3 \text{ per cent}$$

The expected return on security Q is

$$\bar{R}_i = 6 + 1.6\,(15 - 6)$$
$$= 6 + 14.4$$
$$= 20.4 \text{ per cent}$$

Security P with a β of 0.7 has an expected return of 12.3 per cent whereas security Q with a higher beta of 1.6 has a higher expected return of 20.4 per cent.

CAPM represents one of the most important discoveries in the field of finance. It describes the expected return for all assets and portfolios of assets in the economy. The difference in the expected returns of any two assets can be related to the difference in their betas. The model postulates that systematic risk is the only important ingredient in determining expected return. As investors can eliminate all unsystematic risk through diversification, they can be expected to be rewarded only for bearing systematic risk. Thus, the relevant risk of an asset is its systematic risk and not the total risk.

SML AND CML

It is necessary to contrast SML with CML. Both postulate a linear (straight line) relationship between risk and return. In CML the risk is defined as total risk and is measured by standard deviation, while in SML the risk is defined as systematic risk and is measured by β. Capital market line is valid only for efficient portfolios while security market line is valid for all portfolios and all individual securities as well. CML is the basis of the capital market theory while SML is the basis of the capital asset pricing model.

PRICING OF SECURITIES WITH CAPM

The capital asset pricing model can also be used for evaluating the pricing of securities. The CAPM provides a framework for assessing whether a security is underpriced, overpriced or correctly priced. According to CAPM, each security is expected to provide a return commensurate with its level of risk. A security may be offering more returns than the expected return, making it more attractive. On the contrary, another security may be offering less return than the expected return, making it less attractive.

The expected return on a security can be calculated using the CAPM formula. Let us designate it as the theoretical return. The real rate of return estimated to be realised from investing in a security can be calculated by the following formula:

$$R_i = \frac{(P_1 - P_0) + D_1}{P_0}$$

where

P_0 = Current market price.
P_1 = Estimated market price after one year.
D_1 = Anticipated dividend for the year.

This may be designated as the estimated return.

The CAPM framework for evaluation of pricing of securities can be illustrated with Fig. 15.4.

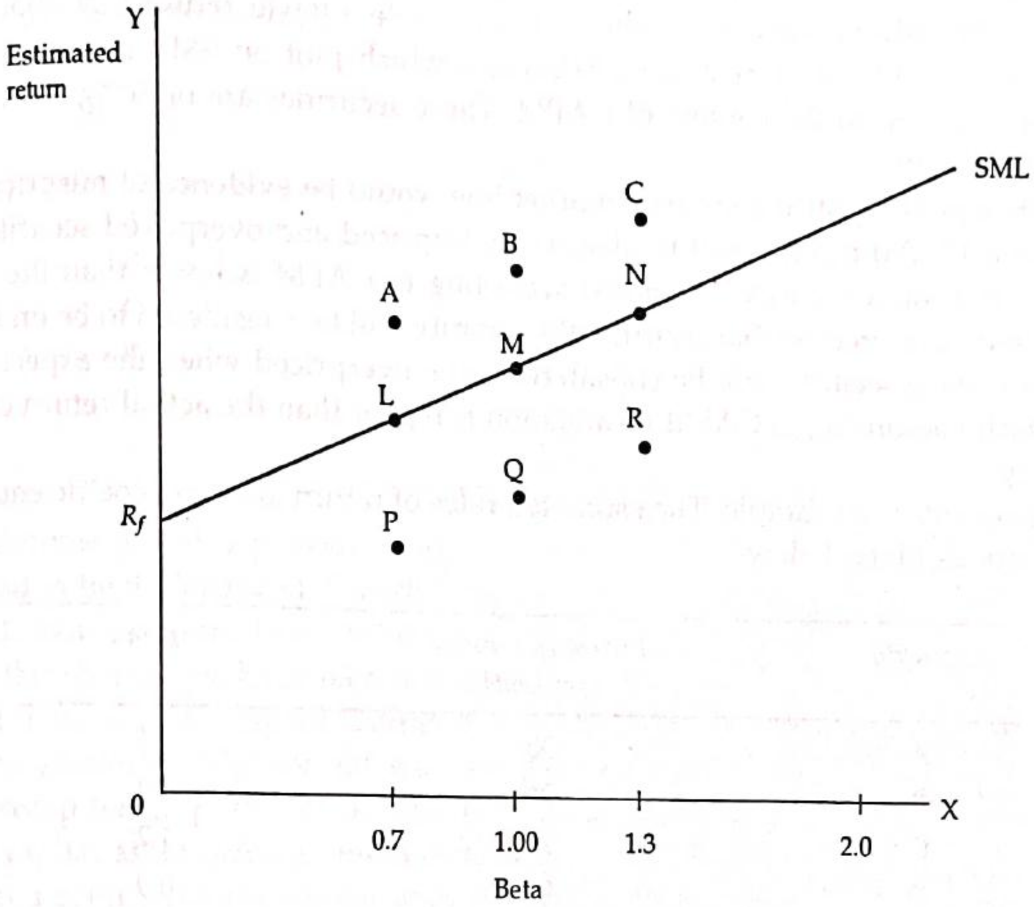

Fig. 15.4 CAPM and security valuation.

Figure 15.4 shows the security market line. Beta values are plotted on the *X* axis, while estimated returns are plotted on the *Y* axis. Nine securities are plotted on the graph according to their beta values and estimated return values.

Securities *A*, *L* and *P* are in the same risk class having an identical beta value of 0.7. The security market line shows the expected return for each level of risk. Security *L* plots on the SML indicating that the estimated return and expected return on security *L* is identical. Security *A* plots above the SML indicating that its estimated return is higher than its theoretical return. It is offering higher return than what is commensurate with its risk. Hence, it is attractive and is presumed to be underpriced. Stock *P* which plots below the SML has an estimated return which is lower than its theoretical or expected return. This makes it undesirable. The security may be considered to be overpriced.

Securities *B*, *M* and *Q* constitute a set of securities in the same risk class. Security *B* may be assumed to be underpriced because it offers more return than expected, while security *Q* may be assumed to be overpriced as it offers lower return than that expected on the basis of its risk. Security *M* can be considered to be correctly priced as it provides a return commensurate with its risk.

Securities *C*, *N* and *R* constitute another set of securities belonging to the same risk class, each having a beta value of 1.3. It can be seen that security *C* is underpriced, security *R* is overpriced and security *N* is correctly priced.

Thus, in the context of the security market line, securities that plot above the line presumably are underpriced because they offer a higher return than that expected from securities with the same risk. On the other hand, a security is presumably overpriced if it plots below the SML because it is estimated to provide a lower return than that expected from securities in the same risk class. Securities which plot on SML are assumed to be appropriately priced in the context of CAPM. These securities are offering returns in line with their riskiness.

Securities plotting off the security market line would be evidence of mispricing in the market place. CAPM can be used to identify underpriced and overpriced securities. If the expected return on a security calculated according to CAPM is lower than the actual or estimated return offered by that security, the security will be considered to be underpriced. On the contrary, a security will be considered to be overpriced when the expected return on the security according to CAPM formulation is higher than the actual return offered by the security.

Let us consider an example. The estimated rates of return and beta coefficients of some securities are as given below:

Security	*Estimated returns (per cent)*	*Beta*
A	30	1.6
B	24	1.4
C	18	1.2
D	15	0.9
E	15	1.1
F	12	0.7

The risk free rate of return is 10 per cent; while the market return is expected to be 18 per cent.

We can use CAPM to determine which of these securities are correctly priced. For this we have to calculate the expected return on each security using the CAPM equation.

$$\bar{R}_i = R_f + \beta_i(\bar{R}_m - R_f)$$

Given that $R_f = 10$ and $\bar{R}_m = 18$

The equation becomes

$$\bar{R}_i = 10 + \beta_i\,(18 - 10)$$

The expected return on security *A* can be calculated by substituting the beta value of security *A* in the equation. Thus,

$$\bar{R}_i = 10 + 1.6\,(18 - 10)$$

$$= 10 + 12.8$$

$$= 22.8 \text{ per cent}$$

Similarly, the expected return on each security can be calculated by substituting the beta value of each security in the equation.

The expected return according to CAPM formula and the estimated return of each security are tabulated below:

Security	*Expected return (CAPM)*	*Estimated return*
A	22.8	30
B	21.2	24
C	19.6	18
D	17.2	15
E	18.8	15
F	15.6	12

Securities *A* and *B* provide more return than the expected return and hence may be assumed to be underpriced. Securities *C, D, E* and *F* may be assumed to be overpriced as each of them provides lower return compared to the expected return.

In this chapter we have seen two equations representing risk return relationships. The first of these was the capital market line which describes the risk return relationship for efficient portfolios. The second was the security market line describing the risk return relationship for all portfolios as well as individual securities. This formula is also known as the **capital asset pricing model or CAPM.** It postulates that every security is expected to earn a return commensurate with its risk as measured by beta. CAPM establishes a linear relationship between the expected return and systematic risk of all assets. This relation can be used to evaluate the pricing of assets.

Example 1 Security J has a beta of 0.75 while security *K* has a beta of 1.45. Calculate the expected return for these securities, assuming that the risk free rate is 5 per cent and the expected return of the market is 14 per cent.

Solution The expected return can be calculated using CAPM

$$\bar{R}_i = R_f + \beta_i(\bar{R}_m - R_f)$$

For security *J*

$$\bar{R}_i = 5 + 0.75\ (14 - 5)$$

$$= 5 + 6.75 = 11.75 \text{ per cent}$$

For security *K*

$$\bar{R}_i = 5 + 1.45\ (14 - 5)$$

$$= 5 + 13.05 = 18.05 \text{ per cent}$$

Example 2 A security pays a dividend of Rs. 3.85 and sells currently at Rs. 83. The security is expected to sell at Rs. 90 at the end of the year. The security has a beta of 1.15. The risk free rate is 5 per cent and the expected return on market index is 12 per cent. Assess whether the security is correctly priced.

Solution To assess whether a security is correctly priced, we need to calculate (a) the expected return as per CAPM formula, (b) the estimated return on the security based on the dividend and increase in price over the holding period.

Expected return

$$R_i = R_f + \beta_i(R_m - R_f)$$
$$= 5 + 1.15\ (12 - 5)$$
$$= 5 + 8.05 = 13.05 \text{ per cent}$$

Estimated return

$$R_i = \frac{(P_1 - P_0) + D_1}{P_0}$$
$$= \frac{(90 - 83) + 3.85}{83}$$
$$= \frac{7 + 3.85}{83} = \frac{10.85}{83} = 0.1307 = 13.07 \text{ per cent}$$

As the estimated return on the security is more or less equal to the expected return, the security can be assessed as fairly priced.

Example 3 The following data are available to you as portfolio manager:

Security	*Estimated return (per cent)*	*Beta*	*Standard deviation (per cent)*
A	30	2.0	50
B	25	1.5	40
C	20	1.0	30
D	11.5	0.8	25
E	10.0	0.5	20
Market index	15	1.0	18
Govt. security	7	0	0

(a) In terms of the security market line, which of the securities listed above are underpriced?

(b) Assuming that a portfolio is constructed using equal proportions of the five securities listed above, calculate the expected return and risk of such a portfolio.

Solution

(a) We can use CAPM to determine which of the securities listed are underpriced. For this we have to calculate the expected return on each security using CAPM equation:

$$\overline{R}_i = R_f + \beta_i(\overline{R}_m - R_f)$$

Given that R_f (Govt. security return rate) = 7 and R_m = 15

The equation becomes

$$\bar{R}_i = 7 + \beta_i(15 - 7)$$

Now,

Security $A = 7 + 2.0\ (15 - 7) = 7 + 16 = 23$ per cent
Security $B = 7 + 1.5\ (15 - 7) = 7 + 12 = 19$ per cent
Security $C = 7 + 1.0\ (15 - 7) = 7 + 8 = 15$ per cent
Security $D = 7 + 0.8\ (15 - 7) = 7 + 6.4 = 13.4$ per cent
Security $E = 7 + 0.5\ (15 - 7) = 7 + 4 = 11$ per cent

The expected return as per CAPM formula and the estimated return of each security can be tabulated.

Security	*Expected return (per cent)*	*Estimated return (per cent)*
A	23.0	30.0
B	19.0	25.0
C	15.0	20.0
D	13.4	11.5
E	11.0	10.0

A security whose estimated return is greater than the expected return is assumed to be underpriced because it offers a higher return than that expected from securities with the same risk.

Accordingly, securities A, B and C are underpriced.

(b) To calculate the expected return and risk $\bar{R}_p$ and β_p, we need to calculate β_p, first

$$\beta_p = \sum_{i=1}^{n} \omega_i \beta_i$$

As the proportion of investment in each security is equal, $\omega_i = 0.20$

$$\beta_p = (0.2)\ (2.0) + (0.2)\ (1.5) + (0.2)\ (1.0) + (0.2)\ (0.8) + (0.2)\ (0.5)$$

$$= (0.2)\ (2.0 + 1.5 + 1.0 + 0.8 + 0.5)$$

$$= (0.2)\ (5.8) = 1.16$$

Expected return of portfolio

$$\bar{R}_p = R_f + \beta_p(\bar{R}_m - R_f)$$

$$= 7 + 1.16\ (15 - 7)$$

$$= 7 + 9.28 = 16.28 \text{ per cent}$$

Systematic risk of the portfolio $\beta_p = 1.16$